BUSINESS MATH

A Step-by-Step Handbook

BASE TEXTBOOK

VERSION 2019 REVISION A

ADAPTABLE | ACCESSIBLE | AFFORDABLE

by J. Olivier

BUSINESS MATH: A Step-By-Step Handbook

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Preface

I have spoken with many math instructors and professors all across Canada. As educators, it is apparent that many of us are experiencing the same challenges with our students. These challenges include, but are not limited to:

• Mathematics has a negative perception amongst most students and is viewed as the "dreaded" course to take; • It is commonly accepted that mathematics is one of the few courses where students make the comment of "I'm not very good at it" and people just accept it as truth; and

• The declining ability of our students with regards to fundamental mathematical skills and problem-solving ability. Compared to when we grew up, our students now face a very different Canada. Today's society is extremely time-pressed and always on the go. Most students are not only attending college or university, but holding down full- or part-time jobs to pay their bills and tuition. There are more mature students returning from industry for upgrading and retraining who have families at home. There are increasing rates of cultural diversity and international students in the classroom. Many students do not go directly home at night and do their homework. Rather, they fit their schoolwork, assignments, and homework into their schedules where they can. In addition to all of this, our students have grown up surrounded by technology at home, in school, and at work. It is the modern way of doing business. Microsoft Office is prevalent across most industries and companies, however LibreOffice is readily available and starting to compete with Microsoft Office. As technology continues to develop and prices become more affordable, we see more students with laptops, iPhones, and even iPads. Our students need us to embrace this technology which is a part of their life in order to help them become the leaders of tomorrow.

What Do We Need To Do?

I set out to write this textbook to answer the question, "What can we, as educators, do to help?" We can't change the way our students are or how they live. Nor can we change the skill sets they bring into our classroom. What we can do though is adapt our textbooks, resources, and the way we teach mathematics. After all, isn't it our job to find teaching strategies that meet with the needs of our students?

You may ask, 'what do students need in a textbook'? The answer requires us to listen to our students both in feedback and the questions that they pose. Term after term, year after year, do these questions sound familiar?

1. How do we approach the mathematical problem (I don't know where to start)?
2. What are the steps needed to arrive at the answer (how do I get there)?
3. Why is this material so repetitive (particularly made with respect to annuities)?
4. Why do we use algebraic symbols that have absolutely no relevance to the variable they represent? 5. How and why does a formula work?
6. Is there a quick way to locate something in the book when I need it?
7. How does this material relate to me personally and my professional career (what's in it for me?)?
8. How does the modern world utilize technology to aid in math computations (does anyone do this by hand)? 9. Where are the most common errors so that I can try to avoid them?

10. Are there are any shortcuts or "trade secrets" that can help me understand the concepts better?
11. How do all the various mathematical concepts fit together when we only cover each piece one at a time? 12. Is there any way to judge whether I conceptually understand the material?

The answer to each of these questions is incorporated into the content, structure, and features of this textbook. Students helped create this textbook. It is written to meet the needs of a twenty-first century student. Through the combination of all of the features of this textbook, it is my goal to enhance your classroom and increase your student success rates. I encourage you to provide feedback about your experiences and how we, as mathematical educators, can constantly improve our textbooks for our students. I look forward to hearing about your challenges and, most importantly, your novel solutions that help your students succeed. Please email me at jpolivier@shaw.ca with your suggestions, questions, comments, and input. Thank you in advance for your support and input!

Jean-Paul Olivier

About The Author

JEAN-PAUL OLIVIER

Education:
University of Manitoba, Bachelor of Commerce (Honours)

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Jean-Paul has been teaching Business and Financial Mathematics, as well as Business Statistics and Quantitative Methods since 1997. He is a dedicated instructor interested in helping his students succeed through multi-media teaching involving PowerPoints, videos, whiteboards, in-class discussions, readings, online software, and homework practice. He regularly facilitates these quantitative courses and leads a team of instructors. You may have met Jean Paul at various mathematical and statistical symposiums held throughout Canada (and the world). He has contributed to various mathematical publications for the major publishers with respect to reviews, PowerPoint development, online algorithmic authoring, technical checks, and co-authoring of textbooks. This text represents Jean-Paul's first venture into solo authoring.

Jean-Paul resides in Winnipeg, Manitoba, Canada, along with his wife and two daughters. When he is not teaching, he loves vacationing in the warm climes of the south with his family in California and Florida.

This book is dedicated to:
My wife, Pamela, who supports me in every way possible,
And to
My daughters, Gabrielle and Madison, who wanted Daddy to help others learn their numbers.

Textbook Features

How To Approach The Mathematical Problem (The PUPP Model)

Our students struggle with completing math exercises from beginning to end. They are unfamiliar with mathematical thought process and the techniques involved. A systematic approach is needed to help students learn how to think. This textbook's core design centers around a structured thinking process termed The PUPP Model (Plan, Understand, Perform, and Present). This process is found throughout the textbook in every guided example to help students develop a step-by-step problem-solving approach.

1. **Plan.** One of the most common difficulties is when students rush into calculations and have no clue what they are solving for! This step focuses and reminds students about what it is they are doing. The goal is to encourage students to recognize what question has been asked and what variable(s) need to be calculated.
2. **Understand.** It is critical to know what you are doing and how you are going to get there. Before performing calculations, this step urges students to think through the entire process required in order to logically arrive at the solution. There are two critical elements:
   i. ***What You Already Know***. Students must read through the question and assign the correct values to appropriate variables. If necessary, diagrams such as timelines are drawn to help visualize the information being provided and what to do with that information.
   ii. ***How You Will Get There.*** Everybody needs a roadmap. This step focuses students on describing the steps, procedures, and formulas required to arrive at the solution.
3. ***Perform.*** This stage is all about the mathematics. Students execute their roadmap and perform the needed calculations and procedures to arrive at the root(s) of the problem.
4. ***Present.*** The final part of the model encourages students to express their solution(s) in the context of the question. It's not good enough just to calculate a number. They have to understand what it means and how it fits into the situation. This also makes it easier to see if the solution makes sense. If not, there is a higher chance of spotting an error. Additionally, visual charts and graphs assist students in understanding how the various numbers fit into the cohesive puzzle.

What Are The Steps Needed To Arrive At The Final Answer? (How It Works)

Have you sometimes known what the destination is but you just couldn't figure out how to get there? Students commonly experience this problem in financial mathematics, particularly when questions become grander in scope. Integrated into the PUPP Model is another core feature of this textbook called "How It Works". In this section, students can find: 1. Detailed, step-by-step sequential procedures to address common business mathematical problems. 2. Handy reference flowchart guides that summarize the key steps required to arrive at the solutions. 3. Where possible, quick and simple examples to further illustrate the procedure.

In every section of the textbook, the "How It Works" is introduced. These steps are then utilized in all guided examples to assist students in taking a problem from beginning to end.

The Repetitiveness Of Annuities

I have heard from instructors, professors, textbook reviewers, and students alike the same concerns about the repetitiveness in teaching the four types of annuities. It is no secret that there are only minor differences between the annuity types. Many have expressed that their courses are highly time-constrained. Other textbooks address most or all annuity types separately. Each annuity type is then solved for the five common unknown variables. This can result in up to twenty different lectures.

An overwhelming number of educators have indicated that they find it better to teach the four annuity types up front. Once students understand how to recognize the annuity type and the key differences, students can solve any annuity for any variable! In this textbook, the four annuity types are thoroughly introduced and discussed, followed by sections discussing the solving for each formula variable. Key knowledge is offered in a single chapter. This textbook also utilizes only four annuity formulas - two for ordinary annuities and another two for annuities due. Incorporated into the design of these formulas are the mathematics needed to

address both simple and general formats. To reduce the chance of error, students are not required to use combinations of annuity formulas nor substitute solutions of one formula into another.

When solving for other annuity variables, students use their algebra skills to substitute and rearrange the four formulas. There are no long lists of formulas that are nothing more than algebraic rearrangements of existing formulas. There are two formulas for ordinary annuities and two formulas for annuities due. Clean and simple.

Relevant Algebraic Symbols

There are two key student concerns here: irrelevant algebraic symbols and algebraic symbols with multiple usages. Both of these concerns are addressed through symbol choices and formulas utilized in this textbook.

Nothing is more frustrating for students than algebraic symbols that have nothing to do with the variable. For example, it is common to let "j" represent the nominal interest rate. To students this is like apples and oranges. What makes more sense is to use relevant algebraic symbols that help students to understand the formulas being presented. Doesn't IY for interest rate per year make much more sense than "j"? This text utilizes representative singular (such as P for profit) or plural (such as PMT for annuity payments) symbols to further student understanding.

Additionally, having identical algebraic symbols with different interpretations is very confusing for students. For example, the symbol for future value (FV) is used to represent the future value of a single payment, an ordinary annuity, or an annuity due. However, many questions require students to execute combinations of single payments and annuities. This requires multiple appearances of the same FV symbol each requiring different computational formulas. Except where industry standards exist, this textbook differentiates algebraic symbols so that each symbol has only one meaning. Thus FV is for the future value of a single payment, FV_{ORD} is the future value of an ordinary annuity payment stream, and FV_{DUE} is the future value of an annuity due payment stream. This means less confusion for our students.

How And Why A Formula Works

We all agree that understanding a formula and not just memorizing it goes a long way in business mathematics. In this respect, this textbook offers annotated and detailed formulas allowing students to simultaneously visualize the formula as a whole and by its various components. Instead of cross-referencing variable definitions and concept explanations, each part of the formula is visually addressed and the elements explained as a cohesive whole.

Handbook Design

It is difficult to get students to look at and use their textbook. An easy reference handbook design forms a cornerstone to aiding our students in two ways.

1. The book offers a friendly, strong visual appeal. It is not just page after page of endless and uninviting formulas. Instead, students find modern graphics, pictures, and an appearance designed to encourage their interaction.
2. Unlike other textbooks, every section and chapter follows an identical structure. The goal is to make material easy to find. Throughout the book there are constant guides and locators.

• In addition to the table of contents, every chapter introduction and summary provides key topic summaries. • If a student needs to know the steps in solving a problem, students always find the details in the "How It Works" section. If they just need a summary or reminder, they can always find a nutshell summary at the end of each chapter.

• If students need calculator function instructions, they always find it in the Technology section of each chapter summary. Call the book a reference manual if you will. It is about students finding what they need when they need it made easy.

Relevance

As educators, we constantly hear, "…and when will I ever use that in my life?" This textbook demonstrates to students both personal and professional applications in discussions, guided examples, case studies, and even their homework questions. 1. On a personal level, students are shown realistic scenarios, companies, and products that they commonly are exposed to. It is always easier to learn something when students can relate to it.

2. On a professional level, this textbook demonstrates business situations in different business fields so that students can see applicability regardless of their chosen career path. Whether students are planning on becoming accountants or marketers, there are applications to be found in this textbook.

This book offers to your student:

• Math organized by business function using products and companies that are familiar to students.

• Both business and personal application sections ranging from HELOCs and Promissory Notes to RRSP planning and purchasing vehicles.

Integration Of Technology & Resources

Texas Instruments BAII+ Calculator

One of the most widely used calculators in Canada, the Texas Instruments BAII+, is illustrated and fully integrated into this textbook.

For those instructors and professors utilizing this calculator, step-by-step calculator button solutions are provided at the bottom of relevant PUPP model examples. Improving this integration is the usage of similar formula and calculator symbols that allows students to make the easy transfer between algebraic variables and calculator buttons.

If calculators are not used in your course or you use an alternate calculator, the layout and design of the textbook strategically places these calculator solutions in an inconspicuous manner in the guided solutions. Thus, they do not interfere with your students' coverage of the material and are easily skipped.

Structured Exercises

Every section, as well as chapter end, has approximately 18 practice questions for the students. Students comment that having more than that appears daunting and discourages doing their homework. Imagine reading a section and finding eighty homework questions at the end! Solutions to all exercises are found at the back of the book. The exercises are broken into three sections: 1. **Mechanics.** This section focuses on fundamental mathematical skills and practicing of the formulas. These questions are straight forward and may appear in a table format that encourages students to become more familiar with the formulas and how to solve them. After completing this section, your students should have the basic mathematical skills however will not be able to demonstrate adequate competency in the subject matter without continuing to the next section.

2. **Applications.** Once students has mastered the mathematics, it is time to put their skills to work and solve various business and personal applications. The questions in this section are in word format and require the students to execute their problem-solving skills. After completing this section, students will be able to demonstrate adequate competency in the subject matter.
3. **Challenge, Critical Thinking, and Other Applications**. This section raises the difficulty bar and asks challenging questions. Solutions may require multi-stage approaches or integration of different concepts. Strong problem-solving skills are needed. Questions may also take students in new directions by having them interrelate different concepts. Students completing this section demonstrate a high level of competency in the subject matter.

Lyryx Integration

This book is partnered with Lyryx Learning, a Canadian company specializing in online homework. Delivered in a positive learning environment, students can repeatedly try randomly generated questions, each time the answers are carefully analyzed and personalized formative feedback is provided to guide them in their learning.

Students are awarded partial marks reflecting not only their efforts on correct answers, but also for "consistent" answers which are incorrect but correctly deduced from previous incorrect answers; this avoids penalizing the student again and helps them to identify exactly where the actual error was committed. Moreover students feel no threat in attempting a question again, as each time only a better mark is recorded, encouraging them to learn by practice.

A proven suggestion is to create a series of 10 or so assignments for a normal semester with a final grade weight of around 10%-20%, which is enough incentive to encourage the students to complete this necessary practice. You may want to only count the best 9 allowing for student schedule flexibility, but on the other hand leaving them open for 2 weeks or so already provides much flexibility to complete them.

Your role as an instructor is to be enthusiastic about Lyryx! Make it a part of your course and your lectures. Bring it into your classroom. Take advantage of its features. Use the software to help identify student problem areas and generate practice questions for in-class hands-on activities. Review your student performance before tests and use Lyryx to pinpoint key review topics

Lyryx not only provides excellent online assessment, but also timely support and services to students and instructor all along the course (see Page ii), visit them directly at http://lyryx.com or contact them by email info@lyryx.com .

Identification Of Common Errors & Finding Shortcuts

Built-in to the design of this book are the standard elements that make students aware of the common errors (Things To Watch Out For) as well as some tips and tricks (Paths To Success) along the way. What makes these different in this book is that they are an integrated component of the textbook and not separated into text boxes that are generally ignored by our students.

How Do All The Pieces Fit Together?

Helping your students integrate their concepts is accomplished in two main ways:
1. **End Of Chapter Exercises.** These questions allow students to explore the various concepts outside of their chapter section. Where possible, various concepts are combined in questions that allow students to integrate their acquired knowledge. 2. **Short Case Studies.** A fictional company called Lightning Wholesale is used throughout this textbook. Almost every chapter features a short case study showing how a single business applies all the various mathematical concepts into its daily operations. Students can learn how various chapter elements combine together into business scenarios.

Am I Understanding The Material? (Give It Some Thought)

The students have read the material, but did they take the time to understand the material? Throughout the textbook, after concepts have been introduced, is a question and answer section called "Give It Some Thought". This is not a mathematical computation section. It features conceptually based questions whereby no calculations are required. Instead, this section offers students periodic checkpoints to determine if they are understanding the relationships of various variables and the concepts being presented.

A Note on Rounding Rules

For some, the method by which a textbook performs rounding in its calculations plays a very important role in the delivery of business math. This textbook addresses the issue of rounding in as simplified manner as possible.

For the instructor:

- The rules utilized in this textbook follow a very basic premise: no rounding until the final answer is achieved unless there is a logical reason to round an interim solution.
- All interim solutions display up to six decimal places, limited only by the graphical display capabilities of the BAII+ calculator. Thus, what the student sees on the screen of the calculator is exactly what is presented in the examples. • Unless there is a standard for rounding the final answer (such as currency), the solution displays up to six decimals in decimal format and four decimals in percentage format.

For the student:

- An introduction to the global rounding rules is presented in the appendix to Chapter 1.
- The basic principles of rounding are discussed in Chapter 2.
- Any nuances, textbook standards, or industry standards are presented as needed throughout the book as various topics are introduced. These key rules are summarized in Appendix C.

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Appendices

Chapter 1: Succeeding In Business Mathematics (How To Use This Textbook)

Math!

You may really dread the "M" word. Not only that, but this course is business mathematics, which some people say is even harder. Is that true, or is it just a rumour?

Take a look at your world from a different perspective—notice that math is everywhere! Before you went to bed last night, you performed some math and solved a complex algebraic equation (and you probably didn't even realize it!). That equation helped you to figure out what time to get up this morning so that you could arrive at your destination on time. You factored in variables such as the required arrival time, commuting time (including the unknown variables of traffic and weather delays), morning routine time, and even snooze button time. When you solved this complex algebraic equation, you then set your alarm clock.

From the moment you woke up this morning, numbers have appeared everywhere. Perhaps you figured out how many calories are in your breakfast cereal. Maybe your car's gas tank was low, so you calculated whether you had enough gas for your commute today. You checked your bank account online to see if you had enough money to pay the bills. At Tim Hortons you figured out what size of coffee you could buy to go with your donut using only the $1.80 in change in your pocket. I hope you get my point.

Students ask, “But why do I want to learn about business math?” The answer is simply this: money. Our world revolves around money. We all work to earn money. We need money to purchase our necessities, such as homes, transportation, food, and utilities. We also need money to enjoy the pleasures of life, including vacations, entertainment, and hobbies. We need money to retire. And businesses need money to survive and prosper.

Quite simply, business math teaches you monetary concepts and how to make smart decisions with your money.

Outline of Chapter Topics

1.1: What Is Business Math?

Business math is the study of mathematics required by the field of business. By the fact that you are reading this textbook, you must be interested in a business field such as accounting, marketing, human resources, or economics. Regardless of your path, you cannot avoid dealing with money and numbers.

Both personally and in your career you certainly use elementary arithmetic such as addition, subtraction, multiplication, and division. However, there is a whole field of mathematics that deals specifically with money. You will work with taxes, gross earnings, product prices, and currency exchange; you will be offered loans, lines of credit, mortgages, leases, savings bonds, and other financial tools. Do you know what these are and how these financial tools can maximize your earnings and minimize your costs? Do you have what it takes to execute smart monetary decisions both personally and for your business? Do you know how interest works and how it gets calculated? If you can answer “yes” to these questions, then you are already off to a great start. If not, by the end of this textbook you will have a better understanding of all of these topics and more.

How Do I Learn about Business Math?

Let's be realistic. In some areas of life and business, you can achieve a reasonable degree of understanding just by reading. However, reading about business mathematics without doing it would be disastrous. To succeed, you must follow a structured approach:

1. Always read the content prior to your professor covering the topic in class.
2. Attend class, ask questions, and explore the topic to advance your understanding.
3. Do the homework and assignments—you absolutely must practice, practice, and practice!
4. Seek help immediately when you need it.

Learning mathematics is like constructing a building. Each floor of the building requires the floor below it to be completed first. In mathematics, each section of a textbook requires the concepts and techniques from the sections that preceded it. If you have trouble with a concept, you must fix it NOW before it causes a large ripple effect on your ability to succeed in subsequent topics. So the bottom line is that you absolutely cannot replace this approach—you must follow it.

1.2: How Can I Use The Features Of This Book To My Advantage?

I wrote this textbook with the help of student input specifically for students like you. It is designed to address common student needs in mathematics. Let's examine each of those needs and learn how you benefit from the features in this textbook.

I Don't Know How to Solve a Math Problem

Do you find it difficult to start a math problem? What do you do first? What is the next step? The reason this is challenging to some students is that they have never been shown problem-solving techniques. This textbook extensively uses an author-created design called the PUPP model in all guided examples. Successfully working through a problem involves four steps: Plan, Understand, Perform, and Present (PUPP). The figure on the next page shows the PUPP model at work. This model provides a structured problem-solving approach that will aid you not only in mathematics but in any problem-solving situation you may encounter.

Example 9.5A: Straightforward Interest Rate Calculation—FV and PV Known When Sandra borrowed $7,100 from Sanchez, she agreed to reimburse him $8,615.19 three years from now including interest compounded quarterly. What interest rate is being charged?

Plan Find the nominal quarterly compounded rate of interest (IY).

What You Already Know

Step 1: The present value, future value, term, and compounding are known, as illustrated in the timeline. CY = quarterly = 4 Term = 3 years

Understand

How You Will Get There

Step 2: Calculate N using Formula 9.2. Step 3: Substitute into Formula 9.3 and rearrange for *i*. Step 4: Substitute into Formula 9.1 and rearrange for IY.

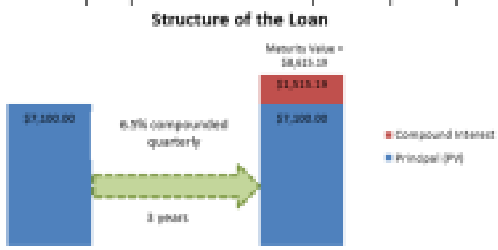
Step 2: N = 4 × 3 = 12
Step 3: $8,615.10 = $7,100(1 + *i*)^{12}

Perform
1.213394 = (1 + *i*)^{12}
$1.213394^{\frac{1}{12}} = 1 +$ ���� $0.016249 = i$

Step 4: $0.016249 = \frac{IY}{4}$ IY = 0.064996 = 0.065 or 6.5%

Calculator Instructions

N	I/Y	PV	PMT	FV	P/Y	C/Y
				5.10	4	4



Present Sanchez is charging an interest rate of 6.5% compounded quarterly on the loan to Sandra.

To understand each of these steps, let's plan a vacation.

1. Plan. If you are heading out on vacation, the first thing you want to know is where you are going! Otherwise, how would you get there? In mathematics, you need to figure out what variable you are solving for before you can proceed. This step focuses your problem solving on the end goal. Many students mistakenly rush through this step and start punching numbers on a calculator or plugging numbers into formulas without understanding what it is they are doing. Spend plenty of time on this step, because the remaining three steps do not help at all if you solve for the wrong variable!
2. Understand. In planning your vacation, once you figure out where you want to go you need to gather information about your destination. There are things you already know, but there are also things that you do not know, so you need a plan to figure these out. This step of the PUPP model helps your problem solving in two ways:
   a. First, you need to assess what information has already been provided to you. You need to assign values to variables. You may need to draw diagrams to understand how all the numbers fit together.
   b. Second, you create a plan by which you will solve your problem. Your roadmap identifies how you go from Point A (some variable is unknown) to Point B (knowing what the variable is).
3. Perform. Once your vacation is planned, all that is left is to go on vacation! In other words, execute your plan. This step involves all of the mechanics and computations of your roadmap that you developed in the previous step. 4. Present. After you return from vacation, you tell everyone about what you just did. In mathematics, this means you must explain solutions in the context of the question asked. If you just calculated x = $700, what exactly does that represent and how should it be interpreted? Calculating a solution without understanding it is not good enough. A further benefit of this last step is that you can detect errors more easily when you have to explain the solution. For example, read this presentation statement: "If an investor puts $1,000 into a savings account that earns 10% interest, I have calculated that

after one year the investor has $700 in her savings account." Did that make sense? If an account is earning interest, the balance should have become bigger, not smaller! Clearly, the solution of x = $700 must be an error!

Once you repeatedly practise this model and ingrain it in your thinking process, you have a structured approach to solving not just math problems but any life problem. When you are first learning the PUPP model you will need to write out every step of the model. As you keep practising, you'll find that the model becomes a way of thinking. You can never skip one of the steps, but you won't have to write them all out anymore because you'll execute some steps with rapid-fire thought!

I Can't Find What I Want When I Need It

When practising your business math through homework and assignments, you will need to go back to look up various formulas or techniques or to review concepts. The challenge of most textbooks is in finding what you need when you need it buried in the vast pages of the textbook.

Browse the table of contents of this book and notice the functional layout. This textbook incorporates a handbook design that aids in finding information. When you need to look something up, you will notice the following: • Every part of every section in every chapter is designed and laid out using an identical structure. While not every part is required in every section, the order is always as follows:

- Concept introduction
- Concept fundamentals and characteristics
- Formula development and explanation
- Steps and procedures required to solve problems
- Important concept elements and technology use
- Common pitfalls and shared tips
- Conceptual understanding questions
- Guided examples
- Homework exercises

• The start of every chapter includes a miniature table of contents allowing you to locate what you need in the chapter without having to flip back to the book's main table of contents.

• End-of-chapter summaries deliver exactly that—each chapter in a nutshell. They contain key concept reviews, key terms, algebraic symbol definitions, formula summaries, and technology discussions on calculator usage. Links to other chapters are also provided as needed. If you need to look something up, this is the place to start!

Are There Any Steps to Help Me Remember?

Students have always asked if they need to follow a logical sequence of steps to solve a problem. The answer, of course, is YES! A section called "How It Works" helps you with this by providing step-by-step procedures, techniques, and formulas you need to solve the problem.

Can This Book Help Me Understand the Formulas Better?

You can't avoid formulas—this is math, after all. What helps, though, is to understand what the formulas do and how each component comes together to produce the solution. If you can understand, you do not need to memorize! This book lays formulas out visually, clearly labelling all symbols. In addition, each component is fully explained (see below). You are not required to cross-reference paragraph-style explanations to formulas, as the visual layout makes it all clear. The goal is for you to understand what exactly the formula does and how, so that you understand enough to recognize when solutions make sense or might be wrong.

N is Net Price: The **net price** is the price of the product after the discount is removed from the list price. It is a dollar amount representing what price remains after you have applied the discount.

Formula 6.1 – Single Discount: $N = L \times (1 - d)$

L is List Price: The **list price** is the normal or regular dollar price of the product before any discounts. It is the **Manufacturer's Suggested Retail Price** (**MSRP** - a price for a product that has been published or advertised in some way), or any dollar amount before you remove the discount.

d is Discount Rate: The discount rate represents the percentage (in decimal format) of the list price that is deducted.

Additionally, you will not find any of those archaic algebraic symbols in this book. Representative symbols make the algebra easier to remember and understand. Those ancient mathematicians just loved to speak Greek (literally!), but this text speaks modern-day English and understands that we have technology here in the twenty-first century! We'll use symbols like N for net price, L for list price, P for profit, and E for expenses, letters that actually remind you of the variable they stand for!

Does Anyone Actually Do Any of This in the Real World?

As you read through the guided examples and do your homework, pause to consider how the question applies to you. Does it demonstrate a business application you can use to be a smarter business professional? How can you extend the question to similar situations? This textbook shows you examples from both business and personal situations that you either have already encountered or probably will encounter in the future. (Remember: Math is all around you!)

- Business. Why do business programs require a business math course in the first term or two? How is it relevant to your future? This textbook provides numerous examples of applications in marketing, finance, economics, accounting, and more. Once you complete this textbook, you should see clearly how business mathematics is relevant and important to your chosen future career.
- Personal. You will see companies and products from your everyday life. Real-world situations show you how to put a few extra dollars in your pocket by making smart mathematical choices. What if you could save $100 per month simply by making smarter choices? Over 40 years at a very conservative rate of interest, that is approximately $100,000 in your pocket. You will also find life lessons. Are you ready to start planning your RRSP? Would you like to know the basics on how to do this? If you are thinking of buying a home, do you know how the

numbers work on your mortgage? To learn how to buy a car at the lowest cost, read on!

Are There Any Secrets?

There are never any secrets. However, I will point out some tricks of the trade and commonly made mistakes throughout this book.

1. TOP SECRET Paths To Success. This feature is designed to make you feel like part of the “in” crowd by letting you take advantage of shortcuts or tricks of the trade, such as some easy way to remember a particular technique or formula. It helps you see how some of the math could be made a little simpler or performed in a different way.

2. Things to Watch Out For. Awareness of common pitfalls and sources of error in mathematics reduces your chances of making a mistake. A reminder also helps when important concepts are about to be used, reused, or combined in a novel way. It gives you a “heads up” when needed.

Is There Any Way to Know That I'm "Getting It"?

Mastering business math requires two key skills:
1. Executing the required techniques and calculations
2. Understanding and successfully integrating various mathematical concepts. (For example, when a variable changes, can you estimate how this affects the final result?)

The second skill is much harder to acquire than the first. A feature called Give It Some Thought, poses scenarios and questions for which no calculations are required. You need to visualize and conceptualize various mathematical theorems. Ultimately, you perform a self-test to see if you are "getting it."

In Today's World, Who Does This By Hand Anymore?

That is a perfectly valid question. You are right: Most people do not do this work by hand and instead use mathematical tools such as calculators, spreadsheet software, and apps. However, as the saying goes, you must learn to walk before you can run. The learning of mathematics requires pencil and paper first. Once you master these, you can then employ technology to speed up the process. This basic version textbook incorporates one technology to aid you in your application of business mathematics:
1. The Calculator. Integrated throughout this book are instructions for one of the financial calculators most widely used in Canada—the Texas Instruments BAII Plus. Calculator functions with step-by-step button sequences are presented with guided examples illustrating how you solve the math problem not only by hand but also using the financial calculator.

Is There Any Way I Can Assess My Ability?

At the end of almost every section of this book you will find approximately 18 practice questions covering the concepts specifically introduced in that section, with final solutions listed to all questions at the end of this textbook. These questions are divided into three categories to help develop your mathematical skills and assess your abilities: 1. Mechanics. This group of questions focuses on fundamental mathematical skills and formula usage. They are your first taste of using the section formulas and working with the mathematical concepts at an introductory level. Successful completion of the Mechanics section means that you:
a. Have at least a basic understanding of the variables at play,

b. Show rudimentary application of concepts, and
c. Can calculate solutions using the formulas.

Do not stop here, though! You do not yet have enough skills to meet the basic requirements of most business math courses. Once you have mastered the Mechanics section, you must progress to the next section, where you learn to apply your knowledge.

2. Applications. These questions are more typical of business math course expectations (check with your instructor). The key difference from the Mechanics section is that you must now execute your problem-solving skills. These questions require you to determine the unknown variable and figure out how the various pieces of the puzzle come together in the solution. Successful completion of these questions means that you
    a. Understand concepts at a satisfactory level,
    b. Can problem solve typical business math applications, and
    c. Can successfully integrate concepts at an acceptable performance level.
3. Challenge, Critical Thinking, and Other Applications. These questions put your knowledge and understanding to the test. The difficulty bar is set high, as these questions commonly require multistep solutions with advanced problem-solving techniques. As well, success might require you to integrate formulas, concepts, or procedures in novel ways. You will be challenged! Successful completion of these questions means that you
    a. Understand concepts at an above-average level,
    b. Can problem solve in difficult situations, and
    c. Can successfully integrate concepts at a higher level of thinking.

How Do I Put It All Together?

You may have noticed in your previous studies that many math textbooks teach one topic at a time. However, they do not necessarily help you see how all the little topics come together and interrelate. This leaves you wondering if there is a master plan.

Throughout this textbook, a single fictional company called Lightning Wholesale illustrates such interconnections. Case studies appear at the end of Chapters 3 through 15. Even if your instructor does not assign these case studies or work through them in class, completing them benefits you in three ways:

1. You learn how to integrate chapter concepts into a cohesive whole.
2. You perform large multitask problems that reflect real-world problem solving. This will make you a much more valuable employee when you graduate.
3. You see how business mathematics works throughout an organization.

1.3: Where Do We Go From Here?

At this point, I hope that you can see that all the features of this textbook are built for you! They are here to help you learn, understand, and perform math. This book gives you every opportunity to succeed!

Now that you know what business mathematics is and why it is important to you, let's get going! Complete the exercises below. Then look at your course syllabus and read your first section. Remember to take advantage of all the features of this textbook along the way.

Section 1.3 Exercises

Mechanics

In questions 1–8, identify where you would look in this textbook if you needed to find the indicated information. 1. The definition of a mathematical concept or term.
2. An explanation of a particular calculator function.
3. Guided solutions and examples to illustrate a mathematical concept.
4. An illustration of how to use the calculator to solve a particular scenario.
5. Step-by-step procedures for solving a mathematical problem.
6. More practice questions.
7. A full discussion of a mathematical concept.
8. What does the acronym PUPP stand for? Why is it important?
9. You just completed the "Mechanics" questions for a particular section. Do you need to do any more questions, or is that enough for you to pass the course?
10. Every chapter includes Review Exercises at the end. Why do you think it is important to

complete these exercises?

Applications

11. Think about your current job. List five activities that you perform that involve mathematics.
12. Talk to family members. Note their individual occupations and ask them about what work activities they perform every day that involve mathematics.

For questions 13–16, gather a few of your fellow students to discuss business mathematics.
13. Identify five specific activities or actions that you need to perform to succeed in your business math course. 14. Consider how you will study for a math test. Develop three specific study strategies.
15. Many students find it beneficial to work in study groups. List three ways that a study group can benefit you in your business math course.
16. For those students who may experience high levels of stress or anxiety about mathematics, identify three coping strategies.
17. Go online and enter the phrase "business math" or "business math help" into a search engine. Look at the table of contents for this book and note any websites that may help you

study, practice, or review each chapter.

Appendix 1: Rounding Rules

How and when to round always is a source of confusion. This Appendix provides you with some rules for rounding your answers. We start with some general guidelines that apply globally throughout the entire textbook, then comment at the rules that apply to some specific sections of text.

Global Rounding Rules

1. All interim solutions never get rounded unless there is a logical reason or business process forcing it to be rounded. 2. When writing nonterminating decimals in this textbook, up to six decimals are written. The horizontal line format is used for repeating decimals. If the number is not a final solution, then it is assumed that all decimals or as many as possible are being carried forward.
3. All final numbers are rounded to four decimals in decimal format and two decimals in percent format unless instructions indicate otherwise.
4. Final solutions are rounded according to common business practices, practical limitations, or specific instructions. 5. Zeroes not required at the end of decimals are generally not written unless required to meet a rounding standard or to visually line up a sequence of numbers.

Topic Specific Rounding Rules

Some specific topics have their own rounding standards. Until you learn about these topics, it does not make any sense to put those standards here. However, the standards are introduced with the relevant topic. They are also summarized into Appendix C: Rounding Rules.

Chapter 2: Back To The Basics

(Shoulder Check)

Where can you go in life and not be exposed to numbers and mathematics? Whether you are figuring out the price of a product (including shipping) on eBay, or balancing your bank accounts, you use your elementary mathematical skills from both your primary and secondary education.

Think about the math you perform every day:

• At the grocery store, you compare products to calculate the best value. One brand of potato chips retails for \$3.99 for 300 g, while the equally satisfying brand beside it is priced at \$3.49 for 250 g. Which offers the better value? • As the host for a large gathering, you are preparing a homemade lasagna and need to triple the original recipe, which calls for 1 ⅔ cups of tomato sauce. In the expanded recipe, how many cups of tomato sauce do you need? • If you are a sports fan, you know plenty of statistics about your favourite players and teams. Many come in percentage form, such as three-point throws for NBA stars or save percentages for NHL goaltenders. What exactly do those percentages mean?

• Many employers pay out bonuses. Perhaps in your company managers get twice as large a bonus as employees. Your company has five managers and 25 employees. If it announces a \$35,000 total bonus, what is your share as an employee?

Mathematics and numbers surround you in the business world, where you must read many numerical reports, interpret how the numbers fit together, and create your own reports showing such metrics as sales and profit projections. Away from work, you must manage your income and pay your bills. This is a mathematical problem you likely solve on a daily basis, ensuring that the money flowing out of your bank account does not exceed the money flowing in. To purchase groceries, vacations, or entertainment, you need numbers.

This chapter gives you a refresher on your basic mathematical skills, which are essential for success in later chapters. Some instructors will review this chapter with you, while others will leave it up to you to complete this chapter independently. Either way, this chapter is important and should be used to test your basic abilities. For example, it would be unfortunate if you got a leasing calculation wrong because you made an error by breaking an order of operation rule even though you understood basic leasing concepts.
Approach this chapter with confidence, and if you encounter any difficulties ensure that you master the concepts before moving on to future chapters.

Outline of Chapter Topics

2.1: Order of Operations (Proceed in an Orderly Manner)
2.2: Fractions, Decimals, and Rounding (Just One Slice of Pie, Please)
2.3: Percentages (How Does It All Relate?)
2.4: Algebraic Expressions (The Pieces of the Puzzle)
2.5: Linear Equations—Manipulating and Solving (Solving the Puzzle)
2.6: Natural Logarithms (How Can I Get that Variable Out of the Exponent?)

2.1: Order of Operations

(Proceed In An Orderly Manner)

You have just won $50,000 in a contest. Congratulations! But before you can claim it, you are required to answer a mathematical skill-testing question, and no calculators are permitted. After you hand over your winning ticket to the redemption agent, she hands you your time-limited skill-testing question: 2 × 5 + 30 ÷ 5. As the clock counts down, you think of the various possibilities. Is the answer 8, 14, 16, or something else altogether? Would it not be terrible to lose $50,000 because you cannot solve the question! If you figured out the solution is 16, you are on the right path. If you thought it was something else, this is a great time to review order of operations.

The Symbols

While some mathematical operations such as addition use a singular symbol (+), there are other operations, like multiplication, for which multiple representations are acceptable. With the advent of computers, even more new symbols have crept into mathematical symbology. The table below lists the various mathematical operations and the corresponding mathematical symbols you can use for them.

Mathematical

Operation	Symbol or Appearance	Comments
Brackets	() or [] or {}	In order, these are known as round, square, and curly brackets.
Exponents	2^3 or 2^3	• In 2^3, 3 is the exponent; an exponent is always written as a superscript. • On a computer, the exponent is recognized with the ^ symbol; 2^3 is represented by 2^3.
Multiplication	× or * or • or 2(2) or (2)(2)	In order, these are known as times, star, or bullet. The last two involve implied multiplication, which is when two terms are written next to each other joined only by brackets.
Division	/ or ÷ or $^{4}{}_{2}$	In order, these are known as the slash, divisor, and divisor line. Note in the last example that the division is represented by the horizontal line.
Addition	+	There are no other symbols.
Subtraction	−	There are no other symbols.

You may wonder if the different types of brackets mean different things. Although mathematical fields like calculus use specialized interpretations for the different brackets, business math uses all the brackets to help the reader visually pair up the brackets. Consider the following two examples:

Example 1: 3 × (4 / (6 - (2 + 2)) + 2)

Example 2: 3 × [4 / {6 - (2 + 2)} + 2]

Notice that in the second example you can pair up the brackets much more easily, but changing the shape of the brackets did not change the mathematical expression. This is important to understand when using a calculator, which usually has only round brackets. Since the shape of the bracket has no mathematical impact, solving example 1 or example 2 would involve the repeated usage of the round brackets.

BEDMAS

In the section opener, your skill-testing question was 2 × 5 + 30 ÷ 5. Do you just solve this expression from left to right, or should you start somewhere else? To prevent any confusion over how to resolve these mathematical operations, there is an agreed-upon sequence of mathematical steps commonly referred to as BEDMAS. **BEDMAS** is an acronym for **B**rackets, **E**xponents, **D**ivision, **M**ultiplication, **A**ddition, and **S**ubtraction.

How It Works

Step 1: Brackets must be resolved first. As brackets can be nested inside of each other, you must resolve the innermost set of brackets first before proceeding outwards to the next set of brackets. When resolving a set of brackets, you must perform the mathematical operations within the brackets by following the remaining steps in this model (EDMAS). If there is more than one set of brackets but the sets are not nested, work from left to right and top to bottom.

Step 2: If the expression has any exponents, you must resolve these next. Remember that an exponent indicates how many times you need to multiply the base against itself. For example, $2^3 = 2 \times 2 \times 2$. More review of exponents is found in Section 2.4.

Step 3: The order of appearance for multiplication and division does not matter. However, you must resolve these operations in order from left to right and top to bottom as they appear in the expression.

Step 4: The last operations to be completed are addition and subtraction. The order of appearance doesn't matter; however, you must complete the operations working left to right through the expression.

Important Notes

Before proceeding with the Texas Instruments BAII Plus calculator, you must change some of the factory defaults, as explained in the table below. To change the defaults, open the Format window on your calculator. If for any reason your calculator is reset (either by removing the battery or pressing the reset button), you must perform this sequence again.

Buttons Pushed	**Calculator Display**	**What It Means**
2nd Format	DEC=2.00	You have opened the Format window to its first setting. DEC tells your calculator how to round the calculations. In business math, it is important to be accurate. Therefore, we will set the calculator to what called a floating display, which means your calculator will carry all of the decimals and display as many as possible on the screen.
9 Enter	DEC=9.	The floating decimal setting is now in place. Let us proceed.
↓	DEG	This setting has nothing to do with business math and is just left alone. If it does not read DEG, press 2nd Set to toggle it.
↓	US 12-31-1990	Dates can be entered into the calculator. North Americans and Europeans use slightly different formats for dates. Your display is indicating North American format and is acceptable for our purposes. If it does not read US, press 2nd Set to toggle it.
↓	US 1,000	In North America it is common to separate numbers into blocks of 3 using a comma. Europeans do it slightly differently. This setting is acceptable for our purposes. If your display does not read US, press 2nd Set to toggle it.
↓	Chn	There are two ways that calculators can solve equations. This is known as the Chain method, which means that your calculator will simply resolve equations as you punch it in without regard for the rules of BEDMAS. This is not acceptable and needs to be changed.
2nd Set	AOS	AOS stands for Algebraic Operating System. This means that the calculator is now programmed to use BEDMAS in solving equations.
2nd Quit	0.	Back to regular calculator usage.

Also note that on the BAII Plus calculator you have two ways to key in an exponent:
1. If the exponent is squaring the base (e.g., 3^2), press 3 x^2. It calculates the solution of 9.
2. If the exponent is anything other than a 2, you must use the y^x button. For 2^3, you press 2 y^x 3 =. It calculates the solution of 8.

Things To Watch Out For

Negative Signs. Remember that mathematics uses both positive numbers (such as +3) and negative numbers (such as −3). Positive numbers do not need to have the + sign placed in front of them since it is implied. Thus +3 is written as just 3. Negative numbers, though, must have the negative sign placed in front of them. Be careful not to confuse the terminology of a negative number with a subtraction or minus sign. For example, 4 + (−3) is read as "four plus negative three" and not "four plus minus three." To key a negative number on a calculator, enter the number first followed by the ± button, which switches the sign of the number.

The Horizontal Divisor Line. One of the areas in which people make the most mistakes involves the "hidden brackets." This problem almost always occurs when the horizontal line is used to represent division. Consider the following mathematical expression:

(4 + 6) ÷ (2 + 3)

If you rewrite this expression using the horizontal line to represent the divisor, it looks like this: $4 + 6$

$2 + 3$

Notice that the brackets disappear from the expression when you write it with the horizontal divisor line because they are implied by the manner in which the expression appears. Your best approach when working with a horizontal divisor line is to reinsert the brackets around the terms on both the top and bottom. Thus, the expression looks like this: $(4 + 6)$

$(2 + 3)$

Employing this technique will ensure that you arrive at the correct solution, especially when using calculators.

Paths To Success

Hidden and Implied Symbols. If there are hidden or implied symbols in the expressions, your first step is to reinsert those hidden symbols in their correct locations. In the example below, note how the hidden multiplication and brackets are reinserted into the expression: $2^2 \times 3\}\{(2+8) \div 2\}$�

4 �$^{3 + 2^2 \times 3}$

$(2+8) \div 2$� transforms into $4 \times$ �$\{3 +$

Once you have reinserted the symbols, you are ready to follow the BEDMAS model.

Calculators are not programmed to be capable of recognizing implied symbols. If you key in "3(4 + 2)" on your calculator, failing to input the multiplication sign between the "3" and the "(4 + 2)," you get a solution of 6. Your calculator ignores the "3" since it doesn't know what mathematical operation to perform on it. To have your calculator solve the expression correctly, you must punch the equation through as “3 × (4 + 2) =”. This produces the correct answer of 18.

Simplifying Negatives. If your question involves positive and negative numbers, it is sometimes confusing to know what symbol to put when simplifying or solving. Remember these two rules:

Rule #1: A pair of the same symbols is always positive. Thus "4 + (+3)" and "4 − (−3)" both become "4 + 3." **Rule #2**: A pair of the opposite symbols is always negative. Thus "4 + (−3)" and "4 − (+3)" both become "4 – 3." A simple way to remember these rules is to count the total sticks involved, where a "+" sign has two sticks and a "−" sign has one stick. If you have an odd number of total sticks, the outcome is a negative sign. If you have an even number of total sticks, the outcome is a positive sign. Note the following examples:

4 + (−3) = → 3 total sticks is odd and therefore simplifies to negative → 4 − 3 = 1

(−2) × (−2) = → 2 total sticks is even and therefore simplifies to positive → (−2) ×(− 2) = +4

Example 2.1A: Solving Expressions Using BEDMAS

Evaluate each of the following expressions.

a. 2 × 5 + 30 ÷ 5

b. $(6 + 3)^2 + 18 \div 2$

c. $4 \times \left\{\frac{3 + 2^2 \times 3}{(2 + 8) \div 2}\right\}$

Plan You need to evaluate each of the expressions. This means you must solve each expression.

Understand What You Already Know

You are provided with the mathematical expressions in a formula format. These expressions are ready for you to solve.

How You Will Get There

Employ the knowledge of BEDMAS to solve each operation in its correct order.

a. c.

b.

Perform

Calculator Instructions

a. 2 × 5 + 30 ÷ 5 =

b. (6 + 3) y^x 2 + 18 ÷ 2 =

c. 4 × ((3 + 2 y^x 2 × 3) ÷ ((2 + 8) ÷ 2)) =

Present The final solutions for the expressions are:

a. 16

b. 90

c. 12

Section 2.1 Exercises

Hint: If a question involves money, round the answers to the nearest cent.

Mechanics

1. 81 ÷ 27 + 3 × 4
2. 100 ÷ (5 × 4 – 5 × 2)
3. $3^3 - 9 + (1 + 7 \times 3)$
4. $(6 + 3)^2 - 17 \times 3 + 70$
5. $100 - (4^2 + 3) - (3 + 9 \times 3 - 4)$

Applications

6. $[(7^2 - \{-41\}) - 5 \times 2] \div (80 \div 10)$
7. $\$1{,}000\left(1 + 0.09 \times \frac{88}{365}\right)$
8. $3[\$2{,}000(1 + 0.003)^8] + \$1{,}500$
9. $\frac{\$20{,}000}{1 + 0.07 \times \frac{7}{12}}$
10. $4 \times [(5^2 + 15)^2 \div (13^2 - 9)]^2$
11. $\$500\left[\frac{(1 + 0.00875)^{43} - 1}{0.00875}\right]$
12. $\$1{,}000\left(1 + \frac{0.12}{6}\right)^{15}$

Challenge, Critical Thinking, & Other Applications

13. $\left[\frac{\$2,500}{1+0.10}\right] + \left[\frac{\$7,500}{(1+0.10)^2}\right] + \left[\frac{-\$1,500}{(1+0.10)^3}\right] + \left[\frac{-\$2,000}{(1+0.10)^4}\right]$

14. $\$175,000(1 + 0.07)^{15} + \$14,000\left[\frac{(1+0.07)^{20}-1}{0.07}\right]$

15. $\$5,000\left[\frac{\{1+[(1+0.08)^{0.5}-1]\}^{75}-1}{(1+0.08)^{0.5}-1}\right]$

16. $\$800\left[\frac{(1+0.07)^{20}-(1+0.03)^{20}}{0.07-0.03}\right]$

17. $\$60,000(1 + 0.0058)^{80} - \$450\left[\frac{(1+0.0058)^{25}-1}{0.0058}\right]$

18. $\frac{0.08}{2}\left[\$1,000\right]\left[\frac{1-\frac{1}{\left(1+\frac{0.08}{2}\right)^{16}}}{\frac{0.08}{2}}\right] + \$1,000\left[\frac{1}{\left(1+\frac{0.08}{2}\right)^{16}}\right]$

19. $\$1,475\left[\frac{\left(1+\frac{0.06}{4}\right)^{16}-1}{\frac{0.06}{4}}\right]$

20. $\$6,250(1 + 0.0525)^{10} + \$325\left[\frac{(1+0.0525)^{10}-1}{0.0525}\right]$

2.2: Fractions, Decimals, & Rounding

(Just One Slice Of Pie, Please)

Your local newspaper quotes a political candidate as saying, “The top half of the students are well-educated, the bottom half receive extra help, but the middle half we are leaving out.”[1] You stare at the sentence for a moment and then laugh. To halve something means to split it into two. However, there are three halves here! You conclude that the speaker was not thinking carefully.

In coming to this conclusion, you are applying your knowledge of fractions. In this section, you will review fraction types, convert fractions into decimals, perform operations on fractions, and also address rounding issues in business mathematics.

Types of Fractions

To understand the characteristics, rules, and procedures for working with fractions, you must become familiar with fraction terminology. First of all, what is a fraction? A **fraction** is a part of a whole. It is written in one of three formats: 1/2 or ½ or $^{1}{}_{2}$

Each of these formats means exactly the same thing. The number on the top, side, or to the left of the line is known as the **numerator**. The number on the bottom, side, or to the right of the line is known as the **denominator**. The slash or line in the middle is the **divisor line**. In the above example, the numerator is 1 and the denominator is 2. There are five different types of fractions, as explained in the table below.

Fraction Terminology Characteristics Result of Division*			
���� ����	Proper	The numerator is smaller than the denominator.	Answer is between and 1
���� ����	Improper	The numerator is larger than the denominator.	Answer is greater than 1
���� ���� ����	Compound	A fraction that combines an integer with either a proper or improper fraction. When the division is performed, the proper or improper fraction is *added* to the integer.	Answer is greater than the integer
��� � �� ��� ��� � �� ��	Complex	A fraction that has fractions within fractions, combining elements of compound, proper, or improper fractions together. It is important to follow BEDMAS in resolving these fractions.	Answer varies depending on the fractions involved
���� ���� and ���� � ���	Equivalent	Two or more fractions of any type that have the same numerical value upon completion of the division. Note that both of these examples work out to 0.5.	Answers are equal

*Assuming all numbers are positive.

How It Works

First, focus on the correct identification of proper, improper, compound, equivalent, and complex fractions. In the next section, you will work through how to accurately convert these fractions into their decimal equivalents. Equivalent fractions require you to either solve for an unknown term or express the fraction in larger or smaller terms.

-[1] Neal, Marcia. Candidate for the 3rd Congressional District Colorado State Board of Education, as quoted in Perez, Gayle. 2008. "Retired School Teacher Seeks State Board Seat." *Pueblo Chieftain.*

Solving For An Unknown Term. These situations involve two fractions where only one of the numerators or denominators is missing. Follow this four-step procedure to solve for the unknown:

Step 1: Set up the two fractions.

Step 2: Note that your equation contains two numerators and two denominators. Pick the pair for which you know both values.

Step 3: Determine the multiplication or division relationship between the two numbers.

Step 4: Apply the same relationship to the pair of numerators or denominators containing the unknown.

Assume you are having a party and one of your friends says he would like to eat one-third of the pizza. You notice the pizza has been cut into nine slices. How many slices would you give to your friend?

Step 1: Your friend wants one out of three pieces. This is one-third. You want to know how many pieces out of nine to give him. Assign a meaningful variable to represent your unknown, so have *s* represent the number of slices to give; you need to give him *s* out of 9 pieces, or *s*/9.

$$\frac{1}{3} = \frac{s}{9}$$

Step 2: Work with the denominators 3 and 9 since you know both of them.

Step 3: Take the larger number and divide it by the smaller number. We have 9 ÷ 3 = 3. Therefore, the denominator on the right is three times larger than the denominator on the left.

Step 4: Take the 1 and multiply it by 3 to get the *s*. Therefore, *s* = 1 × 3 = 3.

$$\frac{1 \times 3}{3 \times 3} = \frac{3}{9}$$

You should give your friend three slices of pizza.

Expressing The Fraction In Larger Or Smaller Terms. When you need to make a fraction easier to understand or you need to express it in a certain format, it helps to try to express it in larger or smaller terms. To express a fraction in larger terms, multiply both the numerator and denominator by the same number. To express a fraction in smaller terms, divide both the numerator and denominator by the same number.

- Larger terms: $\frac{2}{12}$ expressed with terms twice as large would be $\frac{2 \times 2}{12 \times 2} = \frac{4}{24}$
- Smaller terms: $\frac{2}{12}$ expressed with terms half as large would be $\frac{2 \div 2}{12 \div 2} = \frac{1}{6}$

When expressing fractions in higher or lower terms, you do not want to introduce decimals into the fraction unless there would be a specific reason for doing so. For example, if you divided 4 into both the numerator and denominator of $\frac{2}{12}$, you would have $\frac{0.5}{3}$, which is not a typical format. To find numbers that divide evenly into the numerator or denominator (called factoring), follow these steps:

- Pick the smallest number in the fraction.
- Use your multiplication tables and start with 1× before proceeding to 2×, 3×, and so on. When you find a number that works, check to see if it also divides evenly into the other number.

For example, if the fraction is $\frac{12}{18}$, you would factor the numerator of 12. Note that 1 × 12 = 12; however, 12 does not divide evenly into the denominator. Next you try 2 × 6 and discover that 6 does divide evenly into the denominator. Therefore, you reduce the fraction to smaller terms by dividing by 6, or $\frac{12 \div 6}{18 \div 6} = \frac{2}{3}$.

Things To Watch Out For

With complex fractions, it is critical to obey the rules of BEDMAS. As suggested in Section 2.1, always reinsert the hidden symbols before solving. Note in the following example that an addition sign and two sets of brackets were hidden: You should rewrite ���� ���� ���� � ���� as ���� + � � ���� ����� � ���� � before you attempt to solve with BEDMAS.

Paths To Success

What do you do when there is a negative sign in front of a fraction, such as $-\frac{1}{2}$? Do you put the negative with the numerator or the denominator? The common solution is to multiply the numerator by negative 1, resulting in $\frac{(-1)\times 1}{2} = \frac{-1}{2}$. In the special case of a compound fraction, multiply the entire fraction by −1. Thus, $-1\frac{1}{2} = (-1) \times (1 + \frac{1}{2}) = -1 - \frac{1}{2}$.

Example 2.2A: Identifying Types of Fractions

Identify the type of fraction represented by each of the following:

a. $\frac{2}{3}$ b. $6\frac{7}{8}$ c. 12^{4}_{3} �6⁺$_5$ d. $\frac{15}{11}$ e. $\frac{5}{6}$ f. $\frac{3}{4}$ & $\frac{9}{12}$

Plan For each of these six fractions, identify the type of fraction.

Understand

What You Already Know

There are five types of fractions, including proper, improper, compound, complex, or equivalent

How You Will Get There

Examine each fraction for its characteristics and match these characteristics with the definition of the fraction.

a. $\frac{2}{3}$ The numerator is smaller than the denominator. This matches the characteristics of a proper fraction.

c. 12^{4}_{3} �6⁺$_5$

e. $\frac{5}{6}$

Perform

b. $6\frac{7}{8}$

d. $\frac{15}{11}$

This fraction combines an integer with a proper fraction (since the numerator is smaller than the denominator). This matches the characteristics of a compound fraction.

There are lots of fractions involving fractions nested inside other fractions. The fraction as a whole is a compound fraction, containing an integer with a proper fraction (since the numerator is smaller than the denominator). Within the proper fraction, the numerator is an improper fraction ($\frac{4}{3}$) and the denominator is a compound fraction containing an integer and a proper fraction ($6\frac{4}{5}$). This all matches the definition of a complex fraction: nested fractions combining elements of compound, proper, and improper fractions together.

The numerator is larger than the denominator. This matches the characteristics of an improper fraction. The numerator is smaller than the denominator. This matches the characteristics of a proper fraction.

f. $\frac{3}{4}$ & $\frac{9}{12}$ There are two proper fractions here that are equal to each other. If you were to complete the division, both fractions calculate to 0.75. These are equivalent fractions.

Present Of the six fractions examined, there are two proper fractions (a and e), one improper fraction (d), one compound fraction (b), one complex fraction (c), and one equivalent fraction (f).

Example 2.2B: Working with Equivalent Fractions

a. Solve for the unknown term *x*: $\frac{7}{12} = \frac{49}{x}$ b. Express this fraction in lower terms: $\frac{5}{50}$

Plan a. Find the value of the unknown term, x.

b. Take the proper fraction and express it in a lower term.

Understand

What You Already Know The needed fractions in a ready-to-solve format are provided.

How You Will Get There

a. Apply the four-step technique to solving equivalent fractions. The first step has already been done for you, in that the equation is already set up.

b. Find a common divisor that divides evenly into both the numerator and denominator. As only 1 and 5 go into the number 5, it makes sense that you should choose 5 to divide into both the numerator and denominator. Note that 5 factors evenly into the denominator, 50, meaning that no remainder or decimals are left over.

Perform

a.

Step 2: You have both of the numerators, so work with that pair.

Step 3: Take the larger number and divide by the smaller number, or 49 ÷ 7 = 7. Therefore, multiply the fraction on the left by 7 to get the fraction on the right.

Step 4: Applying the same relationship, 12 × 7 = 84.

b. $\frac{5 \div 5}{50 \div 5} = \frac{1}{10}$

Present a. The unknown denominator on the right is 84, and therefore $\frac{7}{12} = \frac{49}{84}$.

b. In lower terms, $\frac{5}{50}$ is expressed as $\frac{1}{10}$.

Converting to Decimals

Although fractions are common, many people have trouble interpreting them. For example, in comparing $\frac{27}{37}$ to $\frac{57}{73}$, which is the larger number? The solution is not immediately apparent. As well, imagine a retail world where your local Walmart was having a $\frac{3}{20}$th off sale! It's not that easy to realize that this equates to 15% off. In other words, fractions are converted into decimals by performing the division to make them easier to understand and compare.

How It Works

The rules for converting fractions into decimals are based on the fraction types.

Proper and Improper Fractions. Resolve the division. For example, $\frac{3}{4}$ is the same as 3 ÷ 4 = 0.75. As well, $\frac{5}{4} = 5 \div 4 = 1.25$.

Compound Fractions. The decimal number and the fraction are joined by a hidden addition symbol. Therefore, to convert to a decimal you need to reinsert the addition symbol and apply BEDMAS:

$$3\tfrac{4}{5} = 3 + 4 \div 5 = 3 + 0.8 = 3.8$$

Complex Fractions. The critical skill here is to reinsert all of the hidden symbols and then apply the rules of BEDMAS:

$$2\frac{\frac{11}{4}}{1\frac{1}{4}} = 2 + \frac{(11 \div 4)}{(1 + 1 \div 4)} = 2 + \frac{(11 \div 4)}{(1 + 0.25)} = 2 + \frac{2.75}{1.25} = 2 + 2.2 = 4.2$$

Example 2.2C: Converting Fractions to Decimals

Convert the following fractions into decimals:

a. $\frac{2}{5}$ b. $6\frac{7}{8}$ c. $12\frac{9}{2}\over{1\frac{2}{10}}$

Plan Take these fractions and convert them into decimal numbers.

Understand What You Already Know

The three fractions are provided and ready to convert.

How You Will Get There

a. This is a proper fraction requiring you to complete the division.

b. This is a compound fraction requiring you to reinsert the hidden addition symbol and then apply BEDMAS.

c. This is a complex fraction requiring you to reinsert all hidden symbols and apply BEDMAS.

Perform

a. $\frac{2}{5} = 2 \div 5 = 0.4$

b. $6\frac{7}{8} = 6 + 7 \div 8 = 6 + 0.875 = 6.875$

c. $12\frac{\frac{9}{2}}{1\frac{2}{10}} = 12 + \left(\frac{9 \div 2}{1 + 2 \div 10}\right) = 12 + \left(\frac{9 \div 2}{1 + 0.2}\right) = 12 + \left(\frac{4.5}{1.2}\right) = 12 + 3.75 = 15.75$

Present In decimal format, the fractions have converted to 0.4, 6.875, and 15.75, respectively.

Rounding Principle

Your company needs to take out a loan to cover some short-term debt. The bank has a posted rate of 6.875%. Your bank officer tells you that, for simplicity, she will just round off your interest rate to 6.9%. Is that all right with you? It shouldn't be!

What this example illustrates is the importance of rounding. This is a slightly tricky concept that confuses most students to some degree. In business math, sometimes you should round your calculations off and sometimes you need to retain all of the digits to maintain accuracy.

How It Works

To round a number off, you always look at the number to the right of the digit being rounded. If that number is 5 or higher, you add one to your digit; this is called *rounding up*. If that number is 4 or less, you leave your digit alone; this is called *rounding down*.

For example, if you are rounding 8.345 to two decimals, you need to examine the number in the third decimal place (the one to the right). It is a 5, so you add one to the second digit and the number becomes 8.35.

For a second example, let's round 3.6543 to the third decimal place. Therefore, you look at the fourth decimal position, which is a 3. As the rule says, you would leave the digit alone and the number becomes 3.654.

Nonterminating Decimals

What happens when you perform a calculation and the decimal doesn't terminate?

1. You need to assess if there is a pattern in the decimals:

- ***The Nonterminating Decimal without a Pattern**:* For example, $\frac{6}{17} = 0.352941176\ldots$ with no apparent ending decimal and no pattern to the decimals.
- ***The Nonterminating Decimal with a Pattern**:* For example, $\frac{2}{11} = 0.18181818\ldots$ endlessly. You can see that the numbers 1 and 8 repeat. A shorthand way of expressing this is to place a horizontal line above the digits that repeat. Thus, you can rewrite 0.18181818 ... as $0.\overline{18}$.

2. You need to know if the number represents an interim or final solution to a problem:

- ***Interim Solution***: You must carry forward all of the decimals in your calculations, as the number should not be rounded until you arrive at a final answer. If you are completing the question by hand, write out as many decimals as possible; to save space and time, you can use the shorthand horizontal bar for repeating decimals. If you are completing the question by calculator, store the entire number in a memory cell.
- ***Final Solution***: To round this number off, an industry protocol or other clear instruction must apply. If these do not exist, then you would make an arbitrary rounding choice, subject to the condition that you must maintain enough precision to allow for reasonable interpretation of the information.

Important Notes

To assist in your calculations, particularly those that involve multiple steps to resolve, your calculator has 10 memory cells. Your display is limited to 10 digits, but when you store a number into a memory cell the calculator retains all of the decimals associated with the number, not just those displaying on the screen. Your calculator can, in fact, carry up to 13 digit positions. It is strongly recommended that you take advantage of this feature where needed throughout this textbook.

Let's say that you just finished keying in $\frac{6}{17}$ on your calculator, and the resultant number is an interim solution that you need for another step. With 0.352941176 on your display, press STO followed by any numerical digit on the keypad of your calculator. STO stands for *store*. To store the number into memory cell 1, for example, press STO 1. The number with 13 digits is now in permanent memory. If you clear your calculator (press CE/C) and press RCL # (where # is the memory cell number), you will bring the stored number back. RCL stands for *recall*. Press RCL 1. The stored number 0.352941176 reappears on the screen.

Example 2.2D: Rounding Numbers

Convert the following to decimals. Round each to four decimals or use the repeating decimal notation. a. $\frac{6}{13}$ b. $\frac{4}{9}$ c. $\frac{4}{11}$ d. $\frac{3}{22}$ e. $5\frac{1}{7}$ $\frac{10}{27}$

Plan Convert each of the fractions into decimal format, then round any final answer to four decimals or use repeating decimal notation.

Understand What You Already Know

The fractions and clear instructions on how to round them have been provided.

How You Will Get There

To convert the fractions to decimals, you need to complete the division by obeying the rules of BEDMAS. Watch for hidden symbols and adhere to the rules of rounding.

Perform

a. $\frac{6}{13} = 0.461538$. The fifth decimal is a 3, so round down. The answer is 0.4615.

b. $\frac{4}{9} = 0.444444$. Note the repeating decimal of 4. Using the horizontal bar, write $0.\overline{4}$.

c. $\frac{4}{11} = 0.363636$. Note the repeating decimals of 3 and 6. Using the horizontal bar, write $0.\overline{36}$.

d. $\frac{3}{22} = 0.136363$. Note the repeating decimals of 3 and 6 after the 1. Using the horizontal bar, write $0.1\overline{36}$.

e. $5\frac{\frac{1}{7}}{\frac{10}{27}} = 5 + \frac{(1 \div 7)}{(10 \div 27)} = 5 + \frac{0.142857}{0.\overline{370}} = 5 + 0.385714 = 5.385714$. Since the fifth digit is a 1, round down. The answer is 5.3857.

Present According to the rounding instructions, the solutions are 0.4615, $0.\overline{4}$, $0.\overline{36}$, $0.1\overline{36}$, and 5.3857, respectively.

Rounding Rules

One of the most common sources of difficulties in math is that different people sometimes use different standards for rounding. This seriously interferes with the consistency of final solutions and makes it hard to assess accuracy. So that everyone arrives at the same solution to the exercises/examples in this textbook, these rounding rules apply throughout the book:

1. Never round an interim solution unless there is a logical reason or business process that forces the number to be rounded. Here are some examples of logical reasons or business processes indicating you should round:

a. You withdraw money or transfer it between different bank accounts. In doing so, you can only record two decimals and therefore any money moving between the financial tools must be rounded to two decimals.

b. You need to write the numbers in a financial statement or charge a price for a product. As our currency is in dollars and cents, only two decimals can appear.

2. When you write nonterminating decimals, show only the first six (or up to six) decimals. Use the horizontal line format for repeating decimals. If the number is not a final solution, then assume that all decimals or as many as possible are being carried forward.
3. Round all final numbers to six decimals in decimal format and four decimals in percentage format unless instructions indicate otherwise.
4. Round final solutions according to common business practices, practical limitations, or specific instructions. For example, round any final answer involving dollar currency to two decimals. These types of common business practices and any exceptions are discussed as they arise at various points in this textbook.
5. Generally avoid writing zeros, which are not required at the end of decimals, unless they are required to meet a rounding standard or to visually line up a sequence of numbers. For example, write 6.340 as 6.34 since there is no difference in interpretation through dropping the zero.

Paths To Success

Does your final solution vary from the actual solution by a small amount? Did the question involve multiple steps or calculations to get the final answer? Were lots of decimals or fractions involved? If you answer yes to these questions, the most common source of error lies in rounding. Here are some quick error checks for answers that are "close": 1. Did you remember to obey the rounding rules laid out above? Most importantly, are you carrying decimals for interim solutions and rounding only at final solutions?
2. Did you resolve each fraction or step accurately? Check for incorrect calculations or easy-to-make errors, like transposed numbers.
3. Did you break any rules of BEDMAS?

Section 2.2 Exercises

Mechanics

1. For each of the following, identify the type of fraction presented.

49 e. $1\frac{2}{3}$

27 g. $10\frac{1}{5}$

a. $\frac{1}{8}$ b. $3\frac{3}{4}$ c. $\frac{10}{9}$ d. $\frac{5}{9}$ 3 f. 56 9 h. $\frac{6}{11}$
34

2. In each of the following equations, identify the value of the unknown term.

a. $\frac{3}{4} = \frac{?}{36}$ b. $\frac{?}{8} = \frac{16}{64}$ c. $\frac{2}{?} = \frac{18}{45}$ d. $\frac{5}{6} = \frac{75}{?}$

3. Take each of the following fractions and provide one example of the fraction expressed in both higher and lower terms. a. $\frac{5}{10}$ b. $\frac{6}{8}$

4. Convert each of the following fractions into decimal format.

a. $\frac{7}{8}$ b. $15\frac{5}{4}$ c. $\frac{13}{5}$ d. $133\frac{17}{2}$ � $\frac{2}{3}$ 5 �

5. Convert each of the following fractions into decimal format and round to three decimals.

a. $\frac{7}{8}$ b. $15\frac{3}{4}$ c. $\frac{10}{9}$ d. $\frac{15}{32}$

6. Convert each of the following fractions into decimal format and express in repeating decimal notation. a. $\frac{1}{12}$ b. $5\frac{8}{33}$ c. $\frac{4}{3}$ d. $\frac{-34}{110}$

Applications

7. Calculate the solution to each of the following expressions. Express your answer in decimal format. a. $\frac{1}{5} + 3\frac{1}{4} + \frac{5}{2}$ b. $1\frac{3}{8}$ � $2 - \frac{11}{40} + 19\frac{1}{2} \times \frac{3}{4}$

8. Calculate the solution to each of the following expressions. Express your answer in decimal format with two decimals. 12 �^{4} b. $1 - 0.05 \times \frac{263}{365}$

a. �$1 + \frac{0.11}{365}$ c. 200 �$1 - \frac{1}{4}$� $\frac{2}{}$ �

�$1 + \frac{0.10}{}$

9. Calculate the solution to each of the following expressions. Express your answer in repeating decimal notation as needed. a. $\frac{1}{11} + 3\frac{1}{9}$ b. $\frac{5}{3} - \frac{7}{6}$

Questions 10–14 involve fractions. For each, evaluate the expression and round your answer to the nearest cent.

10. $\$134{,}000(1 + 0.14 \times 23/365)$

11. $\$10{,}000(1 + \frac{0.0525}{2})^{13}$

12. $\frac{\$535{,}000}{(1 + \frac{0.07}{12})^3}$

13. $\$2{,}995$ �$1 + 0.13 \times \frac{90}{365}$� $- \frac{\$400}{1 + 0.13 \times \frac{15}{365}}$

14. $\frac{\$155{,}600}{(1 + \frac{0.06}{12})^8}$

Challenge, Critical Thinking, & Other Applications

Questions 15–20 involve more complex fractions and reflect business math equations encountered later in this textbook. For each, evaluate the expression and round your answer to the nearest cent.

15. $\frac{\$648}{\frac{0.0575}{12}}$ � �$1 - \frac{1}{(1 + \frac{0.0575}{12})^7}$ � :�$1 - \frac{1}{2}$

17. $\frac{\$2{,}000{,}000}{}$ � $(1 + \frac{0.065}{2})^{12} - 1$ $\frac{0.065}{2}$ �

16. $10,000 1.08�4

(1 + 0.115 0.115
4

1.08)^4 − $4,350

+)^2 + $68

18. $8,500 �1 − � 1

1 − 1

(1 + 0.115

1.08 � + $19,750(1

2 � 10

±)2�

19. $15,000 �(1 + 0.058

�1 + 0.07 0.07
2

�1 + 0.07

±)^{16}−1
0.058
4

� 20. 0.08 2 ($1,000) �

� + $1,000 1 2 � 10

2.3: Percentages

(How Does It All Relate?)

Your class just wrote its first math quiz. You got 13 out of 19 questions correct, or $\frac{13}{19}$. In speaking with your friends Sandhu and Illija, who are in other classes, you find out that they also wrote math quizzes; however, theirs were different. Sandhu scored 16 out of 23, or $\frac{16}{23}$, while Illija got 11 out of 16, or $\frac{11}{16}$. Who achieved the highest grade? Who had the lowest? The answers are not readily apparent, because fractions are difficult to compare.

Now express your grades in percentages rather than fractions. You scored 68%, Sandhu scored 70%, and Illija scored 69%. Notice you can easily answer the questions now. The advantage of percentages is that they facilitate comparison and comprehension.

Converting Decimals to Percentages

A **percentage** is a part of a whole expressed in hundredths. In other words, it is a value out of 100. For example, 93% means 93 out of 100, or $\frac{93}{100}$.

The Formula

% is Percentage: This is the decimal expressed as a percentage. It must always be written with the percent (%) symbol immediately following the number.

100 is Conversion Factor: A percentage is always expressed in hundredths.

Formula 2.1 - Percentage Conversion: $\% = \text{dec} \times 100$

dec is Decimal Number: This is the decimal number needing to be converted into a percentage.

How It Works

Assume you want to convert the decimal number 0.0875 into a percentage. This number represents the dec variable in the formula. Substitute into Formula 2.1:

$$\% = 0.0875 \times 100 = 8.75\%$$

Important Notes

You can also solve this formula for the decimal number. To convert any percentage back into its decimal form, you need to perform a mathematical opposite. Since a percentage is a result of multiplying by 100, the mathematical opposite is achieved by dividing by 100. Therefore, to convert 81% back into decimal form, you take 81% ÷ 100 = 0.81.

Things To Watch Out For

Your Texas Instruments BAII Plus calculator has a % key that can be used to convert any percentage number into its decimal format. For example, if you press 81 and then %, your calculator displays 0.81.

While this function works well when dealing with a single percentage, it causes problems when your math problem involves multiple percentages. For example, try keying 4% + 3% = into the calculator using the % key. This should be the same as 0.04 + 0.03 = 0.07. Notice, however, that your calculator has 0.0412 on the display.

Why is this? As a business calculator, you BAII Plus is programmed to take portions of a whole. When you key 3% into the calculator, it takes 3% of the first number keyed in, which was 4%. As a formula, the calculator sees 4% + (3% × 4%). This works out to 0.04 + 0.0012 = 0.0412.

To prevent this from happening, your best course of action is not to use the % key on your calculator. It is best to key all percentages as decimal numbers whenever possible, thus eliminating any chance that the % key takes a portion of your whole. Throughout this textbook, all percentages are converted to decimals before calculations take place.

Paths To Success

When working with percentages, you can use some tricks for remembering the formula and moving the decimal point.

Remembering the Formula. When an equation involves only multiplication of all terms on one side with an isolated solution on the other side, use a mnemonic called the triangle technique. In this technique, draw a triangle with a horizontal line through its middle. Above the line goes the solution, and below the line, separated by vertical lines, goes each of the terms involved in the multiplication. The figure to the right shows how Formula 2.1 would be drawn using the triangle technique.
To use this triangle, identify the unknown variable, which you then calculate from the other variables in the triangle:

1. Anything on the same line gets multiplied together. If solving for %, then the other variables are on the same line and multiplied as dec × 100.

2. Any pair of items with one above the other is treated like a fraction and divided. If solving for dec, then the other variables are above/below each other and are divided as $\frac{\%}{100}$.

Moving the Decimal. Another easy way to work with percentages is to remember that multiplying or dividing by 100 moves the decimal over two places.

1. If you are multiplying by 100, the decimal position moves two positions to
the right.

2. If you are dividing by 100, the decimal position moves two positions to the left
(see Figure 2.5).

Example 2.3A: Working with Percentages

Convert (a) and (b) into percentages. Convert (c) back into decimal format.

a. $\frac{3}{8}$ b. 1.3187 c. 12.399%

Plan For questions (a) and (b), you need to convert these into percentage format. For question (c), you need to convert it back to decimal format.

Understand

What You Already Know

a. This is a fraction to be converted into a decimal, or dec.

b. This is dec.

c. This is %.

How You Will Get There

a. Convert the fraction into a decimal to have dec. Then apply Formula 2.1: % = dec × 100 to get the percentage.

b. As this term is already in decimal format, apply Formula 2.1: % = dec × 100 to get the percentage.

c. This term is already in percentage format. Using the triangle technique, calculate the decimal number through $\text{dec} = \frac{\%}{100}$.

Perform

a. $\frac{3}{8} = 3 \div 8 = 0.375$ % = 0.375 × 100 % = 37.5%

b. % = 1.3187 × 100 % = 131.87%

c. $\text{dec} = \frac{12.399\%}{100} = 0.12399$

Present In percentage format, the first two numbers are 37.5% and 131.87%. In decimal format, the last number is 0.12399.

Rate, Portion, Base

In your personal life and career, you will often need to either calculate or compare various quantities involving fractions. For example, if your income is $3,000 per month and you can't spend more than 30% on housing, what is your maximum housing dollar amount? Or perhaps your manager tells you that this year's sales of $1,487,003 are 102% of last year's sales. What were your sales last year?

The Formula

Rate is The Relationship: The **rate** is the decimal form expressing the relationship between the portion and the base. Convert it to a percentage if needed by applying Formula 2.1. This variable can take on any value, whether positive or negative.

Portion is The Part Of The Quantity: The portion represents the part of the whole. Compare it against the base to assess the rate.

Formula 2.2 – Rate, Portion, Base: ����������������

=������������ ������������

Base is The Entire Quantity: The base is the entire amount or quantity that is of concern. It represents a whole, standard, or benchmark that you assess the portion against.

���� ����

How It Works

Assume that your company has set a budget of $1,000,000. This is the entire amount of the budget and represents your *base*. Your department gets $430,000 of the budget—this is your department's part of the whole and represents the *portion*. You want to know the relationship between your budget and the company's budget. In other words, you are looking for the *rate*.

- Apply Formula 2.2, where rate $= \frac{\$430{,}000}{\$1{,}000{,}000}$.
- Your budget is 0.43, or 43%, of the company's budget.

Important Notes

There are three parts to this formula. Mistakes commonly occur through incorrect assignment of a quantity to its associated variable. The table below provides some tips and clue words to help you make the correct assignment.

Variable	Key Words	Example
Base	of	If your department can spend 43% of the company's total budget **of $1,000,000**, what is your maximum departmental spending?
Portion	is, are	If your department can spend 43% of the company's total budget of $1,000,000, **what is your maximum departmental spending**?
Rate	%, percent, rate	If your department can spend **43%** of the company's total budget of $1,000,000, what is your maximum departmental spending?

Things To Watch Out For

In resolving the rate, you must express all numbers in the same units—you cannot have apples and oranges in the same sequence of calculations. In the above example, both the company's budget and the departmental budget are in the units of dollars. Alternatively, you would not be able to calculate the rate if you had a base expressed in kilometres and a portion expressed in metres. Before you perform the rate calculation, express both in kilometres or both in metres.

Paths To Success

Formula 2.2 is another formula you can use the triangle technique for. You do not need to memorize multiple versions of the formula for each of the variables. The triangle appears to the right.

Be very careful when performing operations involving rates, particularly in summing or averaging rates.

Summing Rates. Summing rates requires each rate to be a part of the same whole or base. If Bob has 5% of the kilometres travelled and Sheila has 6% of the oranges, these are not part of the same whole and cannot be added. If you did, what does the 11% represent? The result has no interpretation. However, if there are 100 oranges of which Bob has 5% and Sheila has 6%, the rates can be added and you can say that in total they have 11% of the oranges.

Averaging Rates. Simple averaging of rates requires each rate to be a measure of the same variable with the same base. If 36% of your customers are female and 54% have high income, the average of 45% is meaningless since each rate measures a

different variable. Recall that earlier in this chapter you achieved 68% on your test and Sandhu scored 70%. However, your test involved 19 questions and Sandhu's involved 23 questions. These rates also cannot be simply averaged to 69% on the reasoning that (68% + 70%) / 2 = 69%, since the bases are not the same. When two variables measure the same characteristic but have different bases (such as the math quizzes), you must use a weighted-average technique, which will be discussed in Section 3.1.

When can you average rates? Hypothetically, assume Sandhu achieved his 70% by writing the same test with 19 questions. Since both rates measure the same variable and have the same base, the simple average of 69% is now calculable.

Give It Some Thought

Consider the following situations and select the best answer without performing any calculations. 1. If the rate is 0.25%, in comparison to the base the portion is

a. just a little bit smaller than the base.
b. a lot smaller than the base.
c. just a little bit bigger than the base.
d. a lot bigger than the base.

2. If the portion is $44,931 and the base is $30,000, the rate is

a. smaller than 100%.
b. equal to 100%.
c. larger than 100% but less than 200%.
d. larger than 200%.

3. If the rate is 75% and the portion is $50,000, the base is

a. smaller than $50,000.
b. larger than $50,000.
c. the same as the portion and equal to $50,000.

base must be larger) 3. b (the portion represents 75% of the base, meaning the
2. c (the portion is larger than the base, but not twice as large)

1. b (0.25% is 0.0025, resulting in a very small portion)

Solutions:

Example 2.3B: Rate, Portion, Base

Solve for the unknown in the following three scenarios.

1. If your total income is $3,000 per month and you can't spend more than 30% on housing, what is the maximum amount of your total income that can be spent on housing?
2. Your manager tells you that 2014 sales are 102% of 2013 sales. The sales for 2014 are $1,487,003. What were the sales in 2013?
3. In Calgary, total commercial real estate sales in the first quarter of 2008 were $1.28 billion. The industrial, commercial, and institutional (ICI) land sector in Calgary had sales of $409.6 million. What percentage of commercial real estate sales is accounted for by the ICI land sector?

Plan

1. You are looking for the maximum amount of your income that can be spent on housing.
2. You need to figure out the sales for 2013.
3. You must determine the percentage of commercial real estate sales accounted for by the ICI land sector in Calgary.

What You Already Know

Business Math: A Step-by-Step Handbook 2019 Revision A How You Will Get There

1. Look for key words in the question: "what **is** the maximum amount" and "**of** your total income." The total income is the base, and the maximum amount is the portion.
Base = \$3,000 Rate = 30% Portion = maximum amount **Understand**
2. Look for key words in the question: "sales for 2014 **are** \$1,487,003" and "**of** 2013 sales." The 2014 sales is the portion, and the 2013 sales is the base.
Portion = \$1,487,003 Rate = 102% Base = 2013 sales 3. Look for key words in the question: "**of** commercial real estate sales" and "**are** accounted for by the ICI land sector." The commercial real estate sales are the base, and the ICI land sector sales are the portion.
Base=\$1.28 billion Portion=\$409.6 million Rate=percentage 1. Portion = 30% × \$3,000 = 0.3 × \$3,000 = \$900 **Perform**

2. $\text{Base} = \frac{\$1,487,003}{102\%} = \frac{\$1,487,003}{1.02} = \$1,457,846.08$

3. $\text{Rate} = \frac{\$409.6\text{ million}}{\$1.28\text{ billion}} = \frac{\$409,600,000}{\$1,280,000,000} = 0.32 \text{ or } 32\%$

1. Apply Formula 2.2, but rearrange using the triangle technique to have Portion = Rate × Base.
2. Apply Formula 2.2, but rearrange using the triangle technique to have Base=Portion/Rate.
3. Apply Formula 2.2, Rate=Portion/Base.

Present 1. The maximum you can spend on housing is \$900 per month.
2. 2013 sales were \$1,457,846.08.
3. The ICI land sector accounted for 32% of commercial real estate sales in Calgary for the first quarter of 2008.

Section 2.3 Exercises

Mechanics

1. Convert the following decimals into percentages.

a. 0.4638 b. 3.1579 c. 0.000138 d. 0.015

2. Convert the following fractions into percentages.

a. $\frac{3}{8}$ b. $\frac{17}{32}$ c. $\frac{42}{12}$ d. $2\frac{4}{5}$

3. Convert the following fractions into percentages. Round to four decimals or express in repeating decimal format as needed. a. $\frac{46}{12}$ b. $\frac{2}{9}$ c. $\frac{3}{11}$ d. $\frac{48}{93}$

4. Convert the following percentages into decimal form.

a. 15.3% b. 0.03% c. 153.987% d. 14.0005%

5. What percentage of $40,000 is $27,000?

6. What is ½% of $500,000?

7. $0.15 is 4,900% of what number?

Applications

8. In February 2009, 14,676 mortgages were in arrears in Canada, which represented 0.38% of all mortgages. How many total mortgages were in the Canadian market at that time?

9. In 2009, medical experts predicted that one out of two Manitobans would contract some form of the H1N1 virus. If the population of Manitoba in 2009 was 1,217,200, how many Manitobans were predicted to become ill?

10. In August 2004, Google Inc. offered its stocks to the public at $85 per share. In October 2007, the share price had climbed to $700.04. Express the 2007 share price as a percentage of the 2004 price.

11. During Michael Jordan's NBA career (1984–2003), he averaged a free throw completion percentage of 83.5% in regular season play. If Jordan threw 8,772 free throws in his career, how many completed free throws did he make?

12. If total advertising expenditures on television advertising declined 4.1% to $141.7 billion in the current year, how much was spent on television advertising in the previous year? Round your answer to one decimal.

13. If the new minimum wage of $8.75 per hour is 102.9412% of the old minimum wage, what was the old minimum wage?

14. A machine can produce 2,500 products per hour. If 37 of those products were defective, what is the defect rate per hour for the machine?

Challenge, Critical Thinking, & Other Applications

15. In 2011, Manitoba progressive income tax rates were 10.8% on the first $31,000, 12.75% on the next $36,000, and 17.4% on any additional income. If your gross taxable earnings for the year were $85,000, what percentage of your earnings did you pay in taxes?

16. In 2011, the maximum amount that you could have contributed to your RRSP (Registered Retirement Savings Plan) was the lesser of $22,450 or 18% of your earned income from the previous year. How much income do you need to claim a $22,450 contribution in 2011?

17. Maria, a sales representative for a large consumer goods company, is paid 3% of the total profits

earned by her company. Her company averages 10% profit on sales. If Maria's total income for the year was $60,000, what total sales did her company realize?

18. A house was purchased six years ago for $214,000. Today it lists at a price that is 159.8131% of the original purchase price. In dollars, how much has the price of the house increased over the six years?

19. An investor buys 1,000 shares of WestJet Airlines at $10.30 per share. A few months later, the investor sells the shares when their value hits 120% of the original share price. What is the price of a WestJet share when the investor sells these shares? How much money did the investor make?

20. A Honda Insight has fuel economy of 3.2 litres consumed per 100 kilometres driven. It has a fuel tank capacity of 40 litres. A Toyota Prius is rated at 4.2 L per 100 km driven. It has a fuel tank capacity of 45 L. What percentage is the total distance driveable (rounded to the nearest kilometre before calculating) of a Honda Insight compared to that of a Toyota Prius?

2.4: Algebraic Expressions

(The Pieces Of The Puzzle)

If you are like most Canadians, your employer pays you biweekly. Assume you earn $12.00 per hour. How do you calculate your pay cheque every pay period? Your earnings are calculated as follows:

$$\$12.00 \times \text{(Hours worked during the biweekly pay period)}$$

The hours worked during the biweekly pay period is the unknown variable. Notice that the expression appears lengthy when you write out the explanation for the variable. Algebra is a way of making such expressions more convenient to manipulate. To shorten the expression, making it easier to read, algebra assigns a letter or group of letters to represent the variable. In this case, you might choose *h* to represent "hours worked during the biweekly pay period." This rewrites the above expression as follows:

$$\$12.00 \times h \text{ or } \$12h$$

Unfortunately, the word *algebra* makes many people's eyes glaze over. But remember that algebra is just a way of solving a numerical problem. It demonstrates how the pieces of a puzzle fit together to arrive at a solution. For example, you have used your algebraic skills if you have ever programmed a formula into Microsoft Excel. You told Excel there was a relationship between cells in your spreadsheet. Perhaps your calculation required cell A3 to be divided by cell B6 and then multiplied by cell F2. This is an algebraic equation. Excel then took your algebraic equation and calculated a solution by automatically substituting in the appropriate values from the referenced cells (your variables). As illustrated in the figure to the right, algebra involves integrating many interrelated concepts. This figure shows only the concepts that are important to business mathematics, which this textbook will present piece by piece. Your understanding of algebra will become more complete as more concepts are covered over the course of this book.

+/-/x/÷ Rules (2.4)

Exponents and Rules (2.4) Substitution

(2.4) Solving One Linear Equation with One Unknown Variable (2.5)

Solving Two Linear Equations with Two Unknown Variables (2.5)

This section reviews the language of algebra, exponent rules, basic operation rules, and substitution. In Section 2.5 you will put these concepts to

Algebra **Language of** **Algebra** **(2.4)**

Logarithms (2.6)

work in solving one linear equation for one unknown variable along with two linear equations with two unknown variables. Finally, in Section 2.6 you will explore the concepts of logarithms and natural logarithms.

The Language of Algebra

Understanding the rules of algebra requires familiarity with four key definitions.

Algebraic Expression

A mathematical **algebraic expression** indicates the relationship between and mathematical operations that must be conducted on a series of numbers or variables. For example, the expression $12h says that you must take the hourly wage of $12 and multiply it by the hours worked. Note that the expression does not include an equal sign, or "=". It only tells you what to do and requires that you substitute a value in for the unknown variable(s) to solve. There is no one definable solution to the expression.

Algebraic Equation

A mathematical **algebraic equation** takes two algebraic expressions and equates them. This equation can be solved to find a solution for the unknown variables. Examine the following illustration to see how algebraic expressions and algebraic equations are interrelated.

6x+3y is the First Algebraic Expression: This expression on the left-hand side of the equation expresses a relationship between the variables x and y. By itself, any value can be substituted for x and y and any solution can be calculated.

4x+3 is the Second Algebraic Expression: This expression on the right hand side of the equation expresses a relationship between x and a number. By itself, any value can be substituted for x and any solution can be generated.

$$6x+3y = 4x + 3$$

= is The Equality: By having the equal sign placed in between the two algebraic expressions, they now become an algebraic equation. Now there would be a particular value for x and a particular value for y that makes both expressions equal

to each other (try $x = -0.75$ and $y = 1.125$).

Term

In any algebraic expression, **terms** are the components that are separated by addition and subtraction. In looking at the example above, the expression $6x + 3y$ is composed of two terms. These terms are "$6x$" and "$3y$." A **nomial** refers to how many terms appear in an algebraic expression. If an algebraic expression contains only one term, like "\$12.00*h*," it is called a **monomial**. If the expression contains two terms or more, such as "$6x + 3y$," it is called a **polynomial**.

Factor

Terms may consist of one or more **factors** that are separated by multiplication or division signs. Using the $6x$ from above, it consists of two factors. These factors are "6" and "x"; they are joined by multiplication.

- If the factor is numerical, it is called the **numerical coefficient**.
- If the factor is one or more variables, it is called the **literal coefficient**.

The next graphic shows how algebraic expressions, algebraic equations, terms, and factors all interrelate within an equation.

joins the two algebraic expressions, turning this into an algebraic equation.

Second Algebraic Expression: This second polynomial expression consists of the two terms x^3 and $-2y$.

First Algebraic Expression: This first polynomial expression is composed of the two terms ^{7}����$_3$and $4xy^2$.

���������

���� + ����

����$^{����}$ =

���� − ����

First Term: This term consists of two factors, where the numerical coefficient is $^7{}_3$and the

Second Term: This term consists of two factors, where the

Equal Sign: The equal sign

literal coefficient is x.

Exponents

coefficient is −2 and the literal coefficient is y.

numerical coefficient is 4 and the literal coefficients are x and y^2.

First Term: This term consists of two factors, which are the literal coefficient x^3 and a numerical coefficient of 1 (recall that in math you do not write the 1 when writing 1����^{3} since it doesn't change anything).

Second Term: This term consists of three factors where the numerical

Exponents are widely used in business mathematics and are integral to financial mathematics. When applying compounding interest rates to any investment or loan, you must use exponents (see Chapter 9 and onwards). **Exponents** are a mathematical shorthand notation that indicates how many times a quantity is multiplied by itself. The format of an exponent is illustrated below.

Power is the product: This is the result of performing the multiplication indicated, or the answer.

Base is the quantity: This is the quantity that is to be multiplied by itself.

$$\mathbf{Base^{Exponent} = Power}$$

Exponent is the multiplication factor: This is the number that indicates how many times the

base is to be multiplied by itself.

Assume you have $2^3 = 8$. The exponent of 3 says to take the base of 2 multiplied by itself three times, or 2 × 2 × 2. The power is 8. The proper way to state this expression is "2 to the exponent of 3 results in a power of 8."

How It Works

Many rules apply to the simplification of exponents, as shown in the table below.

Rule	Illustration	Explanation
1. Multiplication	$y^a \times y^b = y^{a+b}$	If the bases are identical, add the exponents and retain the unchanged base.
2. Division	$\frac{y^a}{y^b} = y^{a-b}$	If the bases are identical, subtract the exponents and retain the unchanged base.
3. Raising powers to exponents	$(y^b z^c)^a = y^{b\times a} z^{c\times a}$ or $\left(\frac{y^b}{z^c}\right)^a = \frac{y^{b\times a}}{z^{c\times a}}$	If a *single* term is raised to an exponent, each factor must retain its base and you must multiply the exponents for each by the raised exponent. Note that if the expression inside the brackets has more than one term, such as $(y^b + z^c)^a$, which has two terms, you cannot multiply the exponent *a* inside the brackets.
4. Zero exponents	$y^0 = 1$	Any base to a zero exponent will always produce a power of 1. This is explained later in this section when you review the concept of algebraic division.
5. Negative exponents	$y^{-a} = \frac{1}{y^a}$	The negative sign indicates that the power has been shifted between the numerator and denominator. It is commonly used in this textbook to simplify appearances. Note that on the BAII Plus calculator, keying in a negative exponent requires you to key in the value of *y*, press y^x, enter the value of a, press ±, and then press = to calculate.
6. Fractional exponents	$y^{\frac{a}{b}} = \sqrt[b]{y^a}$	A fractional exponent is a different way of writing a radical sign. Note that the base is first taken to the exponent of a, then the root of b is found to get the power. For example, $2^{\frac{4}{2}} = \sqrt{2^4} = \sqrt{16} = 4$. This is the same as $2^{\frac{4}{2}} = 2^2 = 4$. To key in a fractional exponent on the BAII Plus calculator, input the value of *y*, press y^x, open a set of brackets, key in a ÷ b, close the brackets, and press = to calculate.

Important Notes

Recall that mathematicians do not normally write the number 1 when it is multiplied by another factor because it doesn't change the result. The same applies to exponents. If the exponent is a 1, it is generally not written because any number multiplied by itself only once is the same number. For example, the number 2 could be written as 2^1, but the power is still 2. Or take the case of $(yz)^a$. This could be written as $(y^1z^1)^a$, which when simplified becomes $y^{1\times a}z^{1\times a}$ or y^az^a. Thus, even if you don't see an exponent written, you know that the value is 1.

Example 2.4A: Exponents in Algebra

Simplify the following expressions:

a. $h^3 \times h^6$ b. $\frac{h^{14}}{h^8}$ c. �h����5����3 / ����4 � d. 1.49268^0 e. ����2����4 / ���� −2 f. $6^{\frac{3}{5}}$�

Plan You have been asked to simplify the expressions. Notice that expressions (d) and (f) contain only numerical coefficients and therefore can be resolved numerically. All of the other expressions involve literal coefficients and require algebraic skills to simplify.

Understand

What You Already Know You have been provided with the expressions, and you have six rules for simplifying exponents in algebra.

How You Will Get There

a. This expression involves multiplying two powers with the same base. Apply Rule #1. b. This expression involves dividing two powers with the same base. Apply Rule #2. c. This expression involves a single term, with products and a quotient all raised to an exponent. Apply Rule #3. d. This power involves a zero exponent. Apply Rule #4. e. This expression involves multiplication, division, and negative exponents. Apply Rules #1, #2, and #5.

f. This power involves a fractional exponent. Apply Rule #6.

Perform

a. $h^3 \times h^6 = h^{3+6} = h^9$ b. $\frac{h^{14}}{h^8} = h^{14-8} = h^6$

c. �h����5����3 / ����4 �3 = h1×3����5×3����3×3 / ����4×3 = h3����15����9 / ����12

d. $1.49268^0 = 1$ e. ����2����4 / ���� −2 = ����2����4����2 / ���� 1 (Rule #5) = ����2����4+2 / ���� 1 (Rule #1) = ����2����6

Calculator Instructions

d. 1.49268 y^x 0 =

f. 6 y^x (3 ÷ 5) =

$����^{1} = ����^{2-1}����^{6}$(Rule #2) $= ��������^{6}$

f. $6^{3}{}_{5}^{�}$ = 2.930156

PresentHere are the simplified solutions:

a. h^9 b. h^6 c. $^{h3}����^{15}����^{9}$

$����^{12}$ d. 1 e. $��������^{6}$ f. 2.930156

Addition and Subtraction

Simplification of unnecessarily long or complex algebraic expressions is always preferable to increase understanding and reduce the chances of error. For example, assume you are a production manager looking to order bolts for a product that you make. Your company makes three products, all in equal quantity. Product A requires seven bolts, Product B requires four bolts, and Product C requires fourteen bolts. If *q* represents the quantity of products required, you need to order 7*q* + 4*q* + 14*q* bolts. This expression requires four calculations to solve every time (each term needs to be multiplied by *q* and you then need to add everything together). With the algebra rules that follow, you can simplify this expression to 25*q*. This requires only one calculation to solve. So what are the rules?

How It Works

In math, terms with the same literal coefficients are called **like terms**. Only terms with the identical literal coefficients may be added or subtracted through the following procedure:

Step 1: Simplify any numerical coefficients by performing any needed mathematical operation or converting fractions to decimals. For example, terms such as $^{1}{}_{2}$ ���� should become 0.5*y*.

Step 2: Add or subtract the numerical coefficients of like terms as indicated by the operation while obeying the rules of BEDMAS.

Step 3: Retain and do not change the common literal coefficients. Write the new numerical coefficient in front of the retained literal coefficients.

From the previous example, you require 7*q* + 4*q* + 14*q* bolts. Notice that there are three terms, each of which has the same literal coefficient. Therefore, you can perform the required addition.

Step 1: All numerical coefficients are simplified already. Skip to step 2.
Step 2: Take the numerical coefficients and add the numbers: 7 + 4 + 14 equals 25.
Step 3: Retain the literal coefficient of *q*. Put the new numerical coefficient and literal coefficient together. Thus, 25*q*. Therefore 7*q* + 4*q* + 14*q* is the same as 25*q*.

Things To Watch Out For

A common mistake in addition and subtraction is combining terms that do not have the same literal coefficient. You need to remember that the literal coefficient *must* be identical. For example, 7*q* and 4*q* have the identical literal coefficient of *q*. However, 7*q* and $4q^2$ have different literal coefficients, *q* and q^2, and cannot be added or subtracted.

Paths To Success

Remember that if you come across a literal coefficient with no number in front of it, that number is assumed to be a 1. For example, *x* has no written numerical coefficient, but it is the same as 1*x*.

Another example would be $\frac{����}{4}$ is the same as $\frac{1����}{4}$ or $\frac{1}{4}����$. On a similar note, mathematicians also don't write out literal coefficients that have an exponent of zero. For example, $7x^0$ is just 7(1) or 7. Thus, the literal coefficient is always there; however, it has an exponent of zero. Remembering this will help you later when you multiply and divide in algebra.

Give It Some Thought

Examine the following algebraic expressions and indicate how many terms can be combined through addition and subtraction. No calculations are necessary. Do not attempt to simplify.

1. $\frac{3}{2}���� + 4����^2 - 10���� - 2���� + \frac{����}{3}$ 2. $23����^2 - \frac{17����^2}{5} + ����^4 + ����^2 - \frac{2}{3}����^2 - 0.15���� + ����^3$

)

g 2. Four terms (all of the $_2$

)*x* Three terms (all of the 1.

Solutions:

Example 2.4B: Addition and Subtraction in Algebra

Simplify the following three algebraic expressions.

a. 9*x* + 3*y* − $\frac{7}{2}����$ + 4*y* b. $����(1 + 0.11 \times \frac{121}{365}) + \frac{15����}{1 + 0.11 \times \frac{36}{365}}$

c. $����(1 + \frac{0.1}{4})^3 + \frac{����}{(1 + \frac{0.1}{4})^4} - \frac{3����}{(1 + \frac{0.1}{4})^2}$

Plan You have been asked to simplify the three algebraic expressions. Notice that each term of the expressions is joined to the others by addition or subtraction. Therefore, apply the addition and subtraction rules to each expression.

Understand What You Already Know

The three algebraic expressions have already been supplied.

How You Will Get There

Applying the three steps for addition and subtraction you should simplify, combine, and write out the solution.

Perform

Present The algebraic expressions simplify as follows:
a. $5.5x + 7y$ b. $15.15566P$ c. $-0.872602x$

Notice how much easier these are to work with than the original expressions.

Multiplication

Whether you are multiplying a monomial by another monomial, a monomial by a polynomial, or a polynomial by another polynomial, the rules for multiplication remain the same.

How It Works

Follow these steps to multiple algebraic expressions:

Step 1: Check to see if there is any way to simplify the algebraic expression first. Are there any like terms you can combine? For example, you can simplify $(3x + 2 + 1)(x + x + 4)$ to $(3x + 3)(2x + 4)$ before attempting the multiplication.

Step 2: Take *every* term in the first algebraic expression and multiply it by *every* term in the second algebraic expression. This means that numerical coefficients in both terms are multiplied by each other, and the literal coefficients in both terms are multiplied by each other. It is best to work methodically from left to right so that you do not miss anything. Working with the example, in $(3x + 3)(2x + 4)$ take the first term of the first expression, $3x$, and multiply it by $2x$ and then by 4. Then move to the second term of the first expression, 3, and multiply it by $2x$ and then by 4 (see the figure).

It becomes: $6x^2 + 12x + 6x + 12$

Step 3: Perform any final steps of simplification by adding or subtracting like terms as needed. In the example, two terms contain the literal coefficient x, so you simplify the expression to $6x^2 + 18x + 12$.

Important Notes

If the multiplication involves more than two expressions being multiplied by each other, it is easiest to work with only one pair of expressions at a time starting with the leftmost pair. For example, if you are multiplying $(4x + 3)(3x)(9y + 5x)$, resolve $(4x + 3)(3x)$ first. Then take the solution, keeping it in brackets since you have not completed the math operation, and multiply it by $(9y + 5x)$. This means you are required to repeat step 2 in the multiplication procedure until you have resolved all the multiplications.

Things To Watch Out For

The negative sign causes no end of grief for a lot of people when working with multiplication. First, if a numerical coefficient is not written explicitly, it is assumed to be 1. For example, look at $2(4a + 6b) − (2a − 3b)$. This is the same as $2(4a + 6b) + (−1)(2a − 3b)$.

When you multiply a negative through an expression, all signs in the brackets will change. Continuing with the second term in the above example, $−(2a − 3b)$ becomes $−2a + 3b$. The expression then looks like $2(4a + 6b) − 2a + 3b$.

Paths To Success

The order in which you write the terms of an algebraic expression does not matter so long as you follow all the rules of BEDMAS. For example, whether you write $3 × 4$ or $4 × 3$, the answer is the same because you can do multiplication in any order. The same applies to $4 + 3 − 1$ or $3 − 1 + 4$. Now let's get more complex. Whether you write $3x^2 + 5x − 4$ or $5x − 4 + 3x^2$, the answer is the same as you have not violated any rules of BEDMAS. You still multiply first and add last from left to right.

Although the descending exponential format for writing expressions is generally preferred, for example, $3x^2 + 5x + 4$, which lists literal coefficients with higher exponents first, it does not matter if you do this or not. When checking your solutions against those given in this textbook, you only need to ensure that each of your terms matches the terms in the solution provided.

Example 2.4C: Multiplication in Algebra

Simplify the following algebraic expression: (6x + 2 + 2)(3x – 2)

Plan You have been asked to simplify the expression.

Understand What You Already Know

You are multiplying two expressions with each other. Each expression contains two or three terms.

How You Will Get There

Applying the three steps for multiplication of algebraic expressions, you simplify, multiply each term, and combine.

$(6x + 2 + 2)(3x - 2)$ Step 1: Simplify the expression first.

Perform

$(6x + 4)(3x - 2)$
$(6x)(3x) + (6x)(-2) + (4)(3x) + (4)(-2)$
$18x^2 - 12x + 12x - 8$

Step 2: Multiply all terms in each expression with all terms in the other expression.

Step 2: Resolve the multiplications.

Step 3: Perform the final simplifications. You can combine the middle two terms.

$18x^2 - 8$ Final solution.

Present The simplified algebraic expression is $18x^2 - 8$.

Example 2.4D: More Challenging Algebraic Multiplication

Simplify the following algebraic expression: $-(3ab)(a^2 + 4b - 2a) - 4(3a + 6)$

Plan You have been asked to simplify the expression.

Understand What You Already Know

You have been provided with the expression. Notice that the first term consists of three expressions being multiplied together. The second term involves two expressions being multiplied together. Apply the rules of multiplication.

How You Will Get There

Applying the three steps for multiplication of algebraic expressions, you will simplify, multiply each term, and combine.

$-(3ab)(a^2 + 4b - 2a) - 4(3a + 6)$ Step 1: You cannot simplify anything. Be careful with the negatives and write them out.

(−����)(3��������)(����^{2} + 4 − 2����) + (− ����)(3���� + 6)
(−������������)(����^{2} + 4 − 2����) + (− 4)(3���� + 6)
−��������$^{����}$ − ���� ���� ���� + ��������$^{����}$ + (− 4)(3���� + 6)
−3����^{3} − 12���� ^{2} + 6����^{2} +

[−���� ���� − ����]

Perform

Step 2: Work with the first pair of expressions in the first term and multiply.

Step 2: Multiply the resulting pair of expressions in the first term.

Step 2: Work with the pair of expressions in the original second term and multiply.

Step 3: Drop the brackets to simplify.

−3����^{3} − 12���� ^{2} + 6����^{2} − 12���� − 24 Step 3: There are no like terms. You cannot simplify this expression any further.

Present The simplified algebraic expression is −3����^{3} − 12���� ^{2} + 6����^{2} − 12���� − 24.

Division

You are often required to divide a monomial into either a monomial or a polynomial. In instances where the denominator consists of a polynomial, it is either not possible or extremely difficult to simplify the expression algebraically. Only division where denominators are monomials are discussed here.

How It Works

To simplify an expression when its denominator is a monomial, apply these rules:

Step 1: Just as in multiplication, determine if there is any way to combine like terms before completing the division. For example, with 3�������� + 3�������� − 3����^{2} + 9���� ^{2} / 3�������� you can simplify the numerator to 6�������� − 3����^{2} + 9���� ^{2} / 3��������.

Step 2: Take *every* term in the numerator and divide it by the term in the denominator. This means that you must divide both the numerical and the literal coefficients. As with multiplication, it is usually best to work methodically from left to right so that you do not miss anything. So in our example we get:

3�������� − 3����^{2} 6���� ���� +9��� � ^{2}

3���� 3����

���� ����

(2)(1)(1) − (1)(*a*)(1) + (3)(1)(*b*)

$$2 - a + 3b$$

Step 3: Perform any final simplification by adding or subtracting the like terms as needed. As there are no more like terms, the final expression remains 2 − *a* + 3*b*.

Things To Watch Out For

You may have heard of an outcome called "cancelling each other out." For example, in resolving the division $\frac{4x}{4x}$ many people would say that the terms cancel each other out. Many people will also mistakenly interpret this to mean that the quotient is zero and say that $\frac{4x}{4x} = 0$. In fact, when terms cancel each other out the quotient is one, not zero. The numerical coefficient is $\frac{4}{4} = 1$. The literal coefficient is $\frac{x}{x} = 1$. Thus, $\frac{4x}{4x} = (1)(1) = 1$. This also explains why a zero exponent equals one: $\frac{x^1}{x^1} = x^{1-1} = x^0 = 1$.

Paths To Success

A lot of people dislike fractions and find them hard to work with. Remember that when you are simplifying any algebraic expression you can transform any fraction into a decimal. For example, if your expression is $\frac{2x}{5} + \frac{3x}{4}$, you can convert the fraction into decimals: 0.4*x* + 0.75*x*. In this format, it is easier to solve.

Example 2.4E: A Monomial Division

Simplify the following algebraic expression: $\frac{30x^6 + 5x^3 + 10x^3}{5x}$

Plan You have been asked to simplify the expression.

Understand

What You Already Know

Notice that the provided expression is a polynomial divided by a monomial. Therefore, apply the division rules.

How You Will Get There

Applying the three steps for division of an algebraic expression, you will simplify, divide each term, and combine.

$$\frac{30x^6 + 5x^3 + 10x^3}{5x}$$

Step 1: The numerator has two terms with the same literal coefficient (x^3). Combine these using the rules of addition.

30����^{6} + ���� $^{����}$ 5����
30����^{6} 5����+���� $^{����}$ 5����

Perform

Step 2: Now that the numerator is simplified, divide each of its terms by the denominator.

Step 2: Resolve the divisions by dividing both numerical and literal coefficients.

$6x^5 + 3x^2$ Step 3: There are no like terms, so this is the final solution.

Present The simplified algebraic expression is $6x^5 + 3x^2$.

Example 2.4F: A More Challenging Division

Simplify the following algebraic expression: 15����^{2}����^{3}+25��������^{2}−���� +10����^{4}����+5��������^{2} / 5����

Plan You have been asked to simplify the expression.

Understand What You Already Know

Notice that the provided expression is a polynomial divided by a monomial. Therefore, apply the division rules.

How You Will Get There

Applying the three steps for division of algebraic expressions, you will simplify, divide each term, and combine.

���� 5���� −   5����+10����^{4}����   5����
$3xy^2 + 6(1)(y) - 0.2(1)(1) + 2(x^3)(1)$

Perform

15����^{2}����^{3} + 25��������^{2} − ���� + 10����^{4}���� + 5��������^{2} / 5����

15����^{2}����^{3} + �������� $^{����}$ − ���� + 10����^{4}����   5����
15����^{2}����^{3} 5����+30��������^{2}

Step 1: The numerator has two terms with the same literal coefficient (xy^2). Combine these by the rules of addition.

Step 2: Now that the numerator is simplified, divide each of its terms by the denominator.

Step 2: Resolve the divisions by dividing both numerical and literal coefficients.

Step 3: Simplify and combine any like terms.

$3xy^2 + 6y - 0.2 + 2x^3$ There are no like terms, so this is the final solution.

Present The simplified algebraic expression is $3xy^2 + 6y - 0.2 + 2x^3$.

Substitution

The ultimate goal of algebra is to represent a relationship between various variables. Although it is beneficial to simplify these relationships where possible and shorten the algebraic expressions, in the end you want to calculate a solution. **Substitution** involves replacing the literal coefficients of an algebraic expression with known numerical values. Once the substitution has taken place, you solve the expression for a final value.

How It Works

Follow these steps to perform algebraic substitution:

Step 1: Identify the value of your variables. Assume the algebraic equation is $\mathrm{PV} = \frac{\mathrm{FV}}{1 + rt}$. It is known that FV = $5,443.84, r = 0.12, and $t = \frac{270}{365}$. You need to calculate the value of PV.

Step 2: Take the known values and insert them into the equation where their respective variables are located, resulting in $\mathrm{PV} = \frac{\$5{,}443.84}{1 + (0.12)\left(\frac{270}{365}\right)}$.

Step 3: Resolve the equation to solve for the variable. Calculate $\mathrm{PV} = \frac{\$5{,}443.84}{1.088767} = \$5{,}000.00$.

Things To Watch Out For

It is common in algebra to represent a variable with more than just one letter. As you can see from the example above, FV is a variable, and it represents *future value*. This should not be interpreted to be two variables, F and V. Similarly, PMT represents *annuity payment*. When you learn new formulas and variables, take careful note of how a variable is represented.

As well, some literal coefficients have subscripts. For example, you could see d_1 and d_2 in the same formula. What sometimes happens is that there is more than one value for the same variable. As you will learn in Chapter 6 about merchandising, when you buy an item you may receive more than one discount rate (what d stands for). Therefore, the first discount gets a subscript of 1, or d_1, and the second discount gets a subscript of 2, or d_2. This allows you to distinguish between the two values in the equation and substitute the correct value in the correct place.

Paths To Success

If you are unsure whether you have simplified an expression appropriately, remember that you can make up your own values for any literal coefficient and substitute those values into both the original and the simplified expressions. If you have obeyed all the rules and simplified appropriately, both expressions will produce the same answer. For example, assume you simplified $2x + 5x$ into $7x$, but you are not sure if you are right. You decide to let $x = 2$. Substituting into $2x + 5x$, you get $2(2) + 5(2) = 14$. Substituting into your simplified expression you get $7(2) = 14$. Since both expressions produced the same answer, you have direct confirmation that you have simplified correctly.

Example 2.4G: Substitution

Substitute and solve the following equation:
$N = L \times (1 - d_1) \times (1 - d_2) \times (1 - d_3)$
Where $L = \$1,999.99$ $d_1 = 35\%$ $d_2 = 15\%$ $d_3 = 5\%$

Plan You need to get a dollar value for the literal coefficient N.

Understand

What You Already Know

You are provided with the equation and the values of four literal coefficients.

How You Will Get There

Step 1: The values of L, d_1, d_2, and d_3 are known. Step 2: Substitute those values into the equation. Step 3: Solve for N.

Perform Step 1: $L = \$1,999.99$ $d_1 = 0.35$ $d_2 = 0.15$ $d_3 = 0.05$
Step 2: $N = \$1,999.99 \times (1 – 0.35) \times (1 – 0.15) \times (1 – 0.05)$ Step 3: $N = \$1,999.99 \times 0.65 \times 0.85 \times 0.95$
$N = \$1,049.74$

Present The value of N is $1,049.74.

Section 2.4 Exercises

Mechanics

For questions 1–4, simplify the algebraic expressions.

1. $2a - 3a + 4 + 6a - 3$
2. $5b(4b + 2)$
3. 6����3 + 12����^{2} + 13.5���� / 3����
4. $(1 + i)^3 \times (1 + i)^{14}$
5. Evaluate the power $8^{\frac{2}{3}}$�.
6. Substitute the known variables and solve for the unknown variable: I = Prt where P = $2,500, r = 0.06, and $t = \frac{135}{365}$

Applications

For questions 7–11, simplify the algebraic expressions.

7. $(6r^2 + 10 - 6r + 4r^2 - 3) - (-12r - 5r^2 + 2 + 3r)$ 2���� �^{5}
8. �5����9+3����^{9}
9. 2+ 0.75���� − ����^{3} + 5 4 _ 2(+ 3) 4
10. 14(1 + ����) + 21(1 + ����)4 −35(1 + ����)^7 7(1 + ����)
11. ���� / 1 + 0.08 × ~~103~~:365 + 3����(1 + 0.08 × ^{52} 365)

12. Evaluate the power: ��$^2{}_5$�^{2}�^{2}.

For questions 13 and 14, substitute the known variables and solve for the unknown variable. 13. $\mathrm{PV} = \frac{\mathrm{FV}}{(1+����)^{N}}$ where FV = $3,417.24, *i* = 0.05, and N = 6

14. $\mathrm{PMT} = \frac{\mathrm{FV}}{�\frac{(1+����)^{N}-1}{����}�}$ where FV = $10,000, N = 17, and *i* = 0.10

Challenge, Critical Thinking, & Other Applications

For questions 15–17, simplify the algebraic expressions.

15. $\frac{5^3���� {}^{4} �^2 + 6(����^8)^{\frac{1}{2}} �}{�^{10����^2 3}����^4} - (3����^2 + 6)(3����^2 - 3)$

16. $\frac{-(5���� + 4���� + 3)(2���� - 5����) - (10���� - 2����)(2���� + 3)}{5}$

17. $\frac{(-3����)^3 (3����^2)^2}{(2����^3)^{-4}}$

18. Substitute the known variables and solve for the unknown variable.

$\mathrm{FV_{ORD}} = \mathrm{PMT}(1 + \Delta\%)^{N-1} \left[\frac{�(1+����)^{\frac{CY}{PY}}� ^{N}}{(1+\Delta\%)} - 1 \right] \Big/ \left[\frac{(1+����)^{CY/PY}}{(1+\Delta\%)} - 1 \right]$ where PMT = $500, *i* = 0.05, Δ% = 0.02, CY = 2, PY = 4, and N = 20

For questions 19-20, evaluate the expression. 19. $\$50{,}000 \times \left(1 + \frac{0.10}{12}\right)^{-27}$

20. $\$995\ �\frac{1 - (1 + 0.02)^{13}\left(1 + \frac{0.09}{4}\right)^{-13}}{\frac{1+0.09}{4} - 0.02}�$

2.5: Linear Equations: Manipulating and Solving (Solving the Puzzle)

You are shopping at Old Navy for seven new outfits. The price points are \$10 and \$30. You really like the \$30 outfits; however, your total budget can't exceed \$110. How do you spend \$110 to acquire all the needed outfits without exceeding your budget while getting as many \$30 items as possible?

This is a problem of linear equations, and it illustrates how you can use them to make an optimal decision. Let L represent the quantity of clothing at the low price point of \$10, and H represent the quantity of clothing at the high price point of \$30. This results in the following algebraic equations:

$L + H = 7$ (the total number of outfits you need)

$\$10L + \$30H = \$110$ (your total budget)

By simultaneously solving these equations you can determine how many outfits at each price point you can purchase. You will encounter many situations like this in your business career, for example, in making the best use of a manufacturer's production capacity. Assume your company makes two products on the same production line and sells all its output. Each product contributes differently to your profitability, and each product takes a different amount of time to manufacture. What combination of each of these products should you make such that you operate your production line at capacity while also maximizing the profits earned? This section explores how to solve linear equations for unknown variables.

Understanding Equations

To manipulate algebraic equations and solve for unknown variables, you must first become familiar with some important language, including linear versus nonlinear equations and sides of the equation.

4x+3 is the Left Side: Every equation has two sides. Everything to the left side of the equal sign is known as the **left side of the equation**.

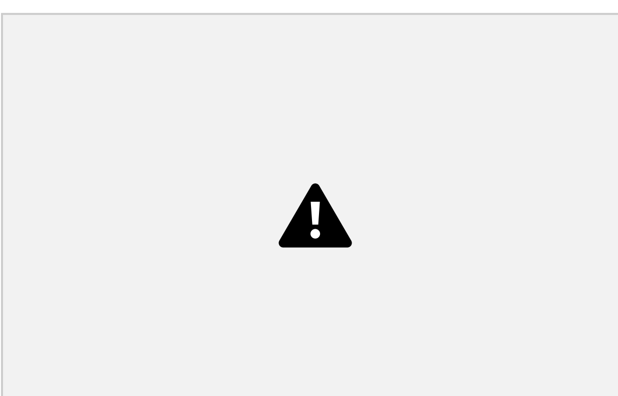
−2*x* − 3 is the Right Side: Everything to the right side of the equal sign is known as the **right side of the equation**.

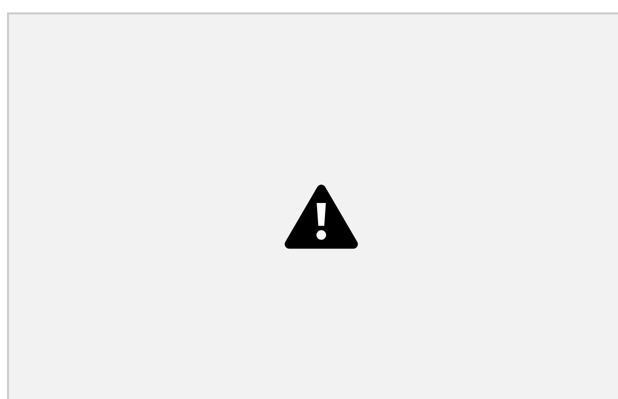
$4x + 3 = -2x - 3$

x is the Variable x: Observe two important features of this variable:

1. Note that the variable has an exponent of 1. So if you were to plot each algebraic expression onto
a Cartesian coordinate system, the expressions would form straight lines (see the left-hand figure

next page). Such equations are called **linear equations**. Because each equation has only one variable, there is only one solution that makes the equation true. In business mathematics, this is the
most common situation you will encounter.
2. If the exponent is anything but a one, the equation is a **nonlinear equation** since the graph of
the expression does not form a straight line. The right-hand figure (next page) illustrates two algebraic expressions where the exponent is other than a 1.





The goal in manipulating and solving a linear equation is to find a value for the unknown variable that makes the equation true. If you substitute a value of $x = -1$ into the above example, the left-hand side of the equation equals the right hand side of the equation (see Figure 2.17). The value of $x = -1$ is known as the **root**, or solution, to the linear equation.

Solving One Linear Equation with One Unknown Variable

In your study of solving linear equations, you need to start by manipulating a single equation to solve for a single unknown variable. Later in this section you will extend from this foundation to the solution of two linear equations with two unknowns.

How It Works

To determine the root of a linear equation with only one unknown variable, apply the following steps:

Step 1: Your first goal is to separate the terms containing the literal coefficient from the terms that only have numerical coefficients. Collect all of the terms with literal coefficients on only one side of the equation and collect all of the terms with only numerical coefficients on the other side of the equation. It does not matter which terms go on which side of the equation, so long as you separate them.

To move a term from one side of an equation to another, take the mathematical opposite of the term being moved and add it to both sides. For example, if you want to move the +3 in $4x + 3 = -2x - 3$ from the left-hand side to the right hand side, the mathematical opposite of +3 is −3. When you move a term, remember the cardinal rule: What you do to one side of an equation you must also do to the other side of the equation. Breaking this rule breaks the equality in the equation.

Step 2: Combine all like terms on each side and simplify the equation according to the rules of algebra.

Step 3: In the term containing the literal coefficient, reduce the numerical coefficient to a 1 by dividing both sides of the equation by the numerical coefficient.

Important Notes

When you are unsure whether your calculated root is accurate, an easy way to verify your answer is to take the original equation and substitute your root in place of the variable. If you have the correct root, the left-hand side of the equation equals the right-hand side of the equation. If you have an incorrect root, the two sides will be unequal. The inequality typically results from one of the three most common errors in algebraic manipulation:

1. The rules of BEDMAS have been broken.
2. The rules of algebra have been violated.
3. What was done to one side of the equation was not done to the other side of the equation.

Things To Watch Out For

When you move a term from one side of the equation to another using multiplication or division, remember that this affects *each and every term on both sides of the equation*. To remove the x from the denominator in the following equation, multiply both sides of the equation by x:

5

���� + 1 ���� = 2 ���� + 2 becomes ���� �5 ���� + 1 ����� = �2 ���� + 2�
���� which then becomes $5 + 1 = 2 + 2$���� Multiplying every term on both sides by *x* maintains the equality.

Paths To Success

Negative numbers can cause some people a lot of grief. In moving terms from a particular side of the equation, many people prefer to avoid negative numerical coefficients in front of literal coefficients. Revisiting $4x + 3 = -2x - 3$, you could move the $4x$ from the left side to the right side by subtracting $4x$ from both sides. However, on the right side this results in $-6x$. The negative is easily overlooked or accidentally dropped in future steps. Instead, move the variable to the left side of the equation, yielding a positive coefficient of $6x$.

Example 2.5A: How to Solve the Opening Example

Take the ongoing example in this section and solve it for *x*: $4x + 3 = -2x - 3$

Plan This is a linear equation since the exponent on the variable is 1. You are to solve the equation and find the root for *x*.

Understand What You Already Know

The equation has already been provided.

$4x + 3 = -2x - 3$

How You Will Get There

Apply the three steps for solving linear equations. To arrive at the root, you must follow the rules of algebra, BEDMAS, and equality.

Perform

$4x + 3 \mathbf{+ 2x} = -2x - 3 \mathbf{+ 2x}$	Step 1: Move terms with literal coefficients to one side and terms with only numerical coefficients to the other side. Let's collect the literal coefficient on the left hand side of the equation. Move $-2x$ to the left-hand side by placing $+2x$ on both sides.
$4x + 3 + 2x = -3$	On the right-hand side, the $-2x$ and $+2x$ cancel out to zero.
$4x + 3 + 2x \mathbf{- 3} = -3 \mathbf{- 3}$	Step 1: All of the terms with the literal coefficient are now on the left. Let's move all of the terms containing only numerical coefficients to the right-hand side. Move the +3 to the right-hand side by placing −3 on both sides.

On the left-hand side, the +3 and −3 cancel out to zero.

$4x + 2x = -3 - 3$	
$\mathbf{6x = -6}$	Step 2: The terms are now separated. Combine like terms according to the rules of algebra.
6����	���� = −6 ����

Step 3: The term with the literal coefficient is being multiplied by the numerical coefficient of 6. Therefore, divide both sides by 6.

The left-hand side numerical coefficients will divide to 1. Resolve the numerical coefficients on the right-hand side.

$x = -1$ This is the root of the equation.

The root of the equation is $x = -1$. To verify the accuracy of your manipulation, take the root of $x = -1$ and substitute it into the original equation:

Present

$$4(-1) + 3 = -2(-1) - 3$$
$$-4 + 3 = 2 - 3$$
$$-1 = -1$$

The left-hand side equals the right-hand side, so the root is correct.

Example 2.5B: Solving a Linear Equation with One Unknown Variable

Solve the following equation for m: $\frac{3m}{4} + 2m = 4m - 15$

Plan This is a linear equation since the exponent on the variable is a 1. You are to solve the equation and find the root for m.

Understand

What You Already Know The equation has already been provided.

$3m$

How You Will Get There Simplify the equations first and then apply the three steps for solving linear equations. To arrive at the root you must follow the rules of algebra, BEDMAS, and equality. You can use an approach that avoids negatives.

$\frac{3m}{4} + 2m = 4m - 15$ First, simplify all fractions to make the equation easier to work with.

0.75m + 2m = 4m − 15
2.75*m* = 4m − 15
2.75m **− 2.75*m*** = 4m − 15 **− 2.75*m***
0 = 4m − 15 **− 2.75*m***
0 **+ 15** = 4m − 15 − 2.75m **+ 15**
0 + **15** = 4m − 2.75m
15 = **1.25*m***
$\frac{15}{1.25} = \frac{1.25m}{1.25}$

Perform

Still simplifying, collect like terms where possible.

Step 1: Collect all terms with the literal coefficient on one side of the equation. Move all terms with literal coefficients to the right-hand side. Step 1: Combine like terms and move all terms with only numerical coefficients to the left-hand side.

On the left-hand side, the +2.75m and −2.75m cancel each other out. Now move the numerical coefficients to the left-hand side.

On the right-hand side, the −15 and +15 cancel each other out. Step 2: Combine like terms on each side.

Step 3: Divide both sides by the numerical coefficient that accompanies the literal coefficient.

Simplify.

$12 = m$ This is the root of the equation.

The root of the equation is *m* = 12.

Present

This makes both sides of the equation, $\frac{3m}{4} + 2m$ and $4m - 15$, equal 33.

Example 2.5C: Solving a Linear Equation with One Unknown Variable Containing Fractions

Solve the following equation for *b* and round your answer to four decimals: $\frac{5}{8}b + \frac{2}{5} = \frac{17}{20} - \frac{b}{4}$

Plan This is a linear equation since the exponent on the variable is a 1. You are to solve the equation and find the root for *b*.

Understand

What You Already Know

The equation has already been provided. Although you could attempt to clear each and every fraction or try to find a common denominator, recall that you can eliminate fractions by converting them to decimals.

How You Will Get There

Simplify the fractions into decimal form. Then apply the three steps for solving linear equations. To arrive at the root, you must follow the rules of algebra, BEDMAS, and equality.

$\frac{5}{8}b + \frac{2}{5} = \frac{17}{20} - \frac{b}{4}$ Simplify the fractions and convert to decimals.

0.625*b* + 0.4 = 0.85 − 0.25*b*
0.625*b* + 0.4 **+ 0.25*b*** = 0.85 − 0.25*b* **+ 0.25*b***
0.625*b* + 0.4 + 0.25*b* = 0.85
0.625*b* + 0.4 +0.25*b* **− 0.4** = 0.85 **− 0.4**
0.625*b* + 0.25*b* = 0.85 − 0.4

0.875*b* = 0.45
$\frac{0.875b}{0.875} = \frac{0.45}{0.875}$
b = 0.514285

Perform

Step 1: Move the literal coefficient terms to the left-hand side. The literal coefficients on the right-hand side cancel each other out. Move the numerical coefficient terms to the right-hand side.

The numerical coefficients on the left-hand side cancel each other out.

Step 2: Combine like terms on each side.

Step 3: Divide both sides by the numerical coefficient that accompanies the literal coefficient.

Simplify.

Round to four decimals as instructed.

$b = 0.5143$ This is the root.

Present The root of the equation, rounded to four decimals, is $b = 0.5143$.

Solving Two Linear Equations with Two Unknown Variables

The manipulation process you have just practiced works well for solving one linear equation with one variable. But what happens if you need to solve two linear equations with two variables simultaneously? Remember when you were at Old Navy purchasing seven outfits earlier in this chapter (equation 1)? You needed to stay within a pricing budget (equation 2). Each equation had two unknown variables representing the number of lower-priced and higher-priced outfits.

The goal is to reduce two equations with two unknowns into a single linear equation with one unknown. Once this transformation is complete, you then identify the unknown variable by applying the three-step procedure for solving one linear equation, as just discussed.

When you work with two linear equations with two unknowns, the rules of algebra permit the following two manipulations:

1. What you do to one side of the equation must be done to the other side of the equation to maintain the equality. Therefore, you can multiply or divide any equation by any number without changing the root of the equation. For example, if you multiply all terms of $x + y = 2$ by 2 on both sides, resulting in $2x + 2y = 4$, the equality of the equation remains unchanged and the same roots exist.
2. Terms that are on the same side of an equation can be added and subtracted between equations by combining like terms. Each of the two equations has a left side and right side. This rule permits taking the left side of the first equation and either adding or subtracting like terms on the left side of

the second equation. When you perform this action, remember the first rule above. If you add the left sides of the equations together, you then must add the right side of both equations together to maintain equality.

How It Works

Follow these steps to solve two linear equations with two unknown variables:

Step 1: Write the two equations one above the other, vertically lining up terms that have the same literal coefficients and terms that have only the numerical coefficient. If necessary, the equations may need to be manipulated such that all of the literal coefficients are on one side with the numerical coefficients on the other side.

Step 2: Examine your two equations. Through multiplication or division, make the numerical coefficient on one of the terms containing a literal coefficient exactly equal to its counterpart in the other equation.

Step 3: Add or subtract the two equations as needed so as to eliminate the identical term

from both equations. **Step 4**: In the new equation, solve for the last literal coefficient.

Step 5: Substitute the root of the known literal coefficient into either of the two original equations. If one of the equations takes on a simpler structure, pick that equation.

Step 6: Solve your chosen equation for the other literal coefficient.

Paths To Success

Sometimes it is unclear exactly how you need to multiply or divide the equations to make two of the
terms identical. For example, assume the following two equations:

$4.9x + 1.5y = 38.3$

$2.7x - 8.6y = 17.8$

If the goal is to make the terms containing the literal coefficient *x* identical, there are two alternative solutions: 1. Take the larger numerical coefficient for *x* and divide it by the smaller numerical coefficient. The resulting number is the factor for multiplying the equation containing the smaller numerical coefficient. In this case, $4.9 \div 2.7 = 1.814$. Multiply all terms in the second equation by 1.814 to make the numerical coefficients for *x* equal to each other, resulting in this pair of equations:

$4.9x + 1.5y = 38.3$

$4.9x - 15.6074y = 32.3037$ (every term multiplied by 1.814)

2. Take the first equation and multiply it by the numerical coefficient in the second equation. Then take the second equation and multiply it by the numerical coefficient in the first equation. In this case, multiply all terms in the first equation by 2.7. Then multiply all terms in the second equation by 4.9.

$13.23x + 4.05y = 103.41$ (every term multiplied by 2.7)

$13.23x - 42.14y = 87.22$ (every term multiplied by 4.9)

Note that both approaches successfully result in both equations having the same numerical coefficient in front of the literal coefficient *x*.

Paths To Success

Ultimately, every pairing of linear equations with two unknowns can be converted into a single equation through substitution. To make the conversion, do the following:
1. Solve either equation for one of the unknown variables.
2. Take the resulting algebraic expression and substitute it into the other equation. This new equation is solvable for one of the unknown variables.
3. Substitute your newfound variable into one of the original equations to determine the value for the other unknown variable.

Take the following two equations:
$a + b = 4$ $2a + b = 6$
1. Solving the first equation for a results in $a = 4 - b$.
2. Substituting the expression for a into the second equation and solving for b results in $2(4 - b) + b = 6$, which solves as $b = 2$.
3. Finally, substituting the root of b into the first equation to calculate a gives $a + 2 = 4$ resulting in $a = 2$. Therefore, the roots of these two equations are $a = 2$ and $b = 2$.

Example 2.5D: Buying Those Outfits

Recall from the section opener that in shopping for outfits there are two price points of \$10 and \$30, your budget is \$110, and that you need seven articles of clothing. The equations below represent these conditions. Identify how many low-priced outfits (L) and high-priced outfits (H) you can purchase.
$L + H = 7$ $\$10L + \$30H = \$110$

Plan You need to determine the quantity of low-price-point items, or L, and high-price-point items, or H, that are within
your limited budget. Note that the exponents on the variables are 1 and that there are two unknowns. So there are two linear equations with two unknowns.

Understand What You Already Know

You require seven articles of clothing and only have a budget of \$110. The equations express the relationships of quantity and budget.

How You Will Get There

Apply the six-step procedure for solving two linear equations with two unknowns.

Perform

$L + H = 7$
$\$10L + \$30H = \$110$

10L + **10**H = **70** \$10L + \$30H = \$110

$10L + 10H = 70$ subtract $\underline{\$10L + \$30H = \$110}$ $\mathbf{-\$20H = -\$40}$
$-\$20$���� $-\$$���� $= -\$40$ $-\$$���� ���� $= 2$
$L + H = 7$ $L + \mathbf{2} = 7$

$L + 2 \mathbf{- 2} = 7 \mathbf{- 2}$

$L = 5$

Step 1: Write the equations one above the other and line them up.

Step 2: Multiply all terms in the first equation by 10 so that L has the same numerical coefficient in both equations.

Step 3: Subtract the equations by subtracting all terms on both sides.

Step 4: Solve for H by dividing

both sides by −20.

Step 5: Substitute the known

value
origina
equati
that or

subtracting
u now have

Present You can purchase five articles of clothing at the low price point and two articles of clothing at the high price point. This allows you to purchase seven articles of clothing and stay within your budget of $110.

Paths To Success

One of the most difficult areas of mathematics involves translating words into mathematical symbols and operations. To assist in this translation, the table below lists some common language and the mathematical symbol that is typically associated with the word or phrase.

Language Math			Symbol		Language Math			Symbol
Sum Addition	In addition to In excess	Increased by Plus	+		Subtract Decreased by Diminished by	Less Minus	Difference Reduced by	−
Divide Division	Divisible Quotient	Per	÷		Multiplied by Times	Percentage of	Product of Of	×
Becomes Is/Was/Were	Will be	Results in Totals	=		More than Greater than			>
Less than	Lower than		<		Greater than or equal to			≥
Less than or equal to			≤		Not equal to			≠

Example 2.5E: Solving Two Linear Equations with Two Unknowns for an Amusement

Park Tinkertown Family Fun Park charges \$15 for a child wrist band and \$10.50 for an adult wrist band. On a warm summer day, the amusement park had total wrist band revenue of \$15,783 from sales of 1,279 wrist bands. How many adult and child wrist bands did the park sell that day?

Plan You need the number of both adult and child wrist bands sold on the given day. Therefore, you must identify two unknowns.

What You Already Know

The price of the wrist bands,total quantity, and sales are known:

Child wrist band price = \$15

Understand

Adult wrist band price = \$10.50

Total revenue = \$15,783

Total unit sales = 1,279

The quantity of adult wrist bands sold and the quantity of child wrist bands sold are unknown:

Adult wrist bands quantity = *a*

Child wrist bands quantity = *c*

a + *c* = 1,279

\$10.50*a* + \$15*c* = \$15,783

How You Will Get There

1. Work with the quantities first. Calculate the total unit sales by adding the number of adult wrist bands to the number of child wrist bands: # of adult wrist bands + # of child wrist bands = total unit sales

a + *c* = 1,279

2. Now consider the dollar figures. Total revenue for any company is calculated as unit price multiplied by units sold. In this case, you must sum the revenue from two products to get the total revenue.

Total adult revenue + Total child revenue = Total revenue

(Adult price×Adult quantity)+(Child price×Child quantity)=Total revenue \$10.50*a* + \$15*c* = \$15,783

3. Apply the six-step procedure for solving two linear equations with two unknowns.

Step 1: Write the equations one above the other and line them up.

10.50*a* + **10.50***c* = **13,429.50** \$10.50*a* + \$15*c* = \$15,783
10.5*a* + 10.5*c* = 13,429.50 Subtract \$10.5*a* + \$15*c* = \$15,783 **−4.5*c* = −2,353.50**
−4.5���� −����.���� = −2,353.50 −����.����·*c* = 523
a + *c* = 1,279 *a* + **523** = 1,279

Perform

a + 523 **− 523** = 1,279 **− 523**

a = 756

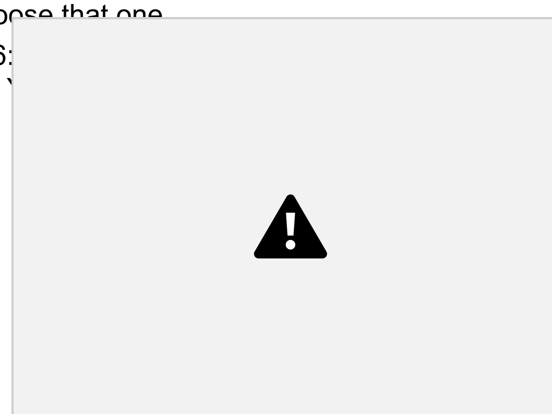
Step 2: Multiply all terms in the first equation by 10.5, resulting in *a* having the same numerical coefficient in both equations.

Step 3: Subtract the equations by subtracting all terms on both sides.

Step 4: Solve for *c* by dividing both sides by −4.5.

Step 5: Substitute the known value for *c* into one of the original equations. The first equation is simple, so choose that one.

Step 6: sides.



Present Tinkertown Family Fun Park sold 523 child wrist bands and 756 adult wrist bands.

Section 2.5 Exercises

Mechanics

Solve the following equations for the unknown variable.

1. 3(x − 5) = 15

2. 12b − 3 = 4 + 5b

3. 0.75(4m + 12) + 15 − 3(2m + 6) = 5(−3m + 1) + 25

Solve each of the following pairs of equations for both unknown variables.

4. x + y = 6
 3x − 2y = 8

5. 4h − 7q = 13
 6h + 3q = 33

6. 0.25���� + 5 2 = 3.5
 3���� 4 − 0.2 = 3

Applications

In questions 7 and 8, solve the equation for the unknown variable.

7. 4���� 1.025 4 + ���� − 2����$(1.05)^2$ = $1,500

8. $2,500(1 + 0.06$t$) + $1,000(1 + 0.04$t$) = $3,553.62

For exercises 9–14, read each question carefully and solve for the unknown variable(s).

9. Pamela is cooking a roast for a 5:30 p.m. dinner tonight. She needs to set a delay timer on her oven. The roast takes 1 hour and 40 minutes to cook. The time right now is 2:20 p.m. How long of a delay must she set the oven for (before it automatically turns on and starts to cook the roast)?

10. In 2010, 266 million North Americans were using the Internet, which represented a 146.3% increase in Internet users over the year 2000. How many North American Internet users were there in 2000?

11. A human resource manager is trying to estimate the cost of a workforce accident. These costs usually consist of direct costs (such as medical bills, equipment damage, and legal expenses) and indirect costs (such as decreased output, production delays, and fines). From past experience, she knows that indirect costs average six times as much as direct costs. If she estimates the cost of an accident to be $21,000, what is the direct cost of the accident?

12. In 2011, Canadian federal tax rates were 0% on the first $10,527 of gross income earned, 15% on the next $31,017, 22% on the next $41,544, 26% on the next $45,712, and 29% on anything more. If a taxpayer paid $28,925.35 in federal tax, what was her gross annual income for 2011?

13. St. Boniface Hospital raises funds for research through its Mega Lottery program. In this program, 16,000 tickets are available for purchase at a price of one for $100 or three for $250. This year, the lottery sold out with sales of $1,506,050. To better plan next year's lottery, the marketing manager wants to know how many tickets were purchased under each option this year.

14. An accountant is trying to allocate production costs from two different products to their appropriate ledgers. Unfortunately, the production log sheet for last week has gone missing. However, from other documents he was able to figure out that 1,250 units in total were produced last week. The production machinery was run for 2,562.5 minutes, and he knows that Product A takes 1.5 minutes to manufacture while Product B takes 2.75 minutes to manufacture. How many units of each product were produced last week?

Challenge, Critical Thinking, & Other Applications

15. Jacob owns 15,000 shares in a corporation, which represents 2% of all issued shares for the company. He sold $\frac{2}{5}$ of his shares to another investor for $7,800. What is the total value for all of the shares issued by the company?

16. Two cellphone companies are offering different rate plans. Rogers is offering $19.99 per month, which includes a maximum of 200 weekday minutes plus $0.35 for every minute above the maximum. TELUS is offering $39.99 for a maximum 300 weekday minutes, but it charges $0.10 for every minute above the maximum. Above how many minutes would TELUS be the better choice?

17. Marianne, William, Hendrick, and Charlotte have all decided to go into business together. They need $175,000 in initial capital funding. William was able to contribute 20% less than Marianne, Hendrick contributed 62.5% more than William, and Charlotte contributed $5,000 less than half as much as Marianne. How much did each partner contribute to the initial funds?

18. A mall is being constructed and needs to meet the legal requirements for parking availability. Parking laws require one parking stall for every 100 square feet of retail space. The mall is designed to have 1,200,000 square feet of retail space. Of the total parking stalls available, 2% need to be handicap accessible, there need to be three times as many small car spaces as handicap spaces, RV spaces need to be one-quarter of the number of small car spaces, and the rest of the spaces are for regular stalls. How many of each type of parking space does the mall require?

19. Simplify the following equation into the format of #*z* = # and find the root. Verify the solution through substitution. ���� �$1 + 0.073 \times \frac{280}{365}$� − $\frac{����}{1 + 0.073 \times \frac{74}{365}}$ $+ \$1,000 = \$2,764.60$

20. Find the roots for the following pairs of equations. Verify the solution through substitution into both equations. $_{3}^{4}{}_{5}$ ���� $+ 0.18$���� $= 12.2398$

-5.13���� $-$ $^{13\text{����}}{}_{5}$ $= -38.4674$

2.6: Natural Logarithms

(How Can I Get That Variable Out of the Exponent?)

Your investment is earning compound interest of 6% per year. You wonder how long it will take to double in value. Answering this question requires you to solve an equation in which the unknown variable is located in the exponent instead of the base position. To find the answer, you need logarithms, which are reviewed in this section and will be applied in Chapters 9 and 11.

Logarithms

A **logarithm** is defined as the exponent to which a base must be raised to produce a particular power. Although the base can assume any value, two values for the base are most often used.

1. ***A Base Value of 10***. This is referred to as a **common logarithm**. Thus, if you have $10^2 = 100$, then 2 is the common logarithm of 100 and this is written as $\log_{10}(100) = 2$, or just $\log(100) = 2$ (you can assume the 10 if the base is not written). The format for a common logarithm can then be summarized as

   log(power) = exponent

   If you have $10^x = y$, then $\log(y) = x$.

2. ***A Base Value of e***. This is referred to as a **natural logarithm**. In mathematics, there is a known constant *e*, which is a nonterminating decimal and has an approximate value of *e* = 2.71828182845. If you have $e^3 = 20.085537$, then 3 is the natural logarithm of 20.085537 and you

write this as ln(20.085537) = 3. The format for a natural logarithm is then summarized as:

ln(power) = exponent

If you have e^4 = 54.59815, then ln(54.59815) = 4.

Common logarithms have been used in the past when calculators were not equipped with power functions. However, with the advent of computers and advanced calculators that have power functions, the natural logarithm is now the most commonly used format. From here on, this textbook focuses on natural logarithms only.

Properties of Natural Logarithms

Natural logarithms possess six properties:

1. **The natural logarithm of 1 is zero**. For example, if 1 is the power and 0 is the exponent, then you have e^0 = 1. This obeys the laws of exponents discussed in Section 2.4 of this chapter.
2. **The natural logarithm of any number greater than 1 is a positive number**. For example, the natural logarithm of 2 is 0.693147, or $e^{0.693147}$ = 2.
3. **The natural logarithm of any number less than 1 is a negative number**. For example, the natural logarithm of 0.5 is −0.693147, or $e^{-0.693147}$ = 0.5. Similar to property 1, this obeys the laws of exponents discussed in Section 2.4, where $e^{-0.693147} = \frac{1}{����^{0.693147}}$, always producing a proper fraction.
4. **A natural logarithm cannot be less than or equal to zero**. Since *e* is a positive number with an exponent, there is no value of the exponent that can produce a power of zero. As well, it is impossible to produce a negative number when the base is positive.
5. **The natural logarithm of the quotient of two positive numbers is ���� � � = ���� ( ) − ���� ( )**. For example, $\ln\left(\frac{\$20{,}000}{\$15{,}000}\right) = \ln(\$20{,}000) - \ln(\$15{,}000)$.

$$\ln(1.3\overline{3}) = 9.903487 - 9.615805$$

$$0.287682 = 0.287682$$

6. **The natural logarithm of a power of a positive base is ���� ( ) = (����)**. This property allows you to relocate the exponent down into the base. You will apply this property especially in Chapters 9 and 11. Demonstrating this principle,

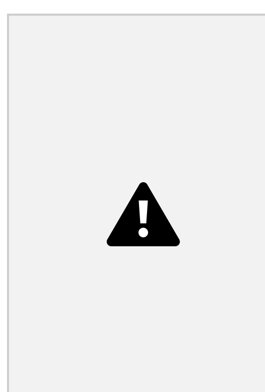
$$\ln(����^{����}) = ����(\ln ����)$$

$$\ln(1.05^6) = 6 \times \ln(1.05)$$

$$\ln(1.340095) = 6 \times 0.048790$$

$$0.292741 = 0.292741$$

Important Notes



Applying natural logarithms on your TI BAII Plus calculator requires two steps.

1. Input the power.
2. With the power still on the display, press the LN key located in the left-hand column of your keypad. The solution on the display is the value of the exponent. If you key in an impossible value for the natural logarithm, an "Error 2" message is displayed on the screen.

If you know the exponent and want to find out the power, remember that e^x = power. This is called the anti-log function. Thus, if you know the exponent is 2, then e^2 = 7.389056. On your calculator, the anti-log can be located on the second shelf directly above the LN button. To access this function, key in the exponent first and then press 2nd e^x.

Paths To Success

You do not have to memorize the mathematical constant value of *e*. If you need to recall this value, use an exponent of 1 and access the e^x function. Hence, e^1 = 2.71828182845.

Give It Some Thought

For each of the following powers, determine if the natural logarithm is positive, negative, zero, or impossible. a. 2.3 b. 1 c. 0.45 d. 0.97 e. −2 f. 4.83 g. 0

g. impossible (property 4)
f. positive (property 2) e. impossible (Property 4) d. negative (property 3)
c. negative (property 3) b. zero (Property 1) a. positive (property 2) 1.
Solutions:

Example 2.6A: Applying Natural Logarithms and Properties

Solve the first two questions using your calculator. For the next two questions, demonstrate the applicable property. a. ln(2.035) b. ln(0.3987) c. $\ln\left(\frac{\$10{,}000}{\$6{,}250}\right)$ d. $\ln[(1.035)^{12}]$

Plan You need to apply the properties of natural logarithms.

Understand

What You Already Know

The properties of natural logarithms are known.

How You Will Get There

a. Apply property 2 and key this through your calculator. b. Apply property 3 and key this through your calculator. c. Apply property 5, ln �����������= ln(����) − ln(����) d. Apply property 6, ln(��������) = ����(ln ����)

Perform

a. ln(2.035) = 0.710496

b. ln(0.3987) = −0.919546

c. $\ln\left(\frac{\$10{,}000}{\$6{,}250}\right) = \ln(\$10{,}000) - \ln(\$6{,}250)$

ln(1.6) = 9.210340 − 8.740336

0.470004 = 0.470004

d. $\ln[(1.035)^{12}] = 12 \times \ln(1.035)$

ln(1.511068) = 12 × 0.034401

0.412817 = 0.412817

Calculator Instructions

Present Organizing your answers into a more common format:

a. $e^{0.710496}$=2.035
b. $e^{-0.919546}$=0.3398

c. ln �$10,000

$6,250 � = 0.470004 (as proven by the property) d. ln[$(1.035)^{12}$] =0.412817 (as proven by the property)

Section 2.6 Exercises

Mechanics

Using your financial calculator, evaluate each of the following. Write your answer using the e^x = power format. Round your answers to six decimals.

1. ln(3.9243)
2. ln(0.7445)
3. ln(1.83921)
4. ln(13.2746)
5. ln(0.128555)

Applications

For each of the following, demonstrate the fifth property of natural logarithms, where ln ����� ���� � = ln(����) − ln(����).

6. ln �$28,500 $19,250�
7. ln �$100,000 $10,000 �
8. ln �$75,000 $12,255 �

For each of the following, demonstrate the sixth property of natural logarithms, where ln(���� ����) = ����(ln ����).

9. ln[$(1.02)^{23}$]
10. ln[$(1.01275)^{41}$]
11. ln[$(1.046)^{34}$]

Using ln�FV PV� ln(1 + ����), substitute the known values and evaluate the expression.

12. 13. 14.

FV	PV	i

$78,23 0	$	$18,97 4	$8,4 95	0.021 75
$233,1 20	$			

Challenge, Critical Thinking, & Other Applications

Using $FV = PV(1 + i)^N$, substitute in the known values and solve for N.

15. 16.

FV		$58,870. 20	$36,880	0.0 1
$18,302. 77	$			

Chapter 2 Summary

Key Concepts Summary

Section 2.1: Order of Operations (Proceed in an Orderly Manner)
• A review of key mathematical operator symbols
• Rules for order of operations are known as BEDMAS
Section 2.2: Fractions, Decimals, and Rounding (Just One Slice of Pie, Please)
• The language and types of fractions
• Working with equivalent fractions by either solving for an unknown or increasing/reducing the fraction • Converting any fraction to a decimal format
• Procedures for proper rounding
• The rounding rules that are used throughout this textbook
Section 2.3: Percentages (How Does It All Relate?)
• Converting decimal numbers to percentages
• Working with percentages in the form of rates, portions, and bases
Section 2.4: Algebraic Expressions (The Pieces of the Puzzle)
• Learning about the language of algebra
• The rules for manipulating exponents
• The rules of algebra for addition and subtraction
• The rules of algebra for multiplication
• The rules of algebra for division
• What is substitution and how it is performed?
Section 2.5: Linear Equations: Manipulating and Solving (Solving the Puzzle)
• A review of key concepts about equations
• The procedures required to solve one linear equation for one unknown variable
• The procedures required to solve two linear equations for two unknown variables
Section 2.6: Natural Logarithms (How Can I Get That Variable Out of the Exponent?)
• A review of the logarithm concept
• The rules and properties of natural logarithms

The Language of Business Mathematics

algebraic equation An equation that takes two algebraic expressions and makes them equal to

each other. **algebraic expression** Indicates the relationship between and mathematical operations that must be conducted on a series of numbers or variables.
base The entire amount or quantity of concern.
BEDMAS An order of operations acronym standing for **B**rackets, **E**xponents, **D**ivision, **M**ultiplication, **A**ddition, and **S**ubtraction.
common logarithm A logarithm with a base value of 10.
complex fraction A fraction that has fractions within fractions, combining elements of compound, proper, and/or improper fractions together.
compound fraction A fraction that combines an integer with either a proper or improper fraction.
denominator Any term by which some other term is divided; commonly the number on the bottom of a fraction. **divisor line** The line that separates the numerator and the denominator in a fraction.
equivalent fractions Two or more fractions of any type that have the same numerical value upon completion of the division.

exponent A mathematical shorthand notation that indicates how many times a quantity is multiplied by itself **factor** Components of terms that are separated from by multiplication or division signs.
fraction A part of a whole.
improper fraction A fraction in which the numerator is larger than the denominator.
left side of the equation The part of an equation that is to the left of the equal sign.
like terms Terms that have the same literal coefficient.
linear equation An algebraic expression in which the variable has an exponent of 1; when plotted, it will form a straight line.
literal coefficient A factor that is a variable.
logarithm The exponent to which a base must be raised to produce a particular power.
monomial An algebraic expression with only one term.
natural logarithm A logarithm with a base value of the mathematical constant *e*.
nomial The number of terms that appear in an algebraic expression.
nonlinear equation An algebraic expression in which the variable has an exponent other than 1; when plotted, it will not form a straight line.
numerator Any term into which some other term is being divided; commonly the number on the top in a fraction. **numerical coefficient** A factor that is numerical.
percentage A part of a whole expressed in hundredths.
polynomial An algebraic expression with two or more terms.
portion Represents part of a whole or base.
proper fraction A fraction in which the numerator is smaller than the denominator.
rate Expresses a relationship between a portion and a base.
right side of the equation The part of an equation that is to the right of the equal sign.
root The value of the unknown variable that will make a linear equation true.
substitution Replacing the literal coefficients of an algebraic expression with their numerical values. **term** In any algebraic expression, the components that are separated by addition and subtraction. **triangle technique** A memorization technique that displays simple multiplication formulae in the form of a triangle. Anything on the same line is multiplied, and items above or below each other are divided to arrive at various solutions.

The Formulas You Need to Know

Symbols Used

% = percentage
dec = any number in decimal format
ln = natural logarithm
Rate = the relationship between the portion and base
Portion = the part of the whole
Base = the whole quantity

Formulas Introduced

Formula 2.1 Percentage Conversion: % = dec × 100 (Section 2.2)

Formula 2.2 Rate, Portion, Base: $\text{Rate} = \frac{\text{Portion}}{\text{Base}}$ (Section 2.2)

Technology

Calculator

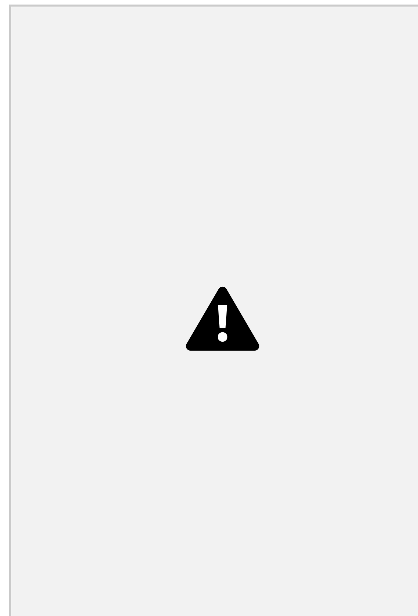
<u>Formatting Instructions</u>

Buttons Pushed	Calculator Display	What It Means
2nd Format	DEC=2.00	You have opened the Format window to its first setting. DEC tells your calculator how to round the calculations. In business math, it is important to be accurate. Therefore, we w set the calculator to what is called a floating display, which means your calculator will carry all of the decimals and display as many as possible on the screen.
9 Enter	DEC=9.	The floating decimal setting is now in place. Let us proceed.
↓	DEG	This setting has nothing to do with business math and is just left alone. If it does not read DEG, press 2nd Set to toggle it.
↓	US 12-31-1990	Dates can be entered into the calculator. North Americans and Europeans use slightly different formats for dates. Your display is indicating North American format and is acceptable for our purposes. If it does not read US, press 2nd Set to toggle it.
↓	US 1,000	In North America it is common to separate numbers into blocks of 3 using a comma. Europeans do it slightly differently. This setting is acceptable for our purposes. If your display does not read US, press 2nd Set to toggle it.

↓ Chn	There are two ways that calculators can solve equations. This is known as the Chain method, which means that your calculator will simply resolve equations as you punch it in without regard for the rules of BEDMAS. This is not acceptable and needs to be changed.
2nd Set AOS	AOS stands for Algebraic Operating System. This means that the calculator is now programmed to use BEDMAS in solving equations.
2nd Quit 0.	Back to regular calculator usage.

Exponents and Signs

x^2 is used for exponents that are squares. 2^2 is keyed in as 2 x^2.
y^x is used for exponents that are not squares. 2^3 is keyed in as 2 y^x 3 =.
± is used to change the sign of a number. To use, key in the number first and then press the ± key.



Memory

STO Store
RCL Recall
0–9 Memory cell numbers (10 in total)
To store a number on the display, press STO # (where # is a memory cell number).
To recall a number in the memory, press RCL # (where # is the memory cell where the number is stored).

Natural Logarithms

The natural logarithm function, LN, is located in the left-most column of your calculator. To use this function, input the power and then press the LN button. The solution displayed is the exponent. To calculate the power, input the exponent and then press 2nd e^x (called the anti-log).

Chapter 2 Review Exercises

Mechanics

1. Solve the following expression: $\left(1 + \frac{0.0725}{4}\right)^{\frac{12}{2}} - 1$

2. Convert the following into a decimal format and express it in an appropriate manner: $4\ {}^{12}{}_{37}$
3. In Canadian five-pin bowling, the maximum score per game is 450. Alexandria just finished bowling a game of 321. What percentage of a perfect score does this represent?
4. Simplify the following algebraic expression: $^{16����}{}_{2}$

$$4���� - \frac{21����^{3}����^{2}}{7����^{2}����^{2}}$$

5. Solve the following equation for *r*: $-2���� - 2.25 = 3.75 - 8����$
6. Demonstrate that $3 \times \ln(2) = \ln(2^3)$.

Applications

7. Solve the following expression: $(0.06)(\$1{,}725)$ $\cancel{\frac{1}{}}�^{-7}$

� 1 − � 1 + 0 . 1 4 �

$\cancel{0.14}$

4

8. Evaluate the following: $�^{1+}$ $\frac{0.0975}{\cancel{4}} \times \$3{,}225$ $\underline{\$255}$ $1+\frac{\cancel{0.0975}}{4}$ �

9. A 240 mL bottle of an orange drink claims that it is made with real fruit juice. Upon examination of the ingredient list, only 15% of the contents is actually fruit juice. How many millilitres of "other ingredients" are in the bottle?

10. On average across all industries, full-time employed Canadians worked 39.5 hours per week. If the typical adult requires eight hours of sleep per day, what percentage of the hours in a single week are left over for personal activities?

11. Substitute the given values in the following equation and solve:

$���� �^{(1+)����-1}�$ where *a* = \$535, *b* = 0.025, and *c* = 6

12. Simplify the following algebraic expression: $�^{15����^{7}{}_{3}����^{2}}$ $5����^{5}���� �^{3} \div �^{9����_{4}6����^{5}}$ ^{3} $\frac{3����^{\cancel{2}2}����}{2}$ ^{3} �

13. Simplify the following algebraic expression: $\mathrm{PMT}\ �^{1-(1+0.0225)_{6}(1+0.044)^{-6}}$ $0.044 - 0.0225$ $� + \mathrm{PMT}\ �^{1}$ $0.044 + 1�$

14. Housing starts in Winnipeg for the first half of the 2009 year were down 46.733% from 2008, when 1,500 new homes were started. How many housing starts were there in 2009?

15. A local baseball team sells tickets with two price zones. Seats behind the plate from first base to third base are priced at $20 per ticket. All other seats down the base lines and in the outfield are priced at $10 per ticket. At last night's game, 5,332 fans were in attendance and total ticket revenue was $71,750. How many tickets in each zone were sold?

Challenge, Critical Thinking, & Other Applications

16. An industry analyst wants to compare the average salaries of two firms, both to each other and to the industry. Firm A's average salary is 93% of the industry average, Firm B's average salary is $58,000, and the industry average salary is 96% of Firm B's average salary.
    a. Determine the industry average salary.
    b. Determine Firm A's average salary.
    c. Express Firm B's average salary as a percentage of Firm A's average salary. Round the percentage to two decimals. $2^{4/12}-1$ $^{4}-1$

$1+$ $1+^{0.07}$

17. Solve the following expression: $500 $2^{4/12}-1$ $1+^{0.07}$

18. Francesca operates an online flower delivery business. She learned that in 2012 there were an estimated 324,000 online orders in her region, which represented a $\frac{1}{12}$ increase over the previous year. Her business received 22,350 online orders in 2012, which represented a $\frac{3}{14}$ increase over her previous year. In 2011, what percentage of online orders were placed with Francesca's business?

19. Goodyear Tires just completed a "Buy Three Get One Free" promotion on its ultra-grip SUV tires, regularly priced at $249.99 per tire. Over the course of the promotion, 1,405 tires were sold resulting in sales of $276,238.95. How many tires were sold at the regular price and how many tires were sold at the special promotional price?

20. The current price of $41.99 for a monthly cellphone rate plan is 105% of last year's rate plan. There are currently 34,543 subscribers to the plan, which is 98% of last year. Express the current total revenue as a percentage of last year's total revenue. Round the final answer to one decimal.

Chapter 3: General Business Management Applications

(Get Your Motor Running)

Basic calculations are so much a part of your everyday life that you could not escape them if you tried. While driving to school, you may hear on the radio that today's forecasted temperature of 27°C is 3°C warmer than the historical average. In the news, consumers complain that gas prices rose 11% in three months. A commercial tells you that the Toyota Prius you are driving uses almost 50% less gas than the Honda Civic Coupe Si. After class you head to Anytime Fitness to cancel a gym membership because you are too busy to work out. You have used only four months of an annual membership for which you paid $449, and now you want to know how large a prorated refund you are eligible for.

On a daily basis you use basic calculations such as averages (the temperature), percent change (gas prices), ratios and proportions (energy efficiency), and prorating (the membership refund). To invest successfully, you must also apply these basic concepts. Or if you are an avid sports fan, you need basic calculations to understand your favourite players' statistics.

And it's not hard to see how these calculations would be used in the business world. Retail management examines historical average daily sales to predict future sales and to schedule the employees who will service those sales. Human resource managers continually calculate ratios between accounting and performance data to assess how efficiently labour is being used. Managers proration a company's budget across its various departments.

This chapter covers universal business mathematics you will use whether your chosen business profession is marketing, accounting, production, human resources, economics, finance, or something else altogether. To be a successful manager you need to understand percent changes, averages, ratios, proportions, and prorating.

Outline of Chapter Topics