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# Introduction to Financial Analysis

# Introduction to Financial Analysis

Kenneth S. Bigel

OPEN TOURO
NEW YORK, NY

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# About the Author

**Dr. Kenneth Bigel**

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    - Campus Chair, Business Department

Dr. Bigel was formerly a fixed income analyst in the International Banking Department of the Bankers Trust Company (now DeutscheBank), analyzing international wholesale loans and debt instruments, and a graduate of its Institutional Credit Training Program. He later was affili-

ated with the Ford Motor Company, conducting investment analysis and planned car profits analysis, annual budgeting, and strategic planning. Subsequently, he worked as a senior portfolio manager attached to the wealth management division of Prudential Securities. He was formerly registered under Series 3, 7, 15, 24, 63, and 65.

As an independent consultant, he was involved in numerous high-profile cases including Enron. Dr. Bigel has conducted executive education programs for Morgan Stanley Capital Markets, Merrill Lynch Capital Markets, UBS, Lehman Brothers, CIBC, G.X. Clarke & Co. (now part of Goldman Sachs), and China CITIC Bank. He currently serves on the Financial Industry Regulatory Association's Board of Arbitrators.

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Dr. Bigel has been interviewed on American radio, was a visiting scholar at Sichuan University and at Xi'an Jiaotong University in China, and appeared on Chinese television. At Touro University, he is a member of the Faculty Senate, The Touro Academy of Leadership and Management, The Assessments Committee, and The Promotions Committee. He chairs the Integrity Committee at LCM.

His wife and their three children reside in New York City. He enjoys reading, playing 60's guitar, seeing his students succeed professionally, and watching his kids grow.

## Educational Background:

- Ph.D., (high honors) New York University, Steinhardt School of Culture, Education, and Human Development (Business Education and Financial Ethics)
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- Hebrew University of Jerusalem (one-year program)

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This work started out as class notes and gradually developed into the product before you. The questions and thoughts of my students are embedded in these pages. I learned new perspectives to the material from students. These questions often resulted in my penning wholly additional pages concerning issues that had not occurred to me and which demanded development. I wrote this book for you. Thanks, guys!

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squeeze in some) and for not tucking them in at night. They came out alright, thanks to the wonderful love and unceasing care of their mother, my dear wife, Mira.

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***Dr. Kenneth S. Bigel***

Touro University
The Lander College for Men
2022 March

# Open Touro Acknowledgements

Open Touro was founded by Touro College Libraries in 2018, as the Open Educational Resources initiative of Touro College & University System (now Touro University). Open Touro aims to reduce the cost of textbooks for students by promoting innovative and equitable open education practices. Open Touro initially launched through the vision and efforts of the Library's Information Literacy Director, Sara Tabaei, and Open Educational Resources (OER) Librarians Juliana Terciotti Magro, and Georgia Westbrook. The initiative is currently co-led by Ms. Tabaei, Scholarly Communication & Data Librarian Timothy Valente, and OER & Instruction Librarian, Kirk Snyder, with the help and support of many others across Touro.

*Introduction to Financial Analysis*by Kenneth S. Bigel, is Open Touro's first original OER publication. Further collaborations with faculty author Dr. Bigel are forthcoming, the next of which, *Corporate Finance*, is currently in production.

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# Preface

Unique Pedagogical Style

This text is written so that the reader can absorb relatively small bytes of information at a time without ever feeling overwhelmed. Each page is short and has a unique topical heading on which the student can focus and easily retain. The paragraphs are kept short, just as you will note they are on this page, to enable the reader to pause, take a breath, and review in his/her mind what s/he just read before going on. Occasionally, concepts and explanation are repeated in consecutive paragraphs in order to present an idea from multiple perspectives and to deepen one's comprehension.

The writing style attempts to avoid jargon, except where necessary. In such cases, the terminology is explained so that the reader may proceed with clarity. Still, there are many words and phrases that one must acquire in Finance. At times, vocabulary words are highlighted in the margins.

Readers are advised to simultaneously press on the "CTRL" and "F" buttons at which time a window will open in which a search word may be entered. This will enable the reader to go back and review concepts at will.

The chapters are organized and written so that the reader get directly to the point, without unnecessary information and "fluff." There is a certain flow to the chapters and sections within the chapters that is intended to make learning smooth and enjoyable.

Concepts and Computations

Financial Analysis, by and large, incorporates Accounting concepts. Thus, it is imperative that the analyst understand the numbers, which s/he will utilize in his/her analysis. This text exposes the reader to the many shortcomings in reading Financial Statements. No prior knowledge of Accounting is assumed. The text, in its early chapters, will tenderly lead the reader through the arcane world of Accounting and point out the many shortcomings in reading the statements and in conducting elementary Financial Analysis.
The reader should have some facility with very basic Algebra, in particular, s/he should be able to solve for an unknown in a simple formula, to be able to transpose a value from one side of an equation to the other, and to be to calculate exponents and roots. The text will make Algebra easy by showing, where necessary, the correct solutions step-by-step.

The text will proceed to Ratio Analysis, the very basic tools of financial analysis, incorporating the previously learned Accounting data. It will then proceed to the notion of the Time Value of Money, which is the central concept in all of finance. The text will conclude with Stock and Bond Valuation, which are based on all the previous information of the text. Thus, the reader will build upon his/her knowledge by going from concept to concept in smooth, linear fashion, ever reaching for higher and higher planes of knowledge.

<span style=”text-decoration: underline;”The Nuances of Financial Analysis

Financial Analysis, at the end of the day, is just common sense, or common business sense. Financial and Mathe-

matical principles must conform to the realities of the field and not the other way around. The mathematics are a tool and not an end. In a sense, a financial analyst is bi-lingual; a student will translate financial principles into formulae when advised and can explain in plain English the meaning and application of such formulae when first presented with one. It is all supposed to make sense. This book tries to capture this dual and essential nature of Finance. As the reader goes through the text, s/he should increasingly gain a sense of empowerment, confidence, and mastery of the subject.

Quotations

The text is replete with quotations from an eclectic myriad of sources including the Bible, the Talmud, great Greek philosophers, famous politicians, modern and popular thinkers from the 18th century onward, and more. The quotations are not intended to postulate or promote a point-of-view, especially where religion is concerned, but instead to inspire the reader to persevere in his/her study, to continue toward the achievement of great heights, and to always consider the social impact of his/her actions. Mastery of Finance presents a person with the opportunity to better oneself in so many ways while simultaneously bettering society-at-large. And that is very cool.

Problem-solving

In virtually each chapter and ends-of chapters, the reader will find formulae to be solved, tables that need to be filled-in, and occasional diagrams to be drawn. The intent is to make the material come alive so that the student can both learn and test him/herself in the process. Solutions are generally provided to all these fill-ins so that the reader may

verify whether s/he resolved the problem correctly and may correct any errors in so doing.

Forthcoming in Dr. Bigel's Basic Finance Series:

*Corporate Finance*

*Securities Markets and Instruments*

*Introduction to Fixed Income Mathematics*

# Part I. Financial Statements and Ratio Analysis, and Forecasting

Part I. Financial Statements and Ratio Analysis, and Forecasting

Chapter 1: Introduction

Chapter 2: Financial Statements Analysis: The Balance Sheet

Chapter 3: Financial Statements Analysis: The Income Statement

Chapter 4: Financial Statements and Finance

# Chapter 1: Introduction

## 1.1 Chapter One: Learning Outcomes

Learning Outcomes

In this chapter, you will:

- **Imagine** what a corporation is, its purpose, and how it is organized.
- **Identify** the place of the Finance function within the corporation.
- **Distinguish** between abstract and concrete reasoning.
- **Formulate** abstract hypotheses and statements.
- **Think** deliberatively as a financial professional.

## 1.2 The Corporation

*There are two ways of being happy: We must either diminish our wants or augment our means – either may do –the result is the same and it is for each man to decide for himself and to do that which happens to be easier.*

-Benjamin Franklin

What is a corporation? You may have noticed that the Latin word "corpus" seems to appear within it. Indeed, it is a body! It is not human or animal, and it has no physical shape. You cannot see it or touch it. It will have components that take physical form, such as a building or inventory, but the corporation itself is non-corporeal.

So, what is it? It is a legal entity. It exists only as a legal construct. As such it is said that the corporation is a "person" under the law. It exists in a legal sense. It can be sued. It can be fined. It is owned by people who purchase ownership interests in it. These interests or "claims" are called "shares" or "stock." Owners are referred to as "shareholders." Shareholders have a claim on the company's profits. We may thus also refer to this type of corporation as a "stock corporation." It is in business – generally speaking –

to make money for its shareholders, although it may serve other more altruistic purposes as well.

The corporation, thus, as an independent person, is legally separate from the owners. The corporation may be sued for damages, but the owners may not be – unless the court determines that the owner is somehow legally liable for a wrong-doing himself and apart from the separate actions of the corporation. Therefore, the owners are protected from legal responsibilities. This does not absolve corporate managers from legal malfeasance if they did other wrongs, e.g., dumping waste illegally.

One downside to the owners is that the corporation itself is a taxable entity. It pays income taxes and then the shareholders, once again, will pay income taxes on any profits distributed to them. These profit distributions are called "dividends." We class this "double taxation."

Of course, there are numerous other means by which a company may be organized in order to avoid double-taxation, but not all will provide the umbrella protection that the corporation provides. You can learn about these business forms in a Management or Tax course.

## 1.3 Business / Corporate Structure: The Management Organization

It is well and fine, and critical, to learn what the Finance discipline's intellectual landmarks are, but the student certainly wants to know how Finance fits into the actual corporate (business) context, which is the focus of this text. Here we shall see.

The corporation will have both an "organizational structure," detailing the manner in which the firm actually operates and a capital structure, which is depicted on the Balance Sheet. We will get to the Balance Sheet soon. First, the organization.

There are four-six basic business functions in the organization:

| | |
|---|---|
| **Marketing**<br>• Market Research<br>• Product Development<br>• Advertising<br>• Sales | **Operations**<br>• Plant Management (Factory/ Warehouse)<br>• Inventory<br>• Cost Accounting<br>• Statistics and Operations Research |
| **Management**<br>• Human Resources<br>• Employee Benefits<br>• Legal and Compliance | **Finance**<br>• Treasurer's Office<br>• Controller's Office<br>(See below for details) |
| **Research and Development (R&D)** | **Management Information Systems (MIS)** |

## 1.4 The Finance Function Within the Corporation

The controller and treasurer of a company serve different functions, although these differences may vary from firm to firm. Both report to the vice president of finance or chief financial officer (CFO). One of the two may also be the CFO, more probably the treasurer. There will be some variance in structure from company to company. The CFO reports to the Chief Operating Officer (COO) or to the Chief Executive Officer (CEO).

Here are some *typical* functions of each.

| Chief Operating Officer / Chief Executive Officer | |
|---|---|
| **Chief Financial Officer** | |
| **Treasurer's Office** | **Controller's Office** |
| Cash Management<br>• Collections<br>• Disbursements (including payroll)<br>• Short Term Cash Management<br>• Credit and Accounts Receivable<br><br>External Financing<br>• Borrowing<br>• Capital Structure decisions<br>• Investor Relations<br><br>Budgeting & Financial Planning<br>• Capital Budgeting<br>• Investment Analysis and Profit Planning<br>• Performance Analysis<br><br>Risk Management<br>• Interest Rate Risk Management<br>• Insurance<br>• Foreign Exchange Risk Management<br><br>Employee Benefits<br>• Retirement<br>• Health<br>• Training and Professional Education<br>• Maintain relations with banks and other lenders | Accounting<br>• External Audit and Financial Reporting<br>• External Audit Relations<br>• Internal Audit<br>• Tax<br>• Budgeting<br>• SEC filings<br>• Management Information Systems<br><br>Payables<br>• Payroll<br>• Accounts Payable |

## 1.5 Capital Structure

Above, we discussed the firm's organizational structure. This is how corporations operate. The firm will also have a "Capital Structure," which will be represented on its Balance Sheet. The Balance Sheet will consist of Assets, Liabilities and Owners' Equity.

Assets are what the company owns, including inventory, plant and equipment, among other items. Liabilities are what the company owes to others including suppliers and lenders. Equity is the value of what the owners have invested in the company.

Companies acquire *Capital* (Liabilities + Equity) in order to, in turn, "finance" (i.e., pay for) the acquisition and maintenance of its assets. Assets, in turn, are exploited to produce sales, which will – hopefully – deliver profits and a return to the shareholders, who are the owners of the corporation.

The basic "accounting equation" is: Assets equals Liabilities plus Equity, or A = L + E. A Balance Sheet must, well, balance, as noted here.

| | |
|---|---|
| Assets | Liabilities |
| | Owners' Equity |

Assets will be on the left and Liabilities plus equity will be on the right – like the Ten Commandments! In general,

the word “Capital” will refer to the right side of the Balance Sheet. The firm’s Capital is not free; it has an economic cost; lenders expect interest on its loans to the company and shareholders expect dividends and the growth of dividends of their equity investment in the firm. The economic cost of the firm’s capital represents the return to lenders and stock investors. Where there is a return to investors (lenders and owners), there must be a cost to the corporation who provides the return. Two sides to the same coin.

In order to be competitive, a corporation must also cover its “Opportunity Cost.” If an investor in the corporation can earn a better return in an equivalent alternative investment, s/he will choose the better alternative. This is a basic principle of Economics. The corporation, in order to be able to attract investment, must therefore cover its Opportunity Costs, i.e., the alternative return an investor gives up when making an investment in this corporation.

We will discuss the Balance Sheet further in depth on the pages to follow. For now, let’s re-wire our brains so that we think like Financial Economists.

## 1.6 Thinking Like an Economist: Abstraction

Economics, and its offspring, Finance, are **abstract** (social) sciences. In order to study economics, it is essential that one understands what an **abstraction** is.

Key Terms:

Abstract

Abstraction

Simplification

An abstraction is an idea, intended to mirror reality in its simplest form. The world is a very complicated place; there are many variables or inputs, some identifiable, others not, that affect an outcome, and which we endeavor to identify. In order to understand the outcome which is generally true, but not necessarily absolutely or always so, one must engage in a process of **simplification** that requires removing minor variables from a general idea in order to reduce the notion to its essential characteristics. Indeed, the Latin word, "abstract," comes from "drawing" or "taking away from."

*The ever-changing kaleidoscope of raw reality*

> *would defeat the human mind by its complexity,*
> *except for the mind's ability to abstract,*
> *to pick out parts and think of them as a whole.*
>
> Thomas Sowell
>
> *A Conflict of Visions* (2002), p. 5

In this process, one is able to derive a broad, general conclusion, based on **first principles** from which is derived a general idea or rule. In economics, this requires a ***ceteris paribus*** assumption, that is to say, holding "**all else equal**." Initially, it is assumed that no other variables matter and are thus ignored away. It takes some discipline to do this at first, but it quickly becomes easy; you must merely keep it in mind.

First Principles

*Ceteris Paribus*

All else equal

In circumstances where little given or known information may be at hand, one must assume reasonable **default assumptions**, i.e., **premises**, which make sense in general and in the simplest, most common form possible. While it may be facile to

Default Assumption

Premise

imagine other considerations, or variables, that may come into play, one must avoid doing this, in order to focus on just a few key variables, which affect the outcome. Here is a relevant comment by Dr. Milton Friedman[1]:

> *A hypothesis is important if it "explains" much by little, that is, if it abstracts the common and crucial elements from the mass of complex and detailed circumstances surrounding the phenomena to be explained and permits valid predictions on the basis ofthem alone. To be important, therefore, a hypothesis must be descriptively false in its assumptions; it takes account of, and accounts for, none of the many other attendant circumstances, since its very success shows them to be irrelevant for the phenomena to be explained….*

Dialectical Reasoning

Mode of Reasoning

Thesis/ Antithesis/ Synthesis

To be sure, Dr. Friedman's comments are not without criticism, but nevertheless are adhered to, by and large, in the social sciences, including, of course, economics. In certain other disciplines, including law, political theory, history, philosophy, and possibly others, we do not engage in this **mode of reasoning**. In these fields, by contrast, we are often engaged in **dialectical reasoning**. There, we first state a **thesis**, then examine its **antithesis**. We go back and forth numerous times until we can arrive at a **synthesis**, which is conclusive or dispositive.

1. Friedman, Milton. (1953). Essays in Positive Economics. Chicago: University of Chicago Press. pp. 14-15.

Those of you that are accustomed to dialectical reasoning must diligently avoid the natural instinct to quickly imagine the antithesis; rather you must remain steadfast to the line of reasoning demanded by the more linear manner of abstract argumentation, based on first principles and ceteris paribus delimitations. (A “delimitation” is a limitation that one person herself imposes on the scope of her reasoning.)

When, in certain instances, we deviate from abstract reasoning, we assume specific, more **descriptive**, circumstances in a **contextual** or **“concrete”** manner. The result may not be **generalizable**. If a conclusion is generalizable, we say that it is true in the overwhelming number of instances, although not necessarily all.

Descrip-
tive

Contextual

Concrete

Generaliz-
able

For instance, it is known via mathematics and reason, that interest rates and stock prices are inversely related. Rates go up, prices go down. This is true beyond any doubt – if we assume ceteris paribus. At the same time, it is more than just interest rates alone that affect stock prices.

One of the beauties of abstract reasoning is that it enables us to employ other, more reasonable assumptions when we find that a model has poor predictive power.

Happy travels!

*Religion without science is blind, science without religion is lame.*

– Albert Einstein

## 1.7 Abstraction: Absurd AND Necessary

You thought we were done talking about abstraction. Sorry – one last discussion. It is that important. Here's a relevant joke:

A chemist, a physicist, and an economist are stranded on a desert island. They have an ample supply of canned food, but alas, no can opener.

The chemist suggests that they should light a fire under the cans so that they would burst open.
The physicist suggests dropping the cans from the cliff to its rocky bottom to smash them open.

The economist declares: "Let's assume we have a can opener."

---

Financial and economic theory makes, what may appear at first, some absurd assumptions. But that is only because the social world, the world inhabited by people, is far more complex, in many ways, than the real world, the world of the hard sciences.

Are the chemist's molecules motivated by fear and greed? Will ambition or altruism affect the trajectory of the falling cans? Economists cannot keep track of every alternative that the human mind may consider, so it abstracts by looking into the outcomes or choices that are usually indepen-

dent of human foibles, or dare I say, are “logical.” Without abstraction, economists would never arrive at any generalizations. We would therefore learn nothing! Zilch.

Suppose you just arrived in New York City for the first time. If you wish to find Times Square, would you use a map (imagine that there is no GPS or Waze) that details all the streets, or just the main arteries? No! You would not be concerned with all the confusing, and mostly useless, details.

A burgeoning field, Behavioral Economics and Finance, deals with the human condition and its interaction with traditional, “objective” economics. However, first things first.

*Being ignorant is not so much a shame, as being unwilling to learn.*

-Benjamin Franklin

*Instruct me and I shall be silent. Make me understand where I have erred.*

-Book of Job (6:24) (Artscroll translation)

# 1.8 Modes of Reasoning: Dialectical versus Analytic

What is Dialectical Reasoning?

1. The process of arriving at the truth by stating a thesis, developing a contradictory antithesis based on concrete possibilities, and combining them into a coherent synthesis, often after numerous variations and iterations.
2. A method of "argument" or exposition that systematically weighs an idea with a view to the resolution of its real or apparent contradictions.

How to Engage in Abstract or Analytic Reasoning

Analytic argumentation differs markedly from Dialectical. The following pertains to the Analytic method only. Keep it in mind.

1. Analytic reasoning commences with a first principle or assumption. On this foundation, an "argument" is built.
    - Assumptions must be reasonable and generally true.
2. An argument is not a debate. Do not start with an oppositional counter-statement. It is not about "winning."

3. Do not spar with the argument. First try to understand it.
    - Taking a contrary position will not assist you in understanding the proposition or argument better.
4. Arguments are often nuanced, not black and white.
5. An argument is not an opinion. The latter is subjective.
6. In college (and later in life), an opinion is not necessarily a "right" to which a student is "entitled." Sound reasoning, i.e., a good argument, trumps opinion.
7. Any position you take must be based on sound argument.

> *The Talmudic method invariably prefers to pose questions in a*
>
> *concrete rather than an abstract form.*
>
> –Rabbi Adin Even-Israel Steinsaltz
>
> *Reference Guide to the Talmud*, p. 131 (2014)

We can safely assert that if one applies a lit match to a piece pf paper, the paper will burn. This is true in general and assumes that all else is equal. BUT what IF it is raining? What IF the paper is treated with a fire retardant? Do these,

somewhat absurd violations of our ceteris paribus postulate negate the veracity of the general principle that paper burns? Here the word “BUT” is oppositional. The word “IF” creates a concrete context, a narrow circumstance, with the, often unconscious, purpose of finding reason to deny a generally sound principle, e.g., that paper burns.

## 1.9 Finance Style

Students very often will come to their first finance class well-read in an impressive host of subjects, including history, literature, and more, but not necessarily in economics or finance.

Finance, as certain other “scientific” subjects, is written in a markedly different style; it is notably terse and succinct. Words are not parsed, and expansive, colorful prose is consistently eschewed. Sparse, precise language is the rule.

Unlike literature, for instance, one may find that s/he has to read the same sentence several times until s/he gets it. You just can’t put your feet up, and slice through many pages in relatively short order.

Do not get frustrated when this happens. Kick around the notions discussed in your head, until you get it. Then kick it over again; you may find that you can see the same thing from different angles with increasing thought. That should invigorate you. Start thinking in Abstract Terms – like a Financial Economist.

Spend extra time on each paragraph and page. You may find that you have to substantially slow down the pace of your reading.

As formulae are presented, carefully check the calculations to be sure you agree. You will find this extremely helpful

in increasing your understanding and insight. Always keep your calculator handy as you read.

Be methodical and take your time. You will find that you will adjust to the new style, and you will find enjoyment in your increasing mastery! Think deliberately. Don’t think too fast!

# Chapter 2: Financial Statements Analysis: The Balance Sheet

## 2.1 Chapter Two: Learning Outcomes

Learning Outcomes

In this chapter, you will:

- **Define** each Balance Sheet account.
- **Calculate** the basic Accounting Equation.
- **Identify** debit and credit entries for the Balance Sheet.
- **Rank** the items in the Financial Claims Hierarchy.
- **Trace** the link from the Income Statement to the Balance Sheet.

## 2.2 The Finance in the Financial Statements

Why do we care about Financial Statements in a Finance course? Finance begins where the Certified Public Accountant's job ends. The accountant's job is to carefully examine the company's financial records (its "books") in order first to determine their accuracy and veracity. The accountant will then simplify the data and summarize them into three Financial Statements: The Balance Sheet, The Income Statement, and the Cash Flow Statement. In this text we will deal only with the first two statements.

The accountant does not have completely free rein regarding the manner in which the financial data are summarized. S/he must abide by "Generally Accepted Accounting Principles", also known simply as GAAP. This is the rulebook for the accounting profession. GAAP rules are set by the accounting profession's rule-making body, the Financial Accounting Standards Board (FASB). The accounting profession in turn, derives its legal status from a federal government organization called the Security and Exchange Commission or "SEC." By law, the SEC empowers the accounting profession to make its own rules and to police the rules – with the SEC's oversight. As many of you may already know, the SEC also oversees the United States' financial markets.

All Financial Statements, including the Balance Sheet, will be provided to lenders who will examine the statements

prior to making any lending determinations. “Public Companies,” i.e., corporations whose stock is “traded” (bought and sold) on a public stock exchange where stock is bought and sold, are required to release their statements to anyone who requests them. Again, this is an SEC requirement.

The skilled financial analyst will then read the statements because s/he is an “interested party” and wants to know whether an investment in the company is well and fine or whether a potential investment may be advised. S/he may represent lenders or equity shareholders; either party may be considered “investors.” Reading the statements requires advanced education concerning how the accountant compiled the statements. GAAP rules are quite complex.

In summary, the accountant is a trained historian of sorts. The financial analyst will read the accountant’s end-product but is more future oriented. The latter is only concerned about how a potential investment will perform in the future.

## 2.3 The Balance Sheet

The Balance Sheet presents a static, unchanging, still-photograph of a company's financial position at a moment in time, i.e., "as of" a certain date, often December 31st. In theory, any of the figures on the balance sheet would be different, either larger or smaller, on the very first day following (or before) the statement date. The Balance Sheet is usually issued every quarter, i.e., every three months. A very simple balance sheet will look something like the following (with the numbers filled in). Take note that any actual Balance Sheet you may examine may differ from this simple example.

XYZ Corp. Balance Sheet as of 12.31.XX

| | | |
|---|---|---|
| Cash and Equivalents | | Accounts Payable |
| Inventories | | Short-term Debt |
| Accounts Receivable | | *Total Current Liabilities* |
| *Total Current Assets* | | Long-Term Debt |
| | | *Total Liabilities* |
| Property, Plant & Equipment | | Preferred Stock |
| Total Long-term Assets | | Common Stock |
| | | Retained Earnings |
| | | *Total Owners Equity* |
| *Total Assets* | | *Total Liabilities + Equity* |

All Asset accounts are "debit balance" accounts. That means that when the account increases (decreases), the amount is recorded on the debit (credit) side of the firm's ledger. Liability and Equity Accounts are "credit balance" accounts. We make these entries into the bookkeeper's ledger's "T-Accounts" (see immediately below). This

mechanistic framework is fundamental to “double-entry book-keeping.”

| Assets | | Liabilities + Equity | |
|---|---|---|---|
| Dr | Cr | Dr | Cr |
| + | - | - | + |

Let’s repeat this: Asset accounts are “debit balance” (debit = “Dr”) accounts, whereas liability and owners’ (or shareholders’) equity are “credit balance” (credit = “Cr”) accounts. Increases (decreases) in asset accounts are characterized by debit (credit) entries; increases (decreases) in liability and equity accounts are characterized by credit (debit) entries.

In “double-entry” bookkeeping, for every debit entry there must be a credit entry. Debits must equal credits, and the balance sheet must balance: Total Assets = Total Liabilities + Equity. This is the basic accounting equation.

<u>Basic accounting equation</u>: The following equations must, by definition, be true:

A = L + E (Assets = Liabilities plus Equity)

A – L = E (Assets minus Liabilities = Equity)

There are also contra-asset accounts (such as accumulated depreciation and allowance for uncollectible accounts receivables), which are “credit balance” (credit = Cr) accounts. These contra-accounts effectively reduce the (net) asset accounts and are credit entries.

The phrase “current,” as in “current assets,” has to do with the life of the asset – or liability. According to the accoun-

tant, any asset (or liability), which is consumed (or paid) within an accounting period (i.e., one year), is current. Any asset – or liability – that has a life exceeding one year is “long-term.” Thus, inventory is placed “above the line,” under current assets. So too is the case with accounts payable. Property is long-term, for example. More on this next….

## 2.4 Sample Bookkeeping Entries

Here are some examples of simple bookkeeping (or "journal" or "ledger") entries, exemplifying double-entry bookkeeping standards. Keep in mind that assets are debit balance accounts, while liabilities and equity are credit balance accounts. Debits must always equal credits. (All the numbers below are in thousands of dollars.)

1. Let's say that a company buys inventory for $1,000 in cash. What are the correct bookkeeping entries?

| Cash | |
|---|---|
| | $1,000 |

| Inventory | |
|---|---|
| $1,000 | |

You will note that cash goes down (credit) and inventory goes up (debit). You will note that the basic accounting equation (A = L + E) remains unchanged.

2. What happens when a company borrows money by issuing long-term debt for $5,000?

| Cash | |
|---|---|
| $5,000 | |

| Long-term Debt | |
|---|---|
| | $5,000 |

First, debt increases (credit) and so too will cash (debit). You will note that both sides of the the basic accounting equation increase.

3. What if the company borrows $7,500 in order to

buy back some of its stock?

| Equity | |
|---|---|
| $7,500 | |

| Long-term Debt | |
|---|---|
| | $7,500 |

Debt increases (credit) and equity goes down (debit). The purchased equity becomes what is called “Treasury Stock,” which is a contra-account and thus a debit balance account. The equity may be reissued again in the future, should the company choose to do so. Another example of a contra-account would be “Allowance for Doubtful Accounts Receivables,” which would be a credit balance account versus accounts receivables.

4. What happens when the company buys $500 in inventory on credit terms?

| Inventory | |
|---|---|
| $500 | |

| Accounts Payable | |
|---|---|
| | $500 |

Inventory rises (debit) and payables also increase (credit).

## 2.5 Current Assets: Inventory and Accounts Receivable

The accountant defines the word, “current,” as in “current asset” – or “current liability” – as an item, which is consumed or exhausted within one calendar year. On the balance sheet, we observe numerous such assets and liabilities. For example, inventory is held by the corporation for sale in the near future and, presumably, is sold in less than one year (in most instances). In fact, the sooner the corporation disposes of its inventory the better, by and large.

“Accounts receivable” is a current asset that bears some further explanation. In many instances, a customer pays for goods purchased at the point of sale. He receives the goods (i.e., the corporation’s inventory) and pays at the same time.That is what usually happens when consumers buy goods at the store. At the time of (a “cash”) sale, the seller reduces inventory (credit) and increases cash (debit).

In many, if not most, large business transactions, the customer may receive the goods, but not pay for them until later. The seller may grant the customer “credit” for the purchase and require “terms of sale” to which the buyer agrees. The terms of sale will dictate that the customer pay for the goods, typically, but not always, within thirty days of delivery. This sale would be referred to as a “credit sale” rather than a “cash sale,” and would be indicated as such on the company’s income statement (see below).

At the time of the credit sale, the seller will record, or "book," an account receivable for the sale on its balance sheet, while the buyer will book an account payable on its. Simultaneously, the seller reduces (i.e., credits) his inventory and the buyer increases (i.e., debits) his.

When payment is made, the accounts receivable are adjusted (as will the cash account). In the case of the seller, the account receivable is reduced (i.e., credited) and the cash account will be increased (i.e., debited).

In the case of the buyer, the account payable will be reduced (i.e., debited) and cash will be reduced (i.e., credited) as well.

## 2.6 Financial Claims Hierarchy

Lenders and owners have different claims on the company's interests. There is a distinct hierarchy in which the corporation's claimants get paid – in order from first to last:

Debtholders (Lenders): They get paid first. This includes the interest on loans, and on the loan's principal when due. Loan payments are made as contracted, and do not increase should the firm become more profitable. Dividends may not be paid to shareholders unless loan payments have been made first, and in full.

Preferred shareholders: These owners (usually) get paid a fixed payment or "dividend," which cannot increase even if the firm's profits increase. If the firm is unable to pay the dividend it may skip it. However, most preferred dividends are "cumulative," which means that no common stock dividends may be paid unless and until all past unpaid dividends that have accumulated "in arrears" are paid in full first.

Common shareholders: These shareholders get their dividends last, and are effective owners of any earnings, which the firm does not pay out, but "retains" and reinvests back into the firm in the manner of added property, plant, equipment, and working capital. In other words, common shareholders have a claim on the firm's net income (after preferred dividends have been paid).

Common shareholders take on the most risk as they are last ones “on the totem pole.” Such is the case whether the firm is an ongoing enterprise or is being liquidated in bankruptcy. They get paid last. Common shareholders have a “residual” claim, or interest, in the company – after all other interests are taken care of.

On the other hand, the common shareholders have the most to gain if the firm is profitable; the dividend may be increased and more earnings may be retained – to their financial benefit, as the firm’s “retained earnings” are owned by the common shareholders.

Again, only common shareholders benefit if profits go up; common share ownership entails more risk, since if interest and preferred dividends are not paid, there may be nothing left for the common shareholders.

Government and Taxes: Let’s not forget that, after interest has been paid, the earnings of the company are then taxed, and that dividends are paid out of Net Income – after taxes have been paid. Taxes are a given, and normally, we do not think of the government as a claimant on the firm, since it is neither lender nor owner.

*Nothing is certain except death and taxes.*

-Benjamin Franklin

## 2.7 Interest Paid on Bonds and Dividends Paid on Stock

- Interest on debt is tax deductible to the corporation; dividends on preferred and common stocks are not tax-deductible, under current law.
    - Interest must be paid before any dividend payments on preferred and common stocks may be made.
- Preferred stock dividends are usually fixed (like the interest on most bonds). Even if the corporation becomes more profitable, the preferred stock dividend cannot be increased. There is no upside, in this case.
    - Only after interest is paid on the corporation's debt, may preferred dividends then be paid. If the dividend is not paid, it is not considered a "default" as with a loan or debt; this is because preferred stock represents ownership interests and not a liability. Preferred stock is thus thought of as a hybrid debt/equity security as it has characteristics of both.
    - Most preferred shares are "cumulative," which means that before any dividends are paid to common share-

holders, all those preferred dividends that have not been paid, and are thus said to have accumulated unpaid or "in arrears,"must first be paid[2].

- Common stock is most risky– first, because its dividends are the last to be paid and secondly, because common shareholders are the last to be paid off in bankruptcy (thus the phrase "residual interest," used above).
    - However, as common shareholders have rights to "residual" profits (i.e., after interest is paid on debt and, second, after preferred stock dividend distributions) that the firm may generate, common shareholders also have the most opportunity to share in positive earnings growth, i.e., they have the most to gain.
    - As a result, common shares usually come with "voting rights," i.e., the ability annually to vote for company management and on certain key issues. Notably preferred shares rarely carry such rights.

## 2.8 Bankruptcy

Many students think that bankruptcy is a death knell. It is not – necessarily. It can actually be a good thing, which ensures the survival of an enterprise that experienced financial stress. There are different forms of bankruptcy. The forms are categorized by "chapters," under Title 11 of the United States Bankruptcy Code. Let's briefly see what the principal bankruptcy forms are, as noted in the Code by chapter, and how each form is different.

Chapter 7:

This bankruptcy form allows debtors to eliminate most or all of their debts over a short period of time, often just a few months. Only student loans, child support payments and some other debts may survive. Here, a trustee is appointed who then liquidates unsecured debts and makes the proper distributions. Collateral on secured debt may be repossessed. Certain assets will be protected, such as social security insurance. To qualify for Chapter 7, the debtor must satisfy a "means test." If the test is not satisfied, the debtor may seek relief under Chapter 13.

Chapter 11:

Here, the debtor retains ownership and control of assets. The debtor is referred to as a "debtor in possession." The "DIP" runs the day-to-day affairs of the business while the creditors work with the bankruptcy court to work out a plan to be made whole. If the creditors come to an agreement,

the business continues operating and certain agreed-upon payments are made. If there is no agreement, the court intercedes; debtors filing a second time are referred to as Chapter 22 filers.

Chapter 13:

In this form, debtors retain ownership and possession of the assets (in contrast to Chapter 7), and will make payments to a trustee from future earnings, which will then be disbursed to creditors. There is a five-year limit in which this process must be completed. Secured creditors may receive larger payments.

**Students' take-away:** Bankruptcy is not always the end of the story.

**Advice:** One should consult with an attorney to obtain detailed, actionable information regarding these complex laws.

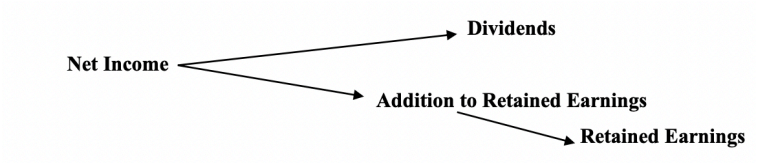
**Question:** In the Banking Crisis of 2008, the United States government rescued, or bailed out, General Motors; it purchased stock in the company, using taxpayer funds. Was that the right move? Would it have been better for Uncle Sam to have allowed the corporation to fail and file under the Bankruptcy Code?

## 2.9 The Balance Sheet, Net Income, and the Common Shareholder

Net income is an Income Statement number, which is also very relevant to the balance sheet.

What happens to net income – or profits – after it is recorded? Let's say the company records net income of $1 million for the year. If the company pays dividends to its shareholders of, say $100,000, those funds are now theirs and not the corporation's. The shareholders are enriched to the extent of $100,000 (less taxes).

This leaves $900,000 in "addition to retained earnings," which at the year's end will be transferred by the accountant from the income statement to the retained earnings account in the equity section of the balance sheet. (This is done when the accountant "closes out the books" at year's end, at which time the income statement reverts to zero.) Of course, as owners of the corporation, all retained earnings, in fact, belong to the common shareholders as well. Thus, from the point of view of the balance sheet, the common shareholders own both the "common stock" and the "retained earnings." The preferred shareholders just own the preferred stock.



## 2.10 Corporate Goals

It is well known that in a Capitalist economy, the corporation is said to serve the interests of the owners. (While there may be additional purposes, we shall assume this simple premise.) The owners wish to earn profits, to "make money." In theory, we shall assume that the owners and, by extension, the corporation has a never-ending appetite for profits and corporate growth. We shall say that the corporation has no interest in vacation, rest, or "doing good," the lack of which premises may, in fact, be false.

"Growth," thus, refers to the increase in a corporation's sales or revenues, which in turn provide increasing levels of profits to the benefit of the corporation's owners. This brings us to the Balance Sheet. No company can produce sales without assets. It is assets (the left side of the Balance Sheet) that produce goods and services for sale. However, assets must be paid for, or more accurately, "financed" when cash itself is unavailable.

The financing of the company's assets comes from raising capital in the form of liabilities, including borrowed money, and injections of cash from owners who purchase shares of equity in the corporation when such shares are offered by the corporation. (This is distinctly different from secondary market trading where shares are bought from selling shareholders with no corporate involvement.) Financing sources also include profits which the company retains – "Retained Earnings."

As the company increases its capital, it is then equipped to acquire more assets with which to increase its sales and profits ad infinitum. Thus, as the balance sheet itself increases, so too does the potential for corporate growth as we define it. Round and round she goes.

## 2.11 Words and Numbers (An Aside)

In business and perhaps in Finance in particular, memorization is less important than active, creative thinking. Some people find math intimidating, but it does not have to be. By and large, Finance does not require the memorization of formulas. However, one must *understand* the mathematics.

This is an important distinction. If you regard formulas as memorization, it may become an unpleasant and, worse, a difficult task. Instead, think of formulas as shorthand explanations of the relationships between important financial concepts, between prices and interest rates, for example. If this is done, the relationships will seem clear and logical, and memorization should prove unnecessary, or at least, less so.

Some people consider themselves word- rather than number-persons. Think of numbers as words! Indeed, they are linguistically meaningful. Mathematical symbols are a shorthand way of saying something that can also be said – at greater length – in words.

Over the course of the semester, the student will be exposed to numerous mathematical formulae.

*Do not worry about your difficulties in mathematics;*

*I can assure you mine are still even greater.*

-Albert Einstein

## 2.12 Chapter 2 Review Questions

Chapter 2: Review Questions

1. Is the Balance Sheet a “flow” or a “static” statement? Do its numbers get bigger (flow), smaller, or neither, as time over the course of the period passes?
2. When, if ever, and, if so, how do Balance Sheet numbers revert to zero?
3. How often, at most, is a public corporation required to issue financial statements?
4. Are assets debit- or credit-balance accounts?
5. Are liabilities and equity debit- or credit-balance accounts?
6. What is the basic accounting equation?
7. List all typical Current Assets and Current Liabilities.
8. When a Credit Sale is booked, what Balance Sheet item is affected?
9. What is meant by “current”?
10. What are all the equity section accounts?
11. Rank these items in terms of who gets paid

first: Preferred stock dividends – interest on debt – common stock dividends.

12. When it is said that "dividends have accumulated in arrears," to which class of stock may we refer – preferred or common? Explain.
13. Trace the link from Net Income to the Balance Sheet.
14. What are the tax ramifications to the corporation regarding interest and dividends paid?
15. To which Capital Component does the word, "Default" apply?
16. Which Capital Component (Debt, Preferred Stock, Common Stock, or Retained Earnings) is riskiest? Why?
17. Is Bankruptcy "the end"?
18. Using the template below, search on-line for a real, true-to-life company's Balance Sheet and transfer the numbers to the template below. Imagine an airline or a retailer, etc. – perhaps Amazon or Apple. What might their numbers look like?

| | | | |
|---|---|---|---|
| Cash and Equivalents | | Accounts Payable | |
| Inventories | | Short-term Debt | |
| Accounts Receivable | | *Total Current Liabilities* | |
| Other Current Assets | | Long-Term Debt | |
| *Total Current Assets* | | *Total Liabilities* | |
| Property, Plant & Equipment | | Preferred Stock | |
| Other | | Common Stock | |
| | | Retained Earnings | |
| Total Long-term Assets | | *Total Owners Equity* | |
| *Total Assets* | | *Total Liabilities + Equity* | |

19. Who owns a company's Retained Earnings?

# Chapter 3: Financial Statements Analysis: The Income Statement

# 3.1 Chapter Three: Learning Outcomes

Learning Outcomes

In this chapter, you will:

- **Explain** all the Income Statement accounts.
- **Convert** the Income Statement to percentage, “Common Size”.
- **Identify** cash versus non-cash expenses.
- **Define** Audit Opinion nuances.
- **List**four financial statement interpretation issues.
- **Calculate** the various asset account costing methods.

## 3.2 The Income Statement

Income statements, in contrast to the balance sheet, are "flow" statements, and are thus not static like the balance sheet; think of it instead as a moving picture, rather than as a photograph. The income statement will reflect cumulative data for a period ending on a certain date. The "period" has a starting and ending date; it may cover a year, half-year, quarter, or even a month.

Over the course of the period, the numbers, whether they be revenues or expenses, will grow only larger. The sole exception to this is the profit figures, including gross profits, earnings before interest and taxes (EBIT), and net income. These data may go either up or down depending on the relative growth of the revenues and expenses that make up those "net" figures. That is, if revenues grew less than expenses over a period, the net profit may go down over time. Take note that expenses are bracketed numbers to indicate that they are subtractions from revenue in order to arrive at profits.

Again, all other entries are the result of the accountant's summarization of revenue and expense bookkeeping entries, which only grow in size over time (with some few exceptions). Profits are merely the calculation of differences between revenues and expenses in the summary income statement. If expenses grow faster than revenues, profits may decrease in time.

In contrast, balance sheet numbers will change, in theory,

daily, and may either increase or decrease. At the year's end, the "books are closed" and all the income statement numbers revert to zero; we start all over again. The balance sheet, in contrast, never reverts to zero; the company always has some assets and liabilities. A very simple income statement will look something like the following:

**XYZ Corp. Income Statement for the Year Ending 12.31.XX**

| | | | |
|---|---|---|---|
| Cash Sales | $1,000 | | |
| Credit Sales | 10,000 | | |
| Total Sales / Revenues | 11,000 | 100% | |
| (Cost of Goods Sold) | (5,000) | 45.4 | |
| Gross Profit | 6,000 | 54.6 | |
| (Selling, General, & Administrative Expenses) | (1,000) | 9.09 | |
| (Depreciation & Amortization) | (500) | 4.6 | |
| Earnings before Interest & Taxes (EBIT) | 4,500 | 40.1 | |
| (Interest) | (500) | 4.6 | |
| (Taxes) [Tax Rate = 30%] | (1,200) | 10.9 | |
| Net Income | 2,800 | 25.5 | 100% |
| (Dividends Paid) | (500) | | 17.9 |
| Addition to retained earnings | 2,300 | | 82.1 |

You must note that expenses are bracketed as they are reductions (subtractions) to revenue in arriving at the profits numbers, i.e., Gross Profits, EBIT, and Net Income.

The key connection between the income statement and balance sheet has largely to do with "addition to retained earnings." When the books are "closed" at year's end, this addition (or deduction) is transferred to "retained earnings" in the bal-

Key Terms:

Common Size Statements

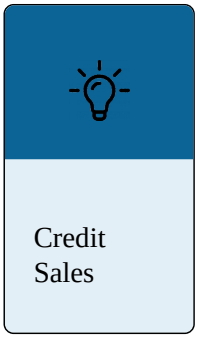
ance sheet; the income statement is “closed out,” and everything in the income statement reverts back to zero. If dividends are paid when there is a loss, the cash used to pay the dividends will have to come from (past years’) retained earnings. “Retained earnings” represent the accumulation of historically retained profits, which were not paid out as dividends, but instead were retained by the corporation, since the corporation’s inception.

As an accountant (or financial analyst), you may think of revenues as credits (right hand) and expenses as debits (left hand) of the “T-accounts” to which reference was made in the prior chapter. Think of the income statement as part of the balance sheet’s equity section, which is also a credit balance account. When numbers are compared to a base figure, in this case either “total revenues” or “Net Income,” the analysis may be referred to as “Vertical Analysis.” This makes the analysis from one company to another simpler. Matters having to do with the relative magnitude of the data from one company to another are thereby neutralized. Such analysis utilizes **Common Size Statements**, where figures are represented in percentage terms, as in the income statement above. Common size statements are also (very occasionally) used for Balance Sheets.

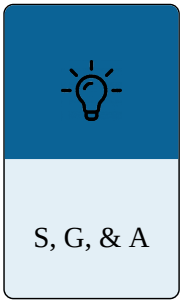
Getting back to debits and credits, at year’s end, the accountant debits the income statement for an amount equal to the “addition to retained earnings.” Remember that the Income Statement is a credit balance statement. This debiting enables the accountant to set the income statement back to zero. (Again, the balance sheet never reverts to zero, except in the extreme case of bankruptcy liquidation.) S/he then credits the balance sheet’s retained earnings account for this same amount. That is how retained

earnings grow (or decrease when there is a loss and retained earnings are thus negative) over time. The additions to earnings should be reinvested by a good management in profitable assets for corporate growth purposes.

"EBIT" may also be referred to as "operating earnings." As you will see, we will differentiate between operating earnings, i.e., earnings generated from the firm's business activities, and other non-operating income. There are a few other basic things that the financial analyst must keep in mind relative to the Income Statement.




**Credit sales** (on the income statement) are recorded as accounts receivable (on the balance sheet) – until the receivables are collected. This is typical of accrual accounting, as opposed to cash accounting, the latter of which recognizes a sale only when cash is exchanged. At the time of a credit sale, credit sales are "credited" on the books, and accounts receivable are debited. When the customer pays, say in 30 days, the account receivable is credited and the cash account is debited.




Selling expenses include advertising, salesperson's salaries, and "freight-out" paid on shipments to customers; an exception is "freight-in" payments, which are paid on shipments to the firm and are included in COGS (cost of goods sold). General and administrative expenses have to do with: clerical and executive offices salaries, outside professional services, telephone and internet, postage, and office equipment depreci-

ation. **Selling, General, and Administrative** expenses are also known as **S, G, & A**.

Depreciation

Imputable

**Depreciation**, if **imputable** to inventory production, will be included in COGS. By "imputable," it is meant that the accountant can objectively ascribe a specified dollar amount of depreciation to a specified dollar amount of production. Imputable means that units produced can be associated and measured with mathematical certainty and precision to units of production. This figure would be added to COGS, rather than reported separately lower down on the Income Statement.

For example, if an equipment manufacturer warrants that a $1 million-dollar machine (at cost) will produce one million units and then need to be scrapped, we can safely say that each unit produced includes one dollar's worth of depreciation. This may happen in the case of depreciable equipment, but not with the plant. Otherwise, i.e., if not imputable, depreciation will be reflected separately and *visibly* below the gross profit line.

This makes matters confusing to the reader. Depreciation may be included "above" (within COGS) and/or "below" the gross profits line (as "depreciation expense"). If the former, it will be included *within* the COGS number; if not, it should be reflected as a separate line item lower down the income statement.

**EBIT** or **Operating Earnings** should be viewed as a kind of dividing line within the income statement. Everything "above the line," has to do with the firm's basic running of the business, or operating revenues and expenses. Those items that appear "below the line" may include numerous non-operating items. Such items may include interest on debt, a profit or loss from the sale of an asset, or the financial outcome of litigation. Interest (on debt) has to do with a financial matter; this is not a matter having to do with the running of the business, but instead, with the decision as to how the firm's assets are financed, in this case with debt rather than with equity. Analytically, it is important to the financial analyst to differentiate between operating and non-operating; we shall depend on this differentiation often.

EBIT

Operating Earnings

Taxes are based on earnings before taxes (EBT), which are not always shown on income statements as a separate line item. Here, EBT, i.e., EBIT ($4,500) less interest expense ($500), but before taxes, equals $4,000. You should also note that taxes were calculated using a 30% tax rate (i.e., a rate arbitrarily chosen for this example), i.e., $4,000 × 30% = $1,200.

| | |
|---|---|
| EBIT= | $4,500 |
| Int.= | (500) |
| EBT= | 4,000 |
| T (@ 30%) = | (1,200) |
| NI= | $2,800 |

## 3.3 On Learning and Studying

*Torah learning is best combined with an occupation/profession, because the effort of both will keep one from sin. Torah study alone without work will, in the end, be nullified and lead to sin.*

-Ethics of the Fathers, 2:2

*He who occupies himself with the study of the law alone is as one who has no God.*

-Rav Huna

Inferred from II Chronicles 15:3, ‘Avodah Zarah 17b

*Ben Zoma says: Who is wise? The one who learns from every person, as it is said: “From all those*

*who taught me I gained understanding" (Psalms 119:99).*

-Ethics of the Fathers, 4:1

# 3.4 Financial Statements: Interpretation

Financial Statements are fraught with interpretive problems, due to accrual accounting methods, emanating from historical bias, arbitrary choice of one or another cost method, and the use of estimates and reserves, all of which are *legitimate applications* of "generally accepted accounting principles." The larger the corporation, the more susceptible its financial statements are to misleading data for the financial analyst to decipher and comprehend.

Accounting presentations complicate analysis, and in particular make difficult the use of analytic financial ratios, which are compiled from the accounting data. Hence, financial analysis is not straightforward, as it would otherwise be if the numerical bases of the accounting data were themselves not subject to variation over time and from firm to firm, and thus to – interpretation!

**Historical bias** – In most, but not all cases, asset cost is recorded by the accountant based on invoiced, historical values.Such costs are determined at a point in time and are not adjusted upward later for changing, and possibly increasing, market values. (These assets may then be depreciated, adding still further to the difficulty in interpretation.)

For example, in the case of long-term assets, a building may be built (or purchased) at a (historical) cost of $1 million dollars. Another, virtually identical building, which

may be built (or purchased) by the same company some years later could have a cost of $2 million; the newer building costs more due to inflation and/or market conditions. The accountant will not necessarily show the details of the original costs of each property and the respective depreciation schedules for each property separately. The accountant will carry the building's value at its original cost, and at a combined sum, in this case of $3 million, leaving the reader of the data with a kind of mixed bag of information. Secondly, the depreciation expense will be based on this original value.

Market values of buildings often, in fact, exceed the recorded, "book" carrying values, i.e., historical cost minus accumulated depreciation. The accountant will not adjust the carrying value of such an asset upward, thereby making it difficult to know the asset's, and therefore the company's, true worth.

This is not an issue where current assets are concerned. It certainly does not apply to cash or accounts receivable; these values do not change. (Under certain narrowly defined circumstances, cash equivalents, a.k.a., "marketable securities" values may be adjusted.) Inventory is generally not marked down in value and never marked up, under accounting rules.

**Estimates** – This too will be illustrated (below) with the use of the (same) example of long-term asset accounting. We will examine both the asset's "salvage value" and the asset's estimated "life," both of which are estimated. This leaves ample room for "judgment" and, hence, earnings manipulation. Other examples may also pertain.

**Reserves** – For example, management has some leeway in setting up loss accounts or "reserves," e.g., for "uncollectible accounts receivable," better known as "allowance for doubtful accounts." We will not demonstrate this phenomenon; suffice it to say that this is a notable problem where accounting "manipulation" is concerned. While reserves have been listed here as a category by itself, the reader may also think of it as an example of an estimate.

**Arbitrary use of cost or other methods** – The accountant (together with company management) has *alternatives* regarding the manner in which s/he may choose to "cost" certain items. This will be illustrated (below) with respect to inventory (a "current asset") and long-term asset accounting, including plant and equipment. You must recall that "property," i.e., land cannot be depreciated.

An external financial analyst must dig below the surface in order to understand the accountant's summary of the corporation's ledgers. He must also have a deep understanding of the nature of the industry in which the corporation operates. A good place to start is by reading the accountant's footnotes and thoroughly understanding the firm's GAAP reporting policies. "GAAP" refers to "generally accepted accounting principles," which are the rules to which accountants must adhere.

## 3.5 The Audit

Now that we know what financial statements are, and what set of problems may ensue in readings statements, it is important to know well what the accountant's role relative to the production of the statements is. Certified Public Accountants (C.P.A.s) examine, i.e., "audit," the books, ledgers, and records of corporations in order to ascertain the data entries' veracity and accuracy, and prepare *summary* financial statements such as those we have seen in our Balance Sheet and Income Statement discussions.The statements are, once again, *summaries* of myriad, and often complex, transactions that occurred within the corporation over the course of the stated period – for the Income Statement, or "as of" a date noted – in the case of the Balance Sheet.

The auditors will present one of four possible "opinions" about the statements. The analyst should examine the opinion for information concerning the accuracy and comprehensiveness of the statements' data prior to reading the statements themselves.

1. **Unqualified Opinion:** This is the ideal opinion and is considered a "complete audit." The results are satisfactory, and the statements are "fairly presented" in the language of the auditor/accountant.
2. **Unqualified Opinion with Explanatory Paragraph:** This may be a complete audit, but the auditor believes that additional information is

required.

3. **Qualified Opinion:** In general, the statements are fairly presented with an important exception that does not affect the statements as a whole. This generally occurs when there is an unjustified deviation from "Generally Accepted Accounting Principles," or GAAP. This opinion falls short of "Adverse."

4. **Adverse Opinion:** The auditor does not feel that the statements taken as a whole fairly present the corporation's financial position in conformity with GAAP accounting.

## 3.6 Perpetual Inventory Accounting

There are two such counting ("Accounting") methods for inventory – "perpetual" and "periodic."A perpetual inventory counting system is usually used where the inventory units are few in number, and high in unit cost. It thus pays for the firm's management to spend some extra resources in knowing exactly how many yachts, for example, it has in the showroom. A one-unit difference is substantial. Under this system, the firm knows – at all times– the dollar value of its inventory. There are certain methods under which a perpetual system accounts for its inventory, including "**specific identification,**" whereby each physical inventory unit is associated with its own specific cost.

In contrast, a periodic inventory system is effective where units are numerous and relatively cheap. Management is not going to be able to justify the extra effort required to know whether there are ten or eleven blue pens in stock, more or less. It will do, instead, a *periodic*, probably annually, physical audit of what's down there in the warehouse. When a physical audit is done, the counting is done in units, rather than in dollars. Thus, the ascription of units to dollars is subject to methodological choice.

When a company uses a periodic system, there are management choices under legitimate accounting rules by which the accountant may record the ending inventory balance and, hence, cost of goods sold figures – in dollars. We will illustrate all this below. Upon the completion of which

analysis, the analyst's relevant interpretive issues should become clear.

The differences in the numbers presented, and the related management strategies as related to the choice of cost method, are potentially markedly different. Let's read on.

# 3.7 Periodic Inventory Analysis: Ending Inventory and Cost of Goods Sold

Let's assume a company's books reflect the following accounting data:

| | | Notes |
|---|---|---|
| Inventory (beginning) | $20,000,000 | Carried over from the last period |
| Purchases | 80,000,000 | Represented by Invoices |
| Goods Available for Sale | $100,000,000 | |
| Inventory (ending) | ($10,000,000) | Subject to Costing Method |
| Cost of Goods Sold | $90,000,000 | |

We know the beginning inventory dollar figure because it is carried over from the prior period's ending value as noted on the Balance Sheet. The purchasing manager will have loads of invoices (bills) for all the purchases made in the period, so that figure is also known. Adding beginning inventory and purchases together, we get the total amount of goods that were available for sale during the period. If we subtract the ending inventory from the goods available number, we get the "cost of goods sold," which will be noted on the Income Statement.

How do we know what the dollar value of the ending inventory is if we are not using a specific identification-type inventory accounting system? When the inventory manager conducts a monthly, or periodic, physical audit, s/he counts inventory units and not dollars. How does the accountant translate units into dollars for valuing the ending inventory?

**Note**: COGS include raw materials, freight-in, electricity, and may also include "imputable" labor and depreciation.

*Do not look at the container, but at what is inside.*

– Famous Hebrew saying

*Don't judge a book by its cover.*

– Another famous saying

# 3.8 Units to Numbers: FIFO and LIFO

If most businesses "do" inventory *periodically* (rather than *perpetually*) for practical reasons, by quite literally, physically counting (ending) inventory units, usually monthly, how does the accountant translate the ending inventory units counted into dollars? As we shall see below, management and its accountants have ample discretion regarding choice of inventory costing method.

In the following illustration for the two most popular costing methods, the company has started the year with one unit of inventory carried at a cost of $100. You will note that the costs go up in time, reflecting an inflationary environment. It is also assumed that only one identical inventory item is being counted.

| | | |
|---|---|---|
| Inventory (beginning) | January | 1 unit @ $100 |
| Purchases | March | 1 unit @ $150 |
| | June | 1 unit @ $200 |
| | September | 1 unit @ $250 |
| | December | 1 unit @ $300 |
| Total Goods Available for Sale | | 5 units / $1,000 (total cost) |
| Actual Sales | | 4 units |

**Alternate Solutions to the Ending Inventory Problem**

In the chart below, you will find alternate solutions to the Ending Inventory Problem when the accountant employs either of two popular costing methods: FIFO (First In, First Out) and LIFO (Last In, First Out). A detailed step-by-step explanation of the chart is found on the next page. Before flipping to the next page, see if you are able to decipher

the table on your own. Immediately following the table is a discussion of the differing results of the two methods. You will note that the choice of method materially affects the Cost of Goods Sold and Ending Inventory values, and therefore all the other values in the table as well.

| | FIFO | LIFO | |
|---|---|---|---|
| | First-in, first-out | Last-in, first-out | Notes |
| Sales | $2,000 | $2,000 | Unaffected by Costing Method |
| Cost of Goods Sold | (700) | (900) | For FIFO, count chronologically.<br>FOR LIFO, count in reverse chronology. |
| Gross Profit | 1,300 | 1,100 | |
| Tax (@40%) | 520 | 440 | This affects "Cash Flow" |
| Inventory (ending) | 300 | 100 | FIFO: Last-in remains<br>LIFO: First-in remains |

Clearly, a firm will adopt LIFO if it wishes to reduce inflated, "windfall" profits, and hence taxes, in an inflationary environment. LIFO will consequently reduce inventory assets on the balance sheet and make relevant ratios, and hence loan prospects, possibly less attractive to banks who wish to see more assets against which loans may be made and, possibly, secured. Management and its accountants have to weigh the offsetting considerations of one or the other costing method.

Take note that inventory is part of current assets and "working capital" (WC). WC = Current Assets less Current Liabilities.

## 3.9 Inventory Costing Calculations: A Closer Look at the COGS and Ending Inventory Computations

First, it must be recognized that the sales amount is independent of any costing method. For every sale, there is an invoice, and the dollar amount is not at all subject to any choice of accounting inventory costing method.

Under FIFO, the first-in were assumed to be the first-out. So, the cost of goods sold (COGS) figure was the sum of the first four (since four units were sold) costs: $100 + 150 + 200 + 250 = $700. The income statement will show a $700 COGS number as a deduction from sales to arrive at the gross profit of $2,000 – 700 = $1,300. This left $300 in the ending inventory on the balance sheet. Notice that $700 + 300 = $1,000 or the entire inventory cost.

Under LIFO, we must take the last-in, first-out order to compute the sum of the units sold and to arrive at the COGS number. Therefore, we add in reverse chronological order: $300 + 250 + 200 + 150 = $900. The ending inventory will be the first unit acquired, costing $100. Again, the two numbers add up to the sum of the inventory cost: $100 + 900 = $1,000. Gross profits will be lower: $2,000 – 900 = $1,100. Taxes are based on profits. In an inflationary environment, taxes payable will therefore be lower.

Assuming inventory costs rise as often happens when there is inflation, FIFO will always produce a higher gross profit.

When corporations expect high inflation (as occurred in the 1970s and in the early 2020s), many will switch to LIFO in order to reduce taxes. Companies pay higher taxes when profits are higher, as might be the case using FIFO, assuming all else equal. To switch to LIFO, the firm must complete IRS Form 970. The corporation cannot go back and forth from one method to the other as the wind blows. You will find the form on the Internet.

The company will prefer LIFO to reduce taxes. It will prefer FIFO to report higher inventory levels and thus show "more" assets to lenders and shareholders. Well, which of the two is preferable in the end? It is permissible for the company to maintain two sets of books, one for filing taxes to the Internal Revenue Service (the IRS) and the other for "reporting." Moreover, it can use varying methods for different inventory items – FIFO for this and LIFO for that. How do you like that?!

Once again, a company may use one inventory costing method for one inventory item, and a different method for another item. In the end, the various items will be lumped together on one inventory line on the Balance Sheet and reflected as such in COGS. There might be a footnote indicating that inventory was valued using both FIFO and LIFO (and perhaps still another method), but further details may be absent. Talk about confusing!

# 3.10 Inventory Accounting Issues: LIFO

Of course, the previous example was "loaded" in that purchase costs rose dramatically over time. In reality, prices tend often to rise with inflation, so the direction of the results in actuality would not be affected. If, in fact, costs declined over the considered periods, the direction of the results would be exactly reversed, and your choice of method, if you anticipated this alternate outcome, would be different accordingly. Here are some relevant upcoming questions.

1. What is meant by "LIFO Base"?
2. What is the effect of costing units sold from the base?
3. There is a technical problem in connection with interim reporting under LIFO (in FIFO too).

Using the foregoing example, imagine that the firm –indefinitely into the future –regularly purchased four units and sold four units. Imagine also that there continued to be some inflation.

Key Terms:

A firm cannot *accurately* provide its LIFO inventory figure until it closes the books on the period. More specifically, the firm cannot figure the Cost of Goods Sold, and therefore its LIFO inventory until it has, quite literally, made the last

inventory purchase and thus knows the costs of units acquired.

LIFO base

It would be conceivable that after some time, inventory costs and sales prices would be quite high, say $20 per unit – theoretically. Still, that one very "old" unit, which is being carried at a historical (ancient!) cost, would reflect a very low "cost basis" of, say $1, representing the inventory's **LIFO Base**. (See the example on the next page.) If many years later, the firm suddenly then sold five units, it would dig into or "cost out" its LIFO base and reflect a very high profit for that item. This now high profit on that unit would be contrary to the firm's motivation for adopting LIFO in the first place, which is to minimize profits and consequently its tax liabilities.

By example, let's say the firm is on a calendar year fiscal cycle. It cannot know in the interim, say, in November, what the December year-end inventory shall be. In our earlier case, the fourth unit purchased is the first out and first charged to COGS. The third acquired is the second to go, etc. We count backwards starting with the last one in! We cannot know what the last one's cost is – until the last one *is* in! (True, this problem pertains to FIFO, but is less worrisome analytically.) An illustration of this problem follows on the next page.

Note:

You should be aware that Cost of Goods Soldrepresents the cost of inventory, which passes through the income state-

ment when sales are recognized, and includes raw materials, “imputable” labor and depreciation, freight-in, and possibly more. “Imputable” refers to measurable costs that can – with certainty – be related to production, in addition to the otherwise explicit inventory costs.

Suppose you have a widget making machine, which costs $1,000,000 and the manufacturer warrants that it has an expected life of producing one million units, after which it must be scrapped. Under this circumstance, the cost accountant may impute $1 of depreciation to each unit of inventory. Otherwise, depreciation would be shown separately, i.e., lower down on the income statement, as illustrated in the sample income statement earlier.

## 3.11 LIFO Base Illustration

XYZ Corp. costs its inventory on a LIFO basis due to the cost and price inflation its product continuously manifests.

In the first year illustrated, let's call it "20X1," the company purchased 5 units. It sold 4 units, leaving the oldest unit to be carried forward as the *LIFO Base Inventory* to the next year, "20×2." See the table below.

Each year going forward, it continues to purchase 4 units and sell 4 units. Therefore, the LIFO Base, initially costed at $1, carries over until "20×5." The revenue the company receives equals the average of the last price in the current and prior years. The last price, i.e., the price at which the last unit was sold, in "20X0" was $2. Due to the high LIFO costing method, the gross profits remain low, thereby minimizing taxes. Had it done its inventory on a FIFO basis, its gross profits would have been slightly higher. (You can figure this out.)

In the year, 20×5, the company sells all 5 units that were available for sale. The last unit sold was at a price of $22 and its cost, based on LIFO, was only $1. The company would therefore book a high, "windfall" profit on that unit, contrary to the intent of LIFO, which is to keep (taxable) profits low.

| | 20x1 | 20x2 | 20x3 | 20x4 | 20x5 |
|---|---|---|---|---|---|
| **Average Price** | $6 | $10 | $14 | $18 | $22 |
| **Newest Layer Cost** | $5 | $9 | $13 | $17 | $21 |
| **Fourth Oldest Cost** | $4 | $8 | $12 | $16 | $20 |
| **Third Oldest Cost** | $3 | $7 | $11 | $15 | $19 |
| **Second Oldest Cost** | $2 | $6 | $10 | $14 | $18 |
| **Oldest Layer Cost** | $1 | $1 | $1 | $1 | $1 |

The purpose of using *LIFO Inventory Accounting* is to reduce gross profits and thus taxes. We have seen that selling product from the *LIFO Base* will negate this objective. To avoid this problem, management should order a 6th unit in 20×5 so that the LIFO Base is not costed out. In doing so, management is "manipulating" the effect of its inventory costing method.

Remember, this is accounting fiction in the sense that the costs are unrelated to the actual age of the units. A good merchandiser would have long ago moved out any product that is subject to physical aging.

## 3.12 Accounting for Long-term Assets: Straight-Line Depreciation (For Reporting Purposes Only)

The following pages represent three depreciation-costing method alternatives for plant (i.e., buildings) or equipment, which are acceptable for reporting purposes (only). Remember that "property," i.e., land, is not depreciated. There is an entirely different set of methods required for tax accounting, which we shall not cover herewith. The following methods are unacceptable for Tax accounting.

The first method we shall outline is called "straight line." Under this method, we expense on the income statement the same amount of depreciation each year. If, for instance, the asset is *estimated* to have a five-year life, as in this example, we shall depreciate 1/5 or 20% of the asset yearly. This ratio will be applied against (i.e., multiplied by) the difference between the property's historical cost and its estimated salvage value. (Note the key word: "estimated," which is used here for the second time.) Herein we shall refer to this difference in the two numbers as the "depreciable amount," a phrase, which we use here, but which you shall not find popularly used elsewhere.

Let's assume the following:

| | |
|---|---|
| Historical Cost: | $1,500,00 |
| Estimated Salvage Value: | 500,000 |
| Estimated Life | 5 years |

As you read this and the next two examples, remember that "Salvage Value" does not apply to buildings but only to equipment. Also remember that property, i.e., land, cannot be depreciated.

In "year zero," which means "now" (see table below), the asset shall be carried on the balance sheet at its original, historical cost of $1,500,000. Each subsequent year, we shall depreciate 1/5, or 20% of the difference between the cost and the salvage value, this difference being: $1,500,000 – $500,000 = $1,000,000. Thus, the depreciation expense is: $1,000,000 ÷ 5 = $200,000 each year.
As the asset balance is depreciated on the balance sheet and the depreciation is expensed on the income statement, the carrying balance of the asset is reduced by $200,000 each year, until – after five years in this case – the building or equipment is sold for its estimated salvage value. Should the asset be sold for more or less than its salvage value, the accountant will record an "unusual" or "non-recurring" profit or loss accordingly.

| Year | Depreciation Expense (Income Statement) | Ending Balance (Balance Sheet) |
|---|---|---|
| 0 | | 1,500,000 |
| 1 | 200,000 | 1,300,000 |
| 2 | 200,000 | 1,100,000 |
| 3 | 200,000 | 900,000 |
| 4 | 200,000 | 700,000 |
| 5 | 200,000 | 500,000 |

# 3.13 Accounting Entries for Depreciation

The following represents the accounting book entries for straight-line depreciation in the second year (from the prior page), by way of example.

(000)

Income Statement Book Entry

Depreciation Expense (dr) $200

Balance Sheet Book Entries (End of second Year)

| | |
|---|---|
| Depreciation (cr) | $200 (year 1) |
| Depreciation (cr) | $200 (year 2) |
| Accumulated Depreciation | $400 |

Note above that the debits are indented to the left, and the credits are more to the right. That's as it should be.

Balance Sheet Appearance

The balance sheet, at the end of the second year, will contain the following items:

| | |
|---|---|
| Gross Plant/Equipment | $1,500 |
| Accumulated Depreciation | ($400) |
| Net Plant/ Equipment | $1,100 |

You will recall that equities are credit balance accounts. As noted earlier, you should think of the income statement as

part of the equity section of the balance sheet. This is not a stretch as retained earnings are derived from the income statement, i.e., when the “addition to retained earnings” are transferred to the balance sheet – at year’s end, at the time that the books are closed. Therefore, think of revenues as credits, and expenses as debits.

*A wise man’s questions contain half the answer.*
-Solomon ibn Gabirol

# 3.14 Accelerated Depreciation Methods: Sum-of-the-Years' Digits (For reporting purposes only)

There are two more methods, which we shall examine, both of which may be referred to as “accelerated” depreciation methods because in the early years there will be more depreciation expense than in the later years.

You’ll remember that the ratio we used in the given example for Straight Line was 20%. Under “Sum-of-the-years’ digits” (SOYD), we apply a new ratio against the $1,000,000 depreciable amount.

In the denominator for the SOYD ratio, we calculate the sum of the years’ digits, which, in this five-year example is: 5 + 4 + 3 + 2 + 1 = 15.

In the numerator, we use the reverse order of the years, starting with five, and then going backwards each year. So, in year one, we expense 5/15 of $1,000,000 = $333,333, and reduce the balance accordingly. In the second year the ratio is 4/15. In the third, year the ratio is 3/15, or, as it happens, the same 20% that we used in the straight-line method; you’ll note that the depreciation expense after this third year will be less than under straight-line. At this rate, you will note that after five years we will have depreciated 15/15 or 100% of the depreciable amount.

| Year | Depreciation Expense | Ending Balance |
|---|---|---|
| 0 | | 1,500,000 |
| 1 | 333,333 | 1,166,667 |
| 2 | 266,667 | 900,000 |
| 3 | 200,000 | 700,000 |
| 4 | 133,333 | 566,667 |
| 5 | 66,667 | 500,000 |

# 3.15 Accelerated Depreciation Methods: Double/Declining Balance (For reporting purposes only)

Under Double/Declining Balance (D/DB), we combine two concepts, hence the slash in the name. That's the way to remember what this method is about. First of all, we depreciate at *double* the straight-line rate, which in this case, would be 2 × 20%, or 40% per year.

If we do this doubling of the annual rate, we would have depreciated the entire asset in half the asset's estimated life, rather than in the full five years; or alternatively, we would have depreciated twice the asset's original cost over the entire five years. Either alternative is clearly incorrect and unacceptable.

Instead, we depreciate twice the straight-line rate, not against the depreciable amount, but against the *declining balance*, ignoring salvage value. That's the second concept: the use of a declining balance. So, in the first year, we depreciate 40% of $1,500,000, or $600,000, leaving a new balance of $900,000. In the second year, we depreciate 40% of the declining balance of $900,000, which is $360,000.

In this example (and not necessarily in all examples), if we continue to depreciate at this rate, we will go below the salvage value in the third year. At this point, the accountant will either depreciate the balance of $40,000 in

one-fell swoop, or, more likely, straight-line the $40,000 balance over the remaining three years. (S/he could also adjust the salvage value lower.) In straight-lining the $40,000 remaining balance, rather than deducting $40,000, one may deduct $13,333 ($40,000 ÷ 3) for each of the last three years.

D/DB is even more accelerated than SOYD, as you may have already noticed. The depreciation expense is greater in the early years.

Your income statement and balance sheet will reflect the following:

| Year | Depreciation Expense | Ending Balance |
|---|---|---|
| 0 | | 1,500,000 |
| 1 | 600,000 | 900,000 |
| 2 | 360,000 | 540,000 |
| 3 | 40,000[3] | 500,000 |
| 4 | 0 | 500,000 |
| 5 | 0 | 500,000 |

# 3.16 Comparative Summary of Depreciation Methods

Given:

| | |
|---|---|
| Historical Cost: | $1,500,000 |
| Estimated Salvage Value: | 500,000 |
| Estimated Life | 5 years |

Straight Line:

| Year | Depreciation Expense | Ending Balance |
|---|---|---|
| 0 | | 1,500,000 |
| 1 | 200,000 | 1,300,000 |
| 2 | 200,000 | 1,100,000 |
| 3 | 200,000 | 900,000 |
| 4 | 200,000 | 700,000 |
| 5 | 200,000 | 500,000 |

Accelerated Depreciation Methods:

- Sum-of-the-Years' Digits:

| | | |
|---|---|---|
| 0 | | 1,500,000 |
| 1 | 333,333 | 1,166,667 |
| 2 | 266,667 | 900,000 |
| 3 | 200,000 | 700,000 |
| 4 | 133,333 | 566,667 |
| 5 | 66,667 | 500,000 |

- Double/Declining Balance:

| | | |
|---|---|---|
| 0 | | 1,500,000 |
| 1 | 600,000 | 900,000 |
| 2 | 360,000 | 540,000 |
| 3 | 40,000[4] | 500,000 |
| 4 | 0 | 500,000 |
| 5 | 0 | 500,000 |

Again, D/DB is the most accelerated of the various methods. The finance student needs to have some sense of the "distortions" that accounting data present to him as a result of (management and the accountants') "choice," and its impact on financial analysis. In addition to the alternate depreciation methods presented in this example, we may also note that the salvage value is an estimate. These arbitrary choices and estimates present alternative "looks"for the financial statements – and interpretative difficulties for the analyst.

Final Note: The three methods covered in the last pages are NOT acceptable for Tax Accounting. There, a wholly different system must be implemented by dint of a 1986 law. The tax system is called "Modified Accelerated Cost Recovery System," or simply "MACRS." This method will not be covered here.

## 3.17 The Balance Sheet versus the Income Statement: A Summary

<u>The Balance Sheet</u>

- Static (photograph)
- "As of" a specified date
- Numbers go up or down
- Numbers never turn back to zero
- "Current" means less than one year – versus "long-term"
- A = L + E or A – L = E

<u>The Income Statement</u>

- Statement of Revenues (Addition) and Expenses (Subtraction)
- Flow (moving picture)
- Cumulative
- Numbers only go up with time – except for net numbers (and adjustments)
- For the period (quarter, half-year, three quarters, or full year) ending….
- The Income Statement is closed out at year-end:
    - The *Addition to Retained Earn-*

*ings* is zeroed out (debited) and transferred to the Balance Sheet (credited) – to Retained Earnings in the Equity section

- Then, the Income Statement's numbers all revert to zero – the "odometer" is returned to zero.
- One may think of the income statement as a subpart of the equity section of the balance sheet.

*A bashful person cannot learn.*
– Ethics of the Fathers 2:6

# 3.18 Chapter Three: Review Questions

Chapter 3: Review Questions

1. Is the Income Statement a “flow” or a “static” statement?
2. Do Income Statement numbers get bigger, smaller, or neither – as time passes?
3. When, if ever, and how, if so, do the Income Statement numbers revert to zero?
4. Using the template below, create an Income Statement in both dollar terms and in “common size,” using the same company as you used in the prior problem set.

| | | | |
|---|---|---|---|
| Cash Sales | | | |
| Credit Sales | | | |
| Total Sales / Revenues | | 100% | |
| (Cost of Goods Sold) | | | |
| Gross Profit | | | |
| (Selling, General, & Administrative Expenses) | | | |
| (Depreciation & Amortization) | | | |
| Earnings before Interest & Taxes (EBIT) | | | |
| (Interest) | | | |
| (Taxes) | | | |
| Net Income | | | 100% |
| (Dividends Paid) | | | |
| Addition to retained earnings | | | |

5. Are Revenues and Expenses respectively debit or credit balance accounts?

6. Differentiate between a “cash” versus a “non-cash expense”? What are the tax implications for each?
7. Which Income Statement item distinguishes between operating matters and all the stuff below it? By what two names is it referred?
8. What is the “Addition to Retained Earnings”? How is it calculated, and what is its relationship to “Retained Earnings”?
9. What is “Depreciation Expense”?
10. Among Property, Plant and Equipment, which is/are depreciated and which not?
11. Explain each of the four interpretation problems financial analysts encounter in reading the accountant’s reports.
12. How are audits opinions potentially different from one another? Does the financial analyst care?
13. You are given the following information. Create a table and calculate the annual depreciation and account balances, using all three reporting methods discussed:

| | |
|---|---|
| Original Cost | $ 10 million |
| Estimated Salvage Value | $500,000 |
| Estimated Life | 5 Years |

14. Calculate the Cost of Goods Sold, given the following inventory data:

| | |
|---|---|
| Beginning Inventory | $75 million |
| Ending Inventory | $50 million |
| Purchases | $100 Million |

15. What does “COGS” include?
16. By what criterion does the accountant “impute” depreciation to COGS?
17. Given the data below, calculate the FIFO and LIFO Ending Inventory, and COGS.

| | |
|---|---|
| | Beginning Inventory = $30 (6 units) |
| Purchases: | January: 10 units @ $5 each |
| | March:15 units @ $7 each |
| | April:20 units @ $8 each |
| | June:25 units @ $10 each |
| | September: 20 units @ $12 each |
| | December: 10 units @ $7 each |
| Total Sales: | 90 units |

18. Define “LIFO Base.”
19. What is the technical interim reporting issue in using LIFO?
20. Can the corporation switch back and forth from FIFO to LIFO?

# Chapter 4: Financial Statements and Finance

# 4.1 Chapter Four: Learning Outcomes

Learning Outcomes

In this chapter, you will:

- **Distinguish** between what the accountant and financial analyst do.
- **Explore** the manner in which management and accountants “manage” earnings.
- **Recognize** possible accounting frauds.
- **Master** the content of Chapters 2-4.

## 4.2 Accounting versus Finance

Here, we summarize some of the key differences between Accounting and Finance.

| **Accounting** | **Finance** |
|---|---|
| Historical | Future-oriented |
| Summary of *all* data | Incremental[1] focus |
| Reporting | Decision-oriented |
| Rules-based | Logic |
| Legalistic | Managerial |

One is not "better" than the other. You just need to know what each is – in terms of its purpose, in order to deal with them.

1. We will discuss the broad meaning of incremental in the Finance context. For now, we mean that the Financial Analyst will focus only on those data, which are relevant to decision-making.

*In the hour when a person is brought before the heavenly court for judgment he is asked:*

*Did you conduct your business affairs honestly?*

-Babylonian Talmud

Tractate Shabbat 31a

# 4.3 Earnings Management: Accrual, Real, and Expectations Management

## Earnings Management:

## Accrual, Real, and Expectations Management[1]

We have already made the point that financial analysts need to be well versed in accounting rules in order to do his/her job well. Analysts rely mightily on accounting data. Thus, we must know what is going on in the corporate accounting world. Reported earnings are subject to all kinds of accounting chicanery, and it is earnings that largely drive stock prices!

Thus, firms have an incentive to "manage earnings" in order to guide analysts' earnings expectations downward and, thereby, report actual earnings that were greater than analysts' previous consensus expectations. Corporations cannot legally inform analysts what their exact earnings expectations are, so they employ various mechanisms to guide the analysts.

Stock prices, in the wake of this management, will react

1. This section is based on Li, S., & Moore, E.A. (2011). Accrual Based Earnings Management , Real Transactions Manipulation and Expectations Management : U . S . and International Evidence. available here.

bullishly to positive earnings announcements, i.e., earnings that exceed expectations. Such stocks will provide higher returns than those of firms who do not beat expectations,[2] as investors may believe that beating expectations signals positive trends in future earnings. Stocks of corporations that miss expected earnings are penalized asymmetrically; markets penalize stocks more for missed earnings than they reward companies for positive surprises.[3]

Not all potentially misleading accounting data are intentionally "managed" by the firm and its accountants. In some cases, GAAP accounting provides no discretionary opportunities for earnings management, e.g., the choice of either FIFO or LIFO, yet the data may nonetheless not represent true values. An example that we discussed had to do with historical bias. In this case, there is only one way in which a long-term asset can be "booked," and hence its reported asset and tax depreciation figures are based on that historical cost. The historical record is unlikely to be realistic.

Key Terms:

Beyond these examples, inventory can be written down on a discretionary basis within reasonable limits. Reported estimates and reserves can be manipulated within reasonable parameters. Indeed, reserves are fertile ground for manipulation. In order to manage earnings upward, the firm may choose to "under-reserve"

2. Bartov E., D. Givoly and C. Hayn. 2002. The rewards to meeting or beating earnings expectations. Journal of Accounting and Economics 33 (2): 173-204.
3. Skinner, D. and R. Sloan. 2002. Earnings surprises, growth expectations and stock returns or Don't let an earnings torpedo sink your portfolio. Review of Accounting Studies 7:289-312

Accrual Earnings Management

the allowance for doubtful accounts. As noted, managed manipulation of accrual accounting methods will lead to varying presentations of the same economic phenomena. AEM, thus, involves the use of the accounting system itself in order to influence a desired result.On the other hand, we have also provided some illustrations of **Accrual Earnings Management** (AEM) in the manner of Inventory Costing and *Reported* Depreciation Accounting Methods (as opposed to tax depreciation methods). AEM has to do with GAAP-permitted choices.

Real Earnings Management

Timing the sale of inventory in order to boost earnings in a targeted period. Another opportunity for earnings management comes from **Real Earnings Management (REM)**. In this case, a firm may engage in certain actual transactions in order to accomplish a particular accounting outcome. Specifically, the firm could structure transactions by setting them at varying discretionary points in time ("timing") in order to achieve a pre-set accounting result. Here are some examples of REM:

1. Delaying fixed asset maintenance expenses.
2. Delaying operating expenses, including advertising and R&D.
3. Putting off salary increases and the payment of bonuses.

4. Delaying taking on potentially profitable capital investment projects.

And… guess what? They can do this time and again, over and over. One must be suspicious of what a CEO says on television. Auditors cannot control or influence this phenomenon as financial statements are not involved! We will let you decide if AEM, REM, and PEM legitimately circumvent accounting rules or not, and whether you may consider its use ethical – or not! Third, firms have an incentive to manage the public's future earnings expectations in such manner as analysts and investors expect less and get more in earnings in reality than had been expected. That is not to say that all companies, at all times, will manage earnings expectations downward. By communicating press releases, meeting with analysts, and giving TV interviews, a firm can engage in **Public Expectations Management (PEM)**. Of course, such manipulations, if that is the correct phrase here, are not affected by the accountant.

Public Expectations Management

Still a final manipulation of earnings may come from fraudulent accounting practices.

In sum, firms can employ any and all of the following four means of accounting chicanery: 1. Accrual Earnings Management; 2. Real Earnings Management; 3. Public Expectations Management; 4. Accounting Fraud. Don't forget: **fraud** is *illegal*!

Fraud

Note:

This discussion was based on a published paper, which may be found at the following link.

- Read the accounting statements' footnotes; also read "Management's Discussion and Analysis"!
- Quality financial statements have repeatable earnings and are not prone to reserves management or any form of AEM; over the long-term, accounting and cash earnings should be approximately the same.

# 4.4 Business Ethics: Examples of Fraudulent Revenue Recognition

## Business Ethics:

## Examples of Fraudulent Revenue Recognition[1]

Below are some more difficulties that you should be aware of. These have more to do with accounting fraud than the legitimate accounting choices enumerated earlier.

***Channel Stuffing*** – Companies sell large amounts of goods to distributors in order to book large profits. Under SEC accounting, it is permissible to book sales for shipped product, less a reasonable allowance for returns. However, *channel stuffers* may not be able to make reasonable estimates and would therefore be required to defer revenue recognition until return estimates may be projected. This may result in deferring revenue for months. Stuffing occurs when vendors ship merchandise in advance of scheduled shipment dates without buyers' acquiescence.

***Bill and Hold*** – Revenue is booked even though the product may not have been shipped and payment may not actually be due for a long period. For example, Sunbeam sold

1. The following section was derived from a report in the New York Times on December 4, 1999, p. C4.

charcoal grills in the winter, although they were not to be shipped until springtime; after a change in management, books were restated to eliminate the relevant revenue and profit. This arrangement would be allowed, under SEC accounting, provided the *buyer* requested it; the buyer must also have a good business reason for the request. Further, the goods must be either already assembled or otherwise ready for shipment before revenue is recognized.

***Internet Sales*** – If an internet site is acting as agent for, as an example, an airline ticket sale, it may only book the revenue it earns. This revenue may be only a commission rather than the full value of the ticket. If it acts as principal, it may book the larger amount as revenue and another amount as expense (the portion which it must return to the airline), showing the net result – which would, in effect, be the commission. Internet companies whose stock prices are sensitive to "top-line" growth prefer to recognize sales as principals. Principal transactions are permitted by the SEC provided the company took title to the product before shipment and "has the risks and rewards of ownership, such as the risk of loss for collection, delivery, or returns…".

Discussion Question: What are the moral and legal differences between "fraud" and the creative use of accounting methods and assumptions?

*Lying speech is an abomination to God, but those who act faithfully please Him.*

– Proverbs 12:22

# 4.5 Business Ethics: Examples of Fraudulent Expense Recognition

## Business Ethics:

## Examples of Fraudulent Expense Recognition[1]

Interpretive accounting issues are clearly not limited to the revenue side. Here are some further issues in connection with expenses. The analyst must always be on the alert.

Stock options provided to executives are not accounted for as expenses, thereby giving earnings a boost because there is no associated expensing to salary. According to a study by Ms. Pat McConnell of Bear Stearns, in 1999 net income of the S & P would have been 6% lower had employee stock options been reflected as an expense. Options will be exercised at below the market's stock prices resulting in an associated cost to the firm. In 1998 and 1997 the reductions would have been 4% and 3% respectively.

The FASB requires that companies include in their footnotes the effect of options granted after 12/15/94. The full effect of this was first felt in 1999 as many companies' plans allow options to vest in five years or less. If compa-

1. The following was derived from a report in the New York Times on August 29, 2000, p. C1.

nies choose not to disclose options as a compensation expense (and the FASB does not require this), the disclosure must be made as a footnote reflecting the adjusted net income and EPS had the cost of options been charged. Obviously, most companies chose the latter. Ms. McConnell found only Boeing and Winn-Dixie, i.e., just two of the 500 S & P companies, that chose to report options as an expense. Earnings at 21 companies fell by more than half, if options were expensed; this list includes McDermott International, Yahoo, Autodesk, and Polaroid. Twelve of these companies would have reported a loss, including Micron Technology. She further cites the following declines in earnings due to options:

| | |
|---|---|
| Health Care Services companies | -38% |
| Computer Networking companies | -24% |
| Commercial and Consumer services | -21% |
| Communication Equipment makers | -19% |

Overall, the S & P 500 growth rate in earnings would decrease from 11% to 9% over the last three years – if options were expensed.

## 4.6 Chapter 4: Review Questions

Chapter 4: Review Questions

1. What are some of the critical differences between Accounting and Finance?
2. In what ways can Accounting data be "managed"?
3. What are the critical differences between the Balance Sheet and the Income Statement?
4. Identify the correct choices: Assets are *dr /cr* balance accounts while Liabilities and Equity are *dr /cr* balance accounts.
5. How are the Additions to Retained Earnings and Retained Earnings different from one another?
6. Who takes the most risk in order to earn the highest return?
7. Give an example where it is permissible to use different accounting methods for reporting versus tax accounting.
8. What are some problems pursuant to using LIFO-based accounting?
9. True or False: Operating Profits include interest

paid.

10. If the company does not pay its dividends on Preferred Stock, has it “defaulted”?
11. True or False: Interest is not tax-deductible whereas Dividends are tax-deductible.
12. What are some of the differences between what the accountant and financial analyst do?
13. If there is some inflation, which will produce higher gross profits – FIFO or LIFO?
14. Provide some examples of and discuss each of the four interpretive problems readers of financial statements may encounter.
15. Solve the following Calculation Problems:

1. Assume the following:

Equipment cost: $12 million

Estimated Life: 8 years

Salvage Value: $1.75 million

What are the second year’s depreciation expenses and asset balances under each of the three reporting methods?

2. Assume the following:

Beginning Inventory: $2 million

Ending Inventory: $4 million

Purchases: $10 million

What is the company’s Cost of Goods Sold?

# Part II: Ratio Analysis and Forecasting Modeling

Part II: Ratio Analysis and Forecasting Modeling

Chapter 5: Financial Ratios and Forecasting; Liquidity and Solvency Ratios

Chapter 6: Financial Ratios: Profitability, Return Ratios, and Turnover

Chapter 7: Financial Ratios: Market Ratios

Chapter 8: Corporate Financial Projections: Accounting Income, Cash Flow and Depreciation

Chapter 9: Corporate Forecasting Models

# Chapter 5: Financial Ratios and Forecasting; Liquidity and Solvency Ratios

# 5.1 Chapter Five: Learning Outcomes

Learning Outcomes

In this chapter you will:

- **Place** Ratio Analysis within the forecasting context
- **Implement** Cross-sectional and Longitudinal Analyses
- **Distinguish** Liquidity from Solvency Issues
- **State** from rote memory all Liquidity and Solvency Ratios
- **Interpret** the meaning of each ratio
- **Apply** each ratio in context
- **Speculate** as to why a ratio may have the value it reflects

## 5.2 Financial Ratios and Forecasting

Now that we are done, for now, with reading and interpreting financial statements, let's discuss why these accounting data are so important and what can be done to make the interpretive process more effective. Keep in mind that the purpose of releasing Financial Statements is to enable effective credit and investment decisions, i.e., decisions regarding the future prospects of a business entity.

Potential and current lenders wish to assess the creditability of the firm and shareholders wish to assess the returns their investments will earn. Will the borrowing corporation be able to make timely interest payments on its debt? Will shareholders receive sufficient dividends and growth in their capital investments, i.e., increases in share prices, to justify owning a portion of this corporation after considering the risks involved?

A good predictor of stock price performance may be book value – assets less liabilities, and earnings. A recent study[1] has found that the usefulness of these predictor variables has become attenuated in recent years, more particularly since the crisis of 2000-2001. A more useful predictor is cash flow. Cash flows are easier to predict than earnings and book values and have greater impact on price than the others.

Yet, regulators and the accounting profession continue producing rulings that make reading statements more com-

1. See *The End of Accounting*, by Baruch Lev and Feng Gu (Wiley, 2016)

plex and therefore opaque. Accrual accounting, despite its flagrant flaws, has become a faith, rather than a science, that sufficiently describes a certain reality. Does one really need to pore through two hundred pages of arcane financial statement information in order to make an investment decision? Isn't the purpose of a statement to simplify and clarify the financial condition and productivity of the corporation and its management?

Much has changed in recent decades. We now live in a much more technological world. Communications are faster. Investment analysis and its implementation are speedier. When we look at financial statements, little, if anything, has changed for the last one hundred years. Statements look the same; the accounts are the same. Double-entry bookkeeping has not advanced since it was developed in Italy by Luca Pacioli in 1494! Intangibles, including brand names, patents, customer lists, R&D-in-process, training, and information systems, are not carried on the balance sheet, but usually expensed only as incurred. More and more, investment decisions are made using data outside of the accounting sphere. Clearly, accounting reporting methods need updating[2].

2. There were two more crises; the Banking Crisis of 2007-2008, and the COVID-19 outbreak (2020) which led to a market crash, which was NOT due to any financial causes.

# On Achievement

*The secret to getting ahead …is getting started.*

-Mark Twain

*The secret of achievement is to hold a picture of a successful outcome in the mind.*

–Henry David Thoreau

*Ninety-nine percent of the failures come from people…*

*who are in the habit of making excuses.*

-George Washington Carver

Scientist and Inventor

*Success is the ability to go from failure to failure without losing your enthusiasm.*

–Winston Churchill

## 5.3 Financial Ratios

**Ratio analysis** is fundamental to finance, and ratios are regularly used in this discipline. Ratios are basic tools of analysis. It is *very* important, therefore, that you master the following.

It is important to understand that *ratios do not provide answers, but instead raise questions*, which the analyst must then resolve. Questions raised must be investigated. Analyst speculations should be based on knowledge of financial statements, the macroeconomic and industry backgrounds in which the company operates, management profiles, and company history. The ratios themselves are derived from accounting data, which are subject to interpretive issues.

A ratio consists of two numbers, which are compared to one another. "2:1" (*two-to-one*) or "1:3" are ratios. In fact, any simple number is a ratio because it can always be compared to one! Fractional numbers also pertain. We may also think of ratios as consisting of a numerator (on the top) and a denominator (on the bottom), e.g., 2/1 and 1/3 above.

No (financial) ratio means anything by itself – it must be compared to some other datum in order to derive inferences. The means by which the comparison may be made are:

**Longitudinal (Vertical)**: This would involve comparing a

ratio for a company at two different points in time. For instance, we may say that IBM's profit margin last year was 10% versus this year's 20%. What are the relevant *trends*? Longitudinal analysis data can be effectively diagrammed.

**Cross-Sectional (Horizontal)**: here we would compare two companies on the basis of a selected ratio at the same point in time. This might lead us to conclude, for instance, that one company is more profitable than another at present.

At times, the construction of a financial ratio may involve comparing a Balance Sheet datum with an Income Statement number. This presents a problem in that the former statement is a static, or "as-of" based statement, whereas the latter is a flow, or moving picture. In order to compare apples to apples, i.e., when comparing an Income Statement with a Balance Sheet number, the analyst may choose to use an average number for the Balance Sheet datum in order to make the Balance Sheet number appear more like a "flow." The average may be calculated in various ways, e.g., by **averaging** two consecutive year-end data, using quarterlies, or, if available, monthlies.

Averaging may be especially warranted in cases where balance sheet data fluctuate widely due to seasonality or other factors. For example, a snow-mobile manufacturer may have much inventory in September and far less inventory in March. In such a case, it may be advised for the analyst to average the inventory number over, say, four quarters. In the pages to follow, we will learn how this averaging is actually done by the analyst – and under what circumstances.

Below are some important financial ratios, listed by category. When would you utilize certain ratios? How may your choice of ratio vary from sector to sector (or industry to industry)? In many, if not most, instances there is no universal – minimum, maximum or average – "ideal" ratio that relates to all situations. The ideal ratio, if there is such a thing, is a function of numerous variables, including the nature of the industry, the company, the management risk profile, and perhaps the shareholders' risk profiles and goals as well.

As we go through the discussion regarding Ratio Analysis, some of you may wish to calculate the generic ratios provided using an example. At the end of Chapter Seven, the reader will find a page entitled "Simple Ratio Analysis Exercise." That page contains a Balance Sheet and Income Statement for a fictitious company; a ratios worksheet follows on the subsequent page, with the solutions following that page. You may work through the ratios there.

Keep in mind that the actual ratios' arithmetic calculations are secondary to the primary and underlying understanding of what the actual ratios are in the abstract, and what they mean. Once you understand the ratios themselves, e.g., what it means when we say that "average collections are 32 days," or that the TIE Ratio should exceed one, the calculations become a matter of a simple arithmetic exercise, a mere changing of the inputted numbers to fixed formulae. That said, you will find a comprehensive exercise, including a Balance Sheet and Income Statement, requiring the calculation of all our Financial Ratios below – in Section #7.5. As you calculate, don't forget the numerous faults with accounting data!

## 5.4 Longitudinal vs. Cross-sectional Analysis (Example)

Definitions:

- **Longitudinal**:
  - Different times
    - Over time
  - Same company
- **Cross-sectional**:
  - Same time
  - Different companies
    - Company-to-company

In all instances the ratios presented will be the same.

Exercise: Describe the longitudinal and cross-sectional relationships for the profitability ratios in the table below.

| | Profitability Ratios | |
|---|---|---|
| | **ABC Inc.** | **XYZ Inc.** |
| **Now** | 5% | 10% |
| **Then** | 3% | 7% |

Description:

- Both ABC and XYZ Corporations are more profitable now than then.
- XYZ is more profitable both now and then than ABC.

# Financial Ratios Do Not Provide Answers

| | |
|---|---|
| Col. Jessup (Jack Nicholson): | You want answers? |
| Lt. Kaffee (Tom Cruise): | I think I'm entitled to… |
| Jessup: | You want answers? |
| Kaffee: | I want the truth! |
| Jessup: | You can't handle the truth! |

-A Few Good Men (1992)

## 5.5 Liquidity and Liquidity Ratios

Note:

Toward the end of chapter 5, the student will find a "Ratio Analysis Exercise." As we go through and explain each of the twenty financial ratios below, the student will, one-by-one, calculate the ratios for the example given in the exercise. You will find that the example is constituted by a fictitious company's Balance Sheet and Income Statement.

We are going to list 20 ratios – covering six categories– in this section. After this discussion, an exercise in calculating all the ratios is presented so that you may apply what you learned.

The first category is **liquidity ratios.** The idea of **liquidity** has to do with the ease and speed with which an asset may be converted into cash – without compromising its "true" worth or "intrinsic value." It must satisfy both conditions in order to be considered liquid.

For instance, it is possible to sell virtually anything at a "fire–sale price," if one needs the cash. The question is whether an item worth, say $100, will fetch $100 – or less. If less, we may say that the item is not liquid, as the seller

did not obtain its true worth, even though s/he was able to obtain some cash for it.

An example of a liquid asset would be IBM stock. Let's say it is trading for $190. Should one wish to sell 100 shares of the stock, s/he would obtain approximately $19,000 for it. However, if I wanted to sell my tie, for which I paid $50, it may be difficult for me to get that amount. Whether you wish to consider the asset's "value" a matter of its original cost or its "current value," is a matter of analytic choice and context.

The notion of liquidity is closely tied to another notion: "marketability." This has to do with the availability of a market in which purchases and sales may be readily transacted. For instance, the stock market provides IBM common shares with a great deal of marketability. Should I however wish to sell my tie, I would not know where to go to find a "used-tie market." Real estate markets are dominated by brokers who advertise frequently, and I may easily sell my car to a used car dealer.

In the case of your typical corporation, it will not own other companies' stock. It will instead produce goods and services for sale and profit. Thus, it will regularly carry inventory and accounts receivable, which are reported as "current assets." The net current assets (current assets less current liabilities), being short-term, are presumably liquid. If the assets are liquid, the firm will be able to quickly convert the assets into cash, which will be used to honor its current liabilities due. If not liquid, the firm will have difficulty paying its current liabilities; it will be said to have a "liquidity problem."

Now that we know what is meant by liquidity and we

understand that the liquidity notion applies to "net current assets," let's quantify the notion by detailing some relevant ratios that put numbers to the idea. The greater the measurable liquidity the less the *risk*that the firm will not have sufficient funds to defray its short-term payment requirements.

**Current Ratio = Current Assets Divided by Current Liabilities = CA ÷ CL**

The current ratio gauges the extent to which a corporation has the liquidity with which it may honor its short-term obligations or liabilities. Does it have the cash to pay its suppliers when accounts payable are due? Are inventories and accounts receivable *sufficient* (and liquid) to cover its current liabilities? We will assume, for now that the inventory and accounts receivable are completely liquid.

It stands to reason that a watermark level for this ratio should be 1:1 or 1x ("one times"); one would not want to see this ratio go below that figure as it would imply that the company does not have sufficient current assets and, hence, the liquidity to meet its current obligations. Many analysts agree that 1.5x or 2x are reasonable figures, but we should be leery of such arbitrary minima. Companies that are very efficient in the manner in which they manage their current assets may have current ratios close to 1. In general, ratios will vary from industry to industry. Note that this ratio is affected by the choice of inventory costing method (e.g., FIFO vs. LIFO) because inventory is a constituent of current assets.

**Negative Working Capital**

We have agreed that a "financially healthy" corporation will have a current ratio exceeding one, which is also to say that its Working Capital is positive (WC = CA – CL > 0). Yet, some, albeit few, good companies have negative Working Capital, but are still financially strong and exhibit strong stock prices. Historically, this list has included Amazon, Dell, and Walmart. Does this not mean that these companies have liquidity problems and may be unable to pay their bills timely?

Not necessarily. It probably means that these companies are collecting cash from sales more quickly than they have to pay their bills, especially to their suppliers. They then, invest in more inventory (increasing their accounts payable, a current liability), fixed assets, and R & D, as their growth paths continue sharply upward. Such investments reduce the company's working capital and current ratio. So, how do they generate such rapid liquidity to pay bills?

In some circumstances, they may honor customer credit card purchases at the point, or time, of sale (even if before shipment of the goods), and collect the cash from the card vendors on the very next day, or long before they have to pay their payables. With continued growth, this phenomenon may go on for a long time.

This means that although growth in both sales and working capital usually USE cash, in such instances, growth actually PROVIDES cash!

**Walmart**

Here is what Walmart has to say about its current ratio (source: Walmart 2017 Annual Report, page 26):

> *The increase in our working capital deficit reflects the company's leverage achieved through savings from procuring merchandise and improved inventory management. We generally operate with a working capital deficit due to our efficient use of cash in funding operations, consistent access to the capital markets, and in providing returns to our shareholders in the form of payments of cash dividends and share repurchases.*

Note: Working Capital = Current Assets less Current Liabilities. If the current ratio is less than one, the working capital will be negative. (See Ratio Exercise.)

## Quick Ratio ("Acid Test") = (CA – Inventory) ÷ CL

Some analysts take a more conservative approach in the assessment of liquidity coverage by *assuming* that inventory is worthless (even though it may, in fact, not be – or not be entirely) and hence that current liabilities will be covered by all current assets except inventory. Indeed, in some industries inventory quickly becomes obsolete due to technological advances; in other cases, inventory may be easily damaged, or lose its fashion and sales appeal.

The current and quick ratios may be thought of as "coverage" ratios in that they indicate the extent to which liquid assets are adequate to "cover," or pay off, current liabilities, especially accounts payable and notes payable (i.e., short-term loans).

**Average Collection Period (ACP) = Days sales outstanding (DSO) = (A/R ÷Credit Sales) (360)**

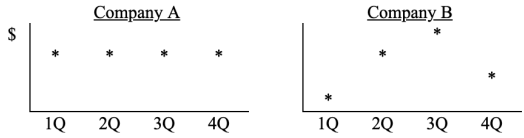
"A/R" stands for Accounts Receivable. This ratio tells the analyst approximately how long it takes the firm to collect its receivables; the ratio is expressed in days. The shorter the collection period, the more liquid the firm's accounts receivable.

Note that we use credit sales because only credit (and not cash) sales become accounts receivable. This ACP figure may then be compared to the firm's typical credit terms. If, for argument's sake, the ACP *exceeds* the firm's credit terms, which may be 30 days, the analyst may first assess whether the firm is having collection problems. Alternatively, the analyst may investigate whether the firm is acting aggressively in its marketing, and choosing to live with the consequences of some late payments as a result. Perhaps the firm expects that the incremental profits will exceed any losses in its collections.

This ratio is our first case, so far, of "apples and oranges" in that, here, we note that both income statement and balance sheet numbers appear within the same ratio. This inconsistency needs to be fixed; we ought not to combine one datum from the Balance Sheet, which is a static figure, with another from the Income Statement, which is a flow kind of number, within one ratio. The two are inconsistent with each other.

In such cases as this, what the analyst may choose to do is “average” the balance sheet number. One way of doing this would be by taking this year’s and last year’s year-end numbers and averaging them(i.e., add the two numbers up and divide by two). We could also use quarterlies or, if available, monthlies. In businesses where receivables fluctuate over the course of the year and may thus be either relatively high or low at certain times of year or during certain seasons, it may be better to average the quarterly numbers, if possible.

This technique would then capture a better sense of movement over the course of the seasonal business cycle and would thus provide a better comparison with the income statement number, which, again, flow over the course of the entire year. If the balance sheet number does not fluctuate – much, or at all – over time, there may be no need for averaging. This choice is at the discretion of the analyst. Compare Companies “A” and “B” in terms of their accounts receivable or inventory levels, as it were, over the course of four quarters. Is anything accomplished by averaging the Balance Sheet numbers for Company A?




You will also note that (usually) Accounts Receivable will display a smaller number than Sales. Normally, we would expect that accounts receivable are collected more or less monthly; indeed, *if* sales terms are 30 days, we

should get about 12 collections per year. This is illustrated on the next page.

We shall also assume a 360-day year, consisting of twelve 30-day rolling months, because most often collections are demanded within 30 days. Some analysts will use a 365-day year on the rationale that Credit Sales accumulate over a 365-day calendar year. It's your choice, Ms/r. Analyst!

Final note: If you are analyzing data for just one quarter, you should use 90, rather than 360, as the multiplier for the ACP. For two or three quarters, you will use 180 and 270 respectively.

**Inventory Turnover = COGS ÷ Inventory**

This ratio tells us how often the inventory "turns over," that is to say, how often the inventory – in its entirety – is replaced. Would you buy fresh tomatoes from a company whose inventory turnover is 10x a year (or every 36 days)? Note that this ratio is affected by the choice of inventory costing method. This ratio has some issues with mixing Balance Sheet and Income Statement numbers, which was discussed the prior section concerning ACP.

## 5.6 The Income Statement versus the Balance Sheet

It is important to understand how related Balance Sheet and Income Statement numbers may compare with one another, especially when used in a particular ratio.

For instance, in the Average Collection Period, we compare MONTHLY credit sales (from the Income Statement) to MONTHLY accounts receivable (from the Balance Sheet). Which number is bigger? Take note that the quarterly data for the Credit Sales are cumulative as reported by the accountant. We are also assuming that the receivables are collected in full at each month's end. Let's see.

***Fictitious Company***
**Fiscal Year Ending (FYE) 12.31.20XX**
**($ Millions)**

| Month | Credit Sales | Accounts Receivable |
|---|---|---|
| Jan | 100 | 100 |
| Feb | 200 | 200 |
| Mar | 150 | 150 |
| 1st Qrtr. | 450 | 150 |
| Apr | 150 | 150 |
| May | 250 | 250 |
| Jun | 300 | 300 |
| 1st Half | 1,150 | 300 |
| Jul | 250 | 250 |
| Aug | 200 | 200 |
| Sept | 150 | 150 |
| 3rd Qrtr. | 1,750 | 150 |
| Oct | 150 | 150 |
| Nov | 100 | 100 |
| Dec | 50 | 50 |
| Full Year | 2,050 | 50 |

In the table above, we show credit sales on a monthly basis for the "Fictitious Company" over the course of the fiscal year ended (FYE) 12.31.20XX. We are assuming that credit terms are 30 days and that all receivables are collected on time. Which number is greater – the credit sales or the accounts receivable?

Clearly, after February, Credit Sales, which are cumulative, will exceed accounts receivable. A similar analysis may be applied to the inventory turnover ratio.

## 5.7 Accounts Receivable Aging Schedule

An aging schedule is an internal, managerial, and confidential document, which shows, account by account, how much money is due the seller from the time of sale, and whether a customer account is “current” or is “past due.” It is a necessary management tool.

The schedule would provide the external analyst with much more than s/he may otherwise know by alternatively calculating Days Sales Outstanding. It is much more detailed. Unfortunately, the schedule is an internal management document and unavailable to outsiders.

The customer accounts are listed row by row, either by name or some some other identifier. Since most credit sales require payment within 30 days, the first column shows how much money is due where the sale occurred less than 30 in the past. As you read to the right, you will see uncollected credit sales figures that are more than 30, 60, and 90 days old, and the respective amounts due. Thus, the schedule might look somewhat like the following. In examining the schedule, what might you tentatively conclude about each of the five listed accounts?

| Customer Account | Number of Days Since Sale | | | | |
|---|---|---|---|---|---|
| | < 30 | 30 - 60 | 61 – 90 | > 90 | Total |
| #1 | $200,00 | 0 | 0 | 0 | $200,000 |
| #2 | $150,00 | $25,000 | 0 | 0 | $175,000 |
| #3 | $75,000 | $25,000 | $25,000 | 0 | $125,000 |
| #4 | 0 | $25,000 | $25,000 | $50,000 | $100,000 |
| #5 | 0 | 0 | $50,000 | $50,000 | $100,000 |
| *Total* | $425,00 | $75,000 | $100,000 | $100,000 | $700,000 |

By examining this schedule, the analyst may uncover some interesting findings. For instance:

1. ACP/DSO may be higher than the company's sales terms due, possibly, to one very large account skewing the average. It may also be greater due to the company's decision to enter a new geographical or product market, knowing in advance that such entry will slow down its collections, but expecting that the added profits will make the expansion worthwhile.
2. The analyst may see how many accounts are current and how many should be written off – analytically – by the analyst. (Remember: this is not accounting, so a real "write-off" would be recorded by the accountant.)
3. This schedule does not tell the reader anything about the profitability of the accounts (but neither does the ACP). It could very well be, for instance, that account #1, which apparently pays well, is also purchasing very low margin products. While not necessarily of great importance to the Accounts Receivable manager, the marketing manager may be very interested in profitability.
4. Overall, it appears that the company

has $425/$700, or 60.7% of its receivables that are “current,” i.e., not past due – assuming 30-days collection terms.

5. The Aging Schedule implies a certain ACP. In order to calculate the ACP, it is necessary to know the Credit Sales figure, which is publicly available from the company’s Income Statement.

## 5.8 Solvency Ratios

**Solvency** has to do with the firm's ongoing ability to generate sufficient liquidity from Operating Earnings (EBIT) in order to meet its continuing debt-service obligations, particularly interest payments, timely and fully. The two most common solvency ratios have to do with "leverage," or the extent to which the firm employs debt as a source of capital as opposed to using equity.

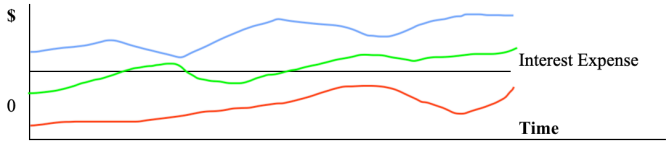
Some debt is desirable as it allows the firm's owners or shareholders to exploit "other people's money" (**OPM**) in order to earn greater profits on a per share basis than they would in the absence of debt, i.e., OPM. Too much debt, however, may be a bad thing as it increases the firm's riskiness by adding on an increasing requirement to pay interest on the debt.

**"TIE" Ratio = Times Interest Earned (or Interest Coverage) = EBIT ÷ Interest Expense**

**EBIT** stands for "Earnings before Interest and Taxes." Over time, a company's operating earnings (EBIT) may fluctuate, the extent of which fluctuation depends on the company's business and its industry. For example, a lawnmower (or snowmobile) manufacturer will have high sales and inventory at certain times of year and low at other times; we refer to this phenomenon as "seasonality." The company's interest expnses is relatively flat over time because the company does not often borrow and its interest expenses are fixed.

In any case, this ratio looks at the extent to which EBIT exceeds the interest expense on its debt over time. Is this margin (or buffer) a “safe” amount? (Some analysts will add back depreciation and amortization to EBIT to arrive at “EBITDA” – operating cash flow – on the rationale that interest is paid with cash.)

In the diagram below, you will find inserted three possible EBIT curves for a fictitious firm; each possibility is represented by a different colored curve.. The “wavier” the EBIT line, the more volatile its operating earnings are/will be over the course of some periods or years. The question is: overall, which curve represents the most secure TIE ratio and why? The implications profoundly relate to the firm’s Solvency!




EBIT may look like a kind of horizontal squiggle, the *squigglyness* of which depends on the volatility of the firm’s operating earnings. The question then is whether interest expense (the straight horizontal line), which should be relatively flat over time, is above, below, or somewhere in the middle of the squiggle. Interest expense would appear as a horizontal line, since it is relatively flat over time because the company’s level of indebtedness changes little, if at all, over time.

Again, the company’s EBIT should be sufficiently above the interest expense line, the extent of which differential

depends on the nature of the company's business and the manner in which the company is managed. A contingency may arise that if EBIT is low relative to (or worse, below) the interest expense, will the firm have the capacity to make the required and timely (cash) payment of interest? Persistent insolvency may lead to bankruptcy.

**Debt ÷ NW**

By "NW," we mean "net worth," or (total) "equity" (remember the basic accounting equation: A – L = NW). Analysts will disagree as to what is meant by "debt." Some will concern themselves only with long-term liabilities; others will include current liabilities among "Debt." In most instances, and for our present use, we will refer to "total liabilities" as debt, a reasonable default assumption since Debt + NW = 100% of assets, thus excluding nothing on the Balance Sheet.

| | |
|---|---|
| $1,000 | $500 |
| | $500 |

While one cannot say what the ideal "debt-to-net-worth ratio" ought to be, it is safe to say that as the ratio increases, the company's ongoing ability to service its debt (i.e., to pay interest in particular) is reduced, and the likelihood of bankruptcy increases. In general, the ratio will change from industry to industry. Some industries are characterized by greater indebtedness, or "leverage" than others. We will have more to say about this later. This ratio may be less than or greater than one. In the Balance Sheet to the right, the Debt-to-Net Worth Ratio is 500:500 or 1:1, "one to one." The Total Assets are $1,000.

**Debt ÷ TA**

The default option for defining "debt" is again "total liabilities." Again, remember that Total Assets = Debt + Equity (TA = D + Eq). In this case, as the ratio increases, the company's solvency becomes increasingly at risk.

This ratio should not be thought of as "coverage" ratios, in the same sense that the current and quick ratios are (or for that matter as the TIE ratio is). In the latter cases, we view the adequacy (or inadequacy) of current assets as sufficient – or not – to pay for the company's current liabilities. The firm needs liquid current assets to pay off its current liabilities.

Such is not the correct manner, in which to interpret the debt ratios. The company does not need its equity to "pay off" its debts! It is incorrect to say that the net worth, or equity, of the company is "adequate" in this sense. The only time that that equity will be used to pay off debt is in the worst case of bankruptcy and (net) asset liquidation! In most cases, you will be analyzing an ongoing enterprise, not a bankrupt one.

Instead, these two debt ratios (D/NW and D/TA) are simply measures of the *magnitude* of a company's indebtedness and, as such, are useful in comparison to other companies in its industry, and to itself over time. Does *XYZ Corporation* have a lot of debt in comparison (or in proportion to) to its industry peers? Is its indebtedness increasing?

A more useful solvency coverage ratio, if you are looking for one that most directly addresses solvency, is the "Times Interest Earned" ratio, above. The "TIE" ratio provides a

gauge of the company's ability to pay (i.e., to "cover") the interest payable (or paid) on its debt.

The D/TA ratio above would be 500/1,000 or 50%. While the D/NW is more popular, the D/TA ratio gives the analyst an *imaginary* range of 0-100% of debt in comparison to total assets. The D/NW ratio gives no such reasonable range. In any case, one ratio can be easily inferred from the other.

**Total Assets ÷ Equity**

Another solvency ratio is D / TA. This ratio will be presented and discussed when we get to the "DuPont Model" somewhat later (see Sections 6.3 and 6.4).

Leverage

In summary, the more debt a company has relative to either its Equity or its Total Assets, the more **leverage** it is said to have, by definition. In Physics, a "lever" allows one to get more output with the same input as compared to not using the lever.

## "If at First...."

*I can accept failure; everyone fails at something.*

*I cannot accept not trying.*

-Michael Jordan

*A person, who never made a mistake, never tried anything new.*

-Albert Einstein

*I have not failed. I've just found 10,000 ways that won't work.*

-Thomas A. Edison

# Chapter 6: Profitability and Return Ratios, and Turnover

## 6.1 Chapter Six: Learning Outcomes

Learning Objectives

In this chapter you will:

- **State** by rote memory all the ratios in this chapter
- **Interpret** each ratio
- **Distinguish** between Operating and Net profits in the use of the ratios
- **Explain** under what certain circumstances Turnover Ratios are used

# 6.2 Profitability, Return and Asset Turnover Ratios

## Profitability:

| | |
|---|---|
| Gross Profit Margin | Pre-Tax Profit Margin |
| Operating Profit Margin | Net Profit Margins |

Certainly, we are all interested in profitability. Each of the foregoing data (without the word "margin") may be divided by "Total Sales" and used cross-sectionally or longitudinally. "Margin" means percentage. For example, gross profit margin = gross profits ÷ sales.

In discussing liquidity ratios, we agreed that in order to manage liquidity risk the current ratio should be at least one.We can also agree that too high a current ratio may be unhealthy. This is because current assets, especially cash, produce no return. Having inventory sitting around does no good.

A question to ask is: Why has the firm accumulated cash and other current assets rather than investing in more productive, fixed assets? There must be a balance between liquidity on the one hand, and return (see below) on the other. Too much of anything may not be good.

Even though the firm may have high profitability, it does not mean that its cash flow position is strong. One must also examine the solvency ratios, especially the TIE ratio.

The company may have onerous obligations due to maturing debt and the need for replacement of long-term assets. A growing company may need a great deal of liquidity, in order to support growing sales, which growth would be reflected in growing inventory and accounts receivable. The analyst should also check the firm's Liquidity.

**Return Measures:**

We will discuss two important return measures. These ratios give the analyst another look at profitability.

**ROA = Return on Assets = EBIT ÷ TA**

How well is a firm employing its assets? Return on Assets attempts to gauge this. While many will use net income in the numerator, EBIT is preferable in the sense that operating profits reflect what the assets produce, whereas net income is affected by the firm's capital structure, i.e., interest expense and particularly taxes, and other possible items not directly associated with the firm's actual business or "operations."

ROA may be a good tool to judge management's operating performance and its "asset utilization," or how efficiently it exploits the firm's assets. Why may two, otherwise identical companies, have different ROAs? Does one company *manage* better? Does it motivate its employees better? Are they better trained? Do they work harder or longer hours? Does it employ better technology?

**ROE = Return on Equity = NI ÷ Equity**

Common shareholders are most interested in ROE because that is what they get for their equity investment. They understand that for each dollar of equity invested, they will have a claim on the company's net income, i.e., its dividends and (additions to) retained earnings.

What kind of return is the firm able to produce on its equity? Equity is the result of, not just the company's past operating performance (EBIT) and its associated accumulation of retained earnings, but management's capital structure decisions (i.e., the debt/equity ratio) when it had to raise capital in the past. The more debt there is, the less equity; the more interest expense, the less net income. EBIT also affects Net Income!

While ROA reflects the firm's effectiveness in producing operating profits, or EBIT, the effective use of leverage, i.e., debt, can increase the firm's ROE above its ROA. This is not guaranteed; borrowing does not ensure in all instances that ROE > ROA. In addition, "leverage," i.e., the use of debt (*OPM*), increases the firm's financial, or solvency, risk. We shall get a glimpse of this when we – soon – look into the DuPont model on the very next page. ROA and Leverage (i.e., the extent to which a firm utilizes debt) are inter-related. Stay tuned.

ROE may be a better tool (than ROA) for assessing a firm's (equity) investment merits, because shareholders are most interested in net income as that will either be distributed to them as dividends and/or added to retained earnings, which are "owned" by shareholders.

Once again, what is management providing the shareholders for their equity investment? Shareholders are interested in earnings, as it is earnings, which are either paid out as

dividends or retained, and that is what benefits the shareholders. Remember: retained earnings belong to the common shareholders.

**Asset Turnover:**

**S ÷ Fixed Assets** and **S ÷ TA**

Here "turnover" is not a physical concept as it was when we spoke about Inventory Turnover. How often do the firm's *Fixed Assets* (i.e., Property, Plant, and Equipment),or total assets, "turn over" relative to sales? In other words, how much Sales does a company produce from its fixed or total assets? If two identical firms differ in only this respect, what might the analyst conclude? Does one firm maximize price and revenues? Does one firm manage production and exploit its assets more efficiently, more productively? What do turnover ratios say about pricing, asset utilization, and management performance?

An analyst may prefer using total assets rather than just fixed assets, in this analysis, if the company being analyzed is a service company with fewer fixed assets to speak of in comparison with a heavy manufacturing firm. An analyst may use Asset Turnover rather than Return Ratios in the case where EBIT = 0.

*If you take shortcuts, you get cut short.*

-Gary Busey, actor

## 6.3 The DuPont Model

The DuPont Model was developed by managers at the DuPont Chemical Corporation for purposes of internally pinpointing strengths and weaknesses within the company's management hierarchy. Imagine you are at the top of the corporation, that you are the chairman of the board or CEO, and you answer to the company's shareholders. Arguably, the one ratio you are most interested in would be the company's return-on-equity ratio. Whether the ROE looks good or bad, you would wish to look further into the performance of the company, make improvements, increase efficiency, and buttress strong areas.

The DuPont formula provides upper management with a top-down look into the company's performance culminating with ROE, which, again, is of the most interest to company shareholders. The CEO looks into three critical areas: 1. Profitability ("Profit Margin" – see below), 2. Operating ("Total Asset Turnover"), and 3. Financial perspectives ("Leverage").

| | | |
|---|---|---|
| Total Costs | Operating Costs + Interest + Depreciation + Taxes | **Profit Margin** may be improved by reducing costs relative to sales. Advertising can increase sales. |
| Net Income | Sales - Total Costs | |
| **Profit Margin** | Net Income ÷ Sales | |

| | | |
|---|---|---|
| Current Assets | Cash + Cash Equivalents + Inventories + Accounts Receivable | **Asset Turnover** can be improved by eliminating unproductive assets, e.g., obsolete inventory. Efficiency can be increased by means of proper machinery maintenance, technology, and training. |
| Total Assets | Current Assets + Net Fixed Assets | |
| **Total Asset Turnover** | Sales ÷ Total Assets | |

| | | |
|---|---|---|
| **ROA** | Profit Margin x Total Asset Turnover = [NI ÷ S] × [S ÷ TA] = (Profit Margin)(Total Asset Turnover) = NI ÷ TA | **ROA** can be improved by increasing profit margin and asset turnover. |

| | | |
|---|---|---|
| **Leverage** | Total Assets ÷ Equity | |
| **ROE** | Return on Assets x Leverage = [NI ÷ TA] × [TA ÷ Eq.] = NI ÷ Eq. | **Leverage / ROE**: Leverage can increase ROE to a level greater than ROA – even if the firm is not utilizing its assets most productively. It must be remembered that leverage introduces greater *financial risk* to the corporation. This means that the chance that the firm will not be able to pay the interest on its debt will increase as the firm borrows more. |

Some Key Points:

1. Good business management produces a favorable ROA.
2. Proper leverage may enhance the investor's return – ROE. Without Leverage, ROA = ROE.
3. You will notice that the ROA and Leverage ratios do not match up with our earlier defini-

tions.

## 6.4 What Does the Dupont Model Show Us?

The Dupont Model enables us to locate the sources of corporate performance. Imagine you are CEO and have to report performance to shareholders.

First, looking top-down, you are interested in ROE. The model shows us the effect of leverage on ROE, given the prior level of ROA.

Then, it ascribes operating performance, as defined by ROA, as being attributable to Profit Margin and Total Asset Turnover. This enables executives to locate areas of weakness within the firm and address them.

Let's illustrate how Leverage benefits an unnamed corporation, given its ROA.

Let's assume the following:

Debt = $700

Equity = $300

Total Assets = $1,000

Net Income = $100

Therefore:

**ROA** = NI / TA = 100 / 1,000 = 0.10

**Leverage** = Total Assets / Equity = 1,000 / 300 = 3.34

**ROE** = ROA x Leverage = (0.10) (3.34) = 0.34

Leverage has increased the corporation's ROE!

Alternatively, let's say the company has no leverage, but the same in Total Assets. In other words, the assets are financed in full by equity only.

ROA = 100 / 1,000 = 0.10

Leverage = 1,000 / 1,000 = 1.0

ROE = (0.10) (1.0) = 0.10

Without any Leverage at all, ROE = ROA.

Of course, Leverage is not something that the company should employ indiscriminately. Too much debt will raise interest expense to possibly unsustainable levels and thus could have a weakening effect on the TIE ratio, leading eventually to insolvency and bankruptcy – in the worst case. The DuPont Model does not address this risk.

The Dupont Model is useful for assessment of past performance. It is a poor planning tool. Looking at this example, one may be encouraged to employ more and more risk. Of course, as just noted, too much debt is not necessarily a good thing: it increases the risk of becoming insolvent – causing the TIE Ratio to be below one.

# 6.5 Financial Ratios in Action

## Liquidity and Profitability: Costco

Costco, when you look at it, is a very basic kind of business. They run stores, organized as warehouses, and sell everyday merchandise. Take a look at their 2019 report[1]. On pages 34-35, you'll find Costco's Balance Sheet and Income Statement. Look at its profitability ratios. What is its current ratio, and Total Liabilities-to-Total Assets Ratio? How much of its current assets are tied up in inventory? Here is Costco's more recent (2021) report:

https://investor.costco.com/static-files/0878117f-7f3f-4a77-a9a5-c11a2534e94d

## Solvency: Some Interesting History

General Motors became insolvent in 2007-2008. Apple Computers borrowed a huge amount of money in 2013. Here are their stories.

**The Bankruptcy of General Motors**

Here is General Motors' Financial Report for 2007[2], just

1. Costco. (2019). 2019 Annual report: Fiscal year ended September 1, 2019. Costco investor relations. https://investor.costco.com/static-files/05c62fe6-6c09-4e16-8d8b-5e456e5a0f7e
2. General Motors Corporation (2007). General Motors Corporation 2007 annual report. Annual reports. https://www.annualreports.com/HostedData/AnnualReportArchive/g/NYSE_GM_2007.pdf

before it declared bankruptcy. Notice the "splash" in its early pages; it is replete with attractive product photographs. Since there is so much stuff in the report to read, try to focus on the following. On page 82, you will notice that it lost money – a lot. On page 83, take a look at the Balance Sheet, especially Total Assets, Total Liabilities, and Equity. Does it come as any surprise that its auditors complain of poor "internal controls" on pages 80-81?

There was a time when it was said that "as GM goes, so goes America." Well, that was long ago. In 2009, GM filed for bankruptcy[3].

**The Case of Apple Computers**

Apple Computers issued a huge amount in debt in 2013 in spite of its having, at the same time, a great, as in really a lot, deal of cash. Here's why they sold (a.k.a., issued or borrowed) debt.[4]

Here are Apple's Balance Sheets from 2005-2019[5]. Notice how Long-term Debt went from zero to $17 Billion from

3. Isidore, Chris (2009 June, 2) GM bankruptcy: End of an era. CNN Money. https://money.cnn.com/2009/06/01/news/companies/gm_bankruptcy/
4. References Balassi, J., & Cox, J. (2013). Apple wows market with record $17 billion bond deal. Retrieved Sep 20, 2021, from https://www.reuters.com/article/us-apple-debt/apple-wows-market-with-record-17-billion-bond-deal-idUSBRE93T10B20130430
5. https://www.macrotrends.net/stocks/charts/AAPL/apple/balance-sheet

2012 to 2013. Calculate its Debt-to-Assets ratio for both years.

# Chapter 7: Market Ratios

# 7.1 Chapter Seven: Learning Outcomes

Learning Outcomes

In this chapter, you will:

- **Consider** "Relative Value"
- **Discriminate** between Price-to-Book Value and the Price-Earnings Ratio as measures of Relative Value
- **Explore** dividends as a discretionary managerial decision
- **Interpolate** financial statements data into all the ratios
- **Connect** the DuPont Model with management performance
- **Place limits** on the power and authenticity of financial ratios

## 7.2 Market Ratios

**PE Ratio = Market Price per share ÷ Earnings per share**

**Note:** Earnings per share ("EPS") = Net Income divided by the number of shares the company has ("Number of shares outstanding"). For example, if a company earned $1 million and has 500,000 shares outstanding, its EPS is $2 per share.

Key Terms:

Value Stock

This, rather than the dollar price, may be used as a measure of "cheapness" or "expensiveness" of a stock. "Value" stocks are generally characterized by low PE ratios. A **Value Stock** may be thought of as such because its price – relative to its earnings – represents a "good buy" for the money, i.e., cheaper. The analyst must be cognizant of the possibility that a low PE may also mean that its future earnings prospects are generally deemed unfavorable.

In contrast, **Growth Stocks** will reflect (very) high PEs on the basis of generally optimistic views of future earnings growth. Does this mean that earnings will necessarily catch up with price? Let's say that a company's PE = $50/$1 = 50 times earnings. That is a very

high ratio. If the market anticipates next year's earnings to double, the stock may be said to be trading at just $50/$2 = 25 times next year's earnings, a more reasonable ratio.

Growth Stock

High PEs are the effect of the high expectations the market has for future growth in a company's earnings. As a company's earnings grow in the future, the multiple that one paid for his shares goes down, making the purchase, with the benefit of hindsight, a good choice.

For example, if a company earns $1 per share and sells for $50, its PE will equal 50x earnings, quite high, quite "expensive" for what you get. If, however, the earnings next year do indeed grow to say, $5 per share, the buyer at last year's price who purchased the stock for a meager $50, or only 10x next year's earnings, got a good deal. Therefore, companies with high growth expectations will manifest higher PE ratios than the boring companies with low growth prospects. A stock analyst might have said that the PE ratio is now 50x, but only 10x *next year's* expected earnings – if s/he so prophesied.

At the time this is being written, average PE ratios for the S& P 500 Index of (publicly-traded) stocks are approximately "20 times earnings." Why would someone pay so much for each dollar of earnings? In contrast, privately-owned stocks, i.e., those not traded on the public-markets, will have much lower PE ratios.

Liquidity Premium

One explanation has to do with the **Liquidity Premium** associated with publicly traded stocks. “Liquidity” has to do with the ease and speed with which a seller can convert an asset, in this case stock, into cash while obtaining a sales price that represents that asset’s true value, i.e., without reduction in price from its true, or “intrinsic” value.

It is generally very easy to buy and sell a stock. All one needs to do is place an order with a stockbroker (and pay for it within the required three-day settlement period for stock). There is an added or premium value in this liquidity. That is why “Private Equity” investments can be so profitable. A private company will sell at a much lower PE ratio due to the lack of liquidity. If the company is later sold in an IPO (“initial public offering” – the point at which time a stock is sold to the public for the first time), there will be much profit to go around!

The true cost of buying a privately owned company, i.e., one whose stock or equity is not traded on the public stock exchanges, is not embedded in the price one pays for buying such private corporations. In order to purchase a privately owned company, one has to search for an appropriate one, one that is also available to purchase. This involved time and time is money. The potential buyer is spending time engaged in the search rather than pursuing other profitable opportunities. This is not an issue where publicly traded stock is concerned. These companies are readily available for rapid purchase and sale.

Moreover, in order to buy a private company, one must employ the services of an attorney to negotiate and close

the deal. Bankers must be involved because private stock is usually purchased with large (and often borrowed) sums as opposed to the purchase of a minimum number of shares of stock in a publicly traded stock corporation with few funds. These other costs must be reflected as opportunity costs and thus, under the "Law of One Price" ought to be reflected in the prices of publicly traded stock. Law of One Price requires that similar assets ought to be priced similarly.

There are certain demerits to the PE Ratio. For instance, Earnings are subject to accrual accounting manipulations. Moreover, companies with very low or negative earnings yield a meaningless PE ratio. A company with low earnings may command a ridiculously high PE. A company with negative earnings will not yield a PE at all – it is not computable!

**Dividend Yield = $D_{per\ share} \div P_{per\ share}$**

Cash-on-cash Return

One may view this as the **Cash-on-cash Return** that a stock may provide. If I pay $10 for Yawr Co. stock and it pays an annual dividend of $1, my "cash-on-cash return" or dividend yield is 10%. Some industries will reflect higher dividend yields than others. Dividends are usually paid from current net income, but if there are losses, dividends would have to be paid from retained earnings. Typically, mature, rather than growth companies, pay high dividends. This yield ignores price changes.

Total Return

It is eminently noteworthy that the dividend yield excludes any capital growth, which is integral to Yawr Co. stock's **Total Return**. In other words, the Total Return equals the dividend yield plus the capital gain (or loss) expressed in percentage terms. If you purchased this stock for $10 and sold it at $15, your capital gain would be 50%. The Total Return is: 10% + 50% = 60%.

Signal

Signaling

It is often considered a positive **Signal** that a company maintains its dividend in the face of losses. This **Signaling** indicates management's optimism about future earnings prospects. The dividend itself thus boils down to a "human decision" by the firm's board of directors, rather than a measure of business performance per se. If the board feels that the future is good, it may choose to pay dividends even if the company is presently losing money.

## Dividend Payout Ratio = PR = $D_{Total}$ ÷ NI = DPS ÷ EPS

This ratio indicates the percentage of net income, which is paid out to common shareholders as dividends. If a firm earns $1,000,000 and pays $100,000 in dividends, its payout ratio is 10%. If there is preferred stock, the preferred dividends

would first have to be deducted from the net income. The PR formula would be adjusted: PR = (Common Stock Dividends) ÷ (Net Income – Preferred Stock Dividends).

**Retention Rate = RR = 1 – PR = (NI – $D_{Total}$ ) ÷ NI = ARE ÷ NI**

This ratio indicates how much of the company's earnings are retained internally for future growth and, as such, together with the firm's ROE impact the firm's growth potential. On the next page, we discuss the relationship between earnings retention and firm's growth. (This will be explicated further below in the "EFN Model.") Naturally, PR + RR = 1.0 = 100%.

## 7.3 Earnings Retention and Growth

Our working assumption is that the firm has a never-ending appetite for growth. In order to grow its sales, and hopefully its profits thereby, the firm must retain some of its earnings and invest them in productive assets that can be exploited to increase sales in the future.

Let's examine the following company:

| | | |
|---|---|---|
| Assume: | Net Income (Last Year) | $100,000 |
| And assume: | Equity (Last Year) | $1,000,000 |
| Therefore: | Return-on-Equity (Last Year) | 10% |
| If: | Total Dividends | $10,000 |
| Then: | Addition to Retained Earnings | $90,000 |

If the company's ROE is assumed to be constant, i.e., one of those ceteris paribus assumptions, then the numbers next year will be:

| | | |
|---|---|---|
| Next Year's Equity = | Last Year's Equity Plus the A.R.E. = $1,000,000 + $90,000 = | $1,090,000 |
| Assume again: | Return-on-Equity (Next Year) = | 10% |
| Therefore: | Net Income (Next Year) = (.10) ($1,090,000) = | $109,000 |
| Growth in Net Income = | (109 ÷ 100) – 1 = | 9% |

So, as we see, earnings retention is helpful for growth. Had the company not invested its A.R.E. in productive assets,

its ROE would have declined, as would its prospective growth rate. Companies that have great growth prospects will therefore pay no dividends due to its hunger for using the entirety of its Net Income as Additions to Retained Earnings in order to increase its assets.

# 7.4 The P/BV (Price-to-Book Value) and P/E (Price-Earnings) Ratios

The price of a stock alone does not tell you anything about its value. Take two stocks, one of which is trading at $20 and has 1,000,000 shares outstanding, while the other trades at $10 and has 2,000,000 shares outstanding. Which one has greater value? They are both the same!

Instead, we will look at relative value, i.e., price relative to either of two measures: Book Value (BV) per share and Earnings per share (EPS). Book Value per share is measured by the accountant's assessment of the company's equity (excluding preferred stock, if any) divided by the company's number of (common) equity shares outstanding (NOSO). We already know how to calculate EPS.

Let's take the example of two companies that have the same share price, the same number of shares outstanding, but different book values per share (i.e., assets minus liabilities divided by the number of shares outstanding). Which stock is a better "value"? Which is a better investment?

| | Number of Shares Outstanding | Accounting Book Value per Share | Stock Market Share Price |
|---|---|---|---|
| **LCM Co.** | 1,000,000 | $75 | $100 |
| **TC Co.** | 1,000,000 | $50 | $100 |

At the same price, it *seems* that LCM is a better buy. LCM provides greater BV for each dollar of share price. Clearly, while the market price alone does not necessarily tell you

whether the stock is a "good buy" or not (both stocks here have the same price), there is also some question as to whether Book Value is a reasonable measure.

The balance sheet perspective (i.e., "accounting book value," or the company's net assets) depicted above is fraught with the many interpretive accrual accounting problems enumerated earlier (specifically, we enumerated four issues). There is likely more historical bias embedded in book value than in earnings. On the other hand, Price-to-Book (P/BV) tends to be a more stable measure than the PE ratio. Further, P/BV ignores how well management utilizes the assets in producing earnings for shareholders. Earnings certainly matter! Earnings are what most shareholders focus on.

**Summary: Price-to-Book Value**

**Pro**

- More stable than PE

**Con**

- Subject to accrual accounting idiosyncrasies
- Most shareholders focus on earnings
- Ignores management effectiveness
    - Asset utilization

- Earnings production

Another way of making the assessment of whether the stock price is "fair" or not, would be by looking at the stock's P/E ratio, i.e., Price (per share) ÷ Earnings (per share). How much does the prospective (or current) shareholder pay for a share of the stock in order for him to "earn" so much in terms of EPS? Remember that the shareholder is most interested in earnings per share because that is the source of his dividends and retained earnings, a portion of which he owns. In contrast to the first approach, this approach is income statement oriented. **You should be able to calculate EPS and PE independently.**

| | | LCM | TC |
|---|---|---|---|
| Market Price (per share) | **P** | $100 | $100 |
| Net Income | **NI** | $1.0 million | $100,000 |
| Number of shares outstanding | **NOSO** | 1 million | 1 million |
| **Earnings per share (= NI ÷ Sh.)** | **EPS** | **1,000/1,000 = $1** | **100/1,000 = $0.10** |
| **P/E ratio (= P ÷ E)** | **P/E** | **100/1 = 100x** | **100/0.10 = 1,000x** |

EPS are $1 and $0.10 per share for LCM and TC respectively. Clearly, however, LCM is "cheaper," with a PE ratio of 100x earnings, whereas TC reflects a ratio of 1,000x its earnings – if you accept PE as a valid valuation/pricing measure. (The PE ratios presented in this example are much higher than will be typically found in the markets – even in good times.)

When an investor purchases a stock, s/he is making an investment in the company's ability to generate profits. (This is not to say that the assets have no value.) The profits, in turn will be either distributed to the shareholder as a (cash) dividend – or retained by the corporation and then reinvested in additional assets that should produce even more profits going forward.

We have already cited the basic problems in utilizing (legitimate) accounting data in the course of financial analysis. Was "book value" useful in this example? In recent years, management and its accountants have been very "creative" in the presentation of accounting summaries. Is EPS reliable accounting-wise? The analyst has to be cognizant of alternative accounting presentations – both legitimate and not.

Note regarding EPS calculation:

The calculation for EPS presented above (i.e., net income ÷ the number of shares outstanding) is simplistic and should be used only when better data are not available.

In fact, the accountant will present both "Basic" and "Diluted" EPS, the calculations for which are quite involved. The student analyst must choose whether to utilize the Basic EPS datum in calculating the PE ratio or the Diluted version.

The Diluted datum assumes that convertible and other "complex" securities have been converted to stock; this results in a lower EPS number. The theoretical supposition inherent in the Diluted earnings figure may make it inap-

propriate for use in PE valuation in the eyes of some analysts.

## 7.5 Ratio Analysis Exercise

You are given the balance sheet and income statements for ABC Corporation. Notice the balance sheet is not presented side-by-side. Calculate each of the 20 financial ratios provided above for "This Year" on the next page. Remember to "average" in the appropriate places. No company description or footnotes accompany the statements. You will note that this company has preferred stock, thus complicating certain ratio calculations. Instructions regarding adjustments for Preferred Stock are found in Section 7.8 below.

**($ Millions)**

| | This Year | Last Year |
|---|---|---|
| Cash and Equivalents | 1,000 | 500 |
| Inventories | 5,000 | 6,000 |
| Accounts Receivable | 2,000 | 2,500 |
| *Total Current Assets* | 8,000 | 9,000 |
| Property, Plant & Equipment | 9,000 | 9,500 |
| Other | 1,000 | 1,100 |
| Total Long-term Assets | 10,000 | 10,600 |
| *Total Assets* | 18,000 | 20,600 |

| | | |
|---|---|---|
| Accounts Payable | 4,500 | 6,350 |
| Short-term Debt | 2,000 | 2,200 |
| *Total Current Liabilities* | 6,500 | 8,550 |
| Long-Term Bonds | 7,000 | 7,300 |
| Mortgage Bonds | 1,000 | 1,300 |
| *Total Long-term Liabilities* | 8,000 | 8,600 |
| Preferred Stock (1 million shares issued & outstanding | 1,000 | 1,000 |
| Common Stock (50 thousand shares issued & outstanding) | 500 | 500 |
| Retained Earnings | 2,000 | 1,950 |
| *Total Owners Equity* | 3,500 | 3,450 |
| *Total Liabilities + Equity* | 18,000 | 20,600 |

| | This Year |
|---|---|
| Cash Sales | 15,000 |
| Credit Sales | 30,000 |
| Total Sales / Revenues | 45,000 |
| (Cost of Goods Sold) | (35,000) |
| **Gross Profit** | 10,000 |
| (Selling, General, and Admin. Expenses) | (8,300) |
| (Depreciation and Amortization) | (1,500) |
| **Earnings Before Interest and Taxes (EBIT)** | 200 |
| (Interest Expense) | (100) |
| **Earnings Before Taxes (EBT)** | 100 |
| (Taxes at 30%) | (30) |
| **Net Income** | 70 |
| (Dividends Paid to Preferred Shareholders) | (25) |
| (Dividends Paid to Common Shareholders) | (10) |
| Addition to Retained Earnings | 35 |

# 7.6 Solution Template for Ratio Analysis Problem

1. For the market ratios below, assume a market price per common share of $10.
2. What questions does each of the ratios raise? Name at least one question per ratio.
3. **This template is made for the typical company, which has no preferred stock**. Note that, in completing this worksheet, adjustments will have to be made for Preferred Stock – given our example!

| Ratio | Formula | Solution |
|---|---|---|
| Current Ratio | CA / CL | |
| Quick Ratio | (CA – Invt'y) / CL | |
| Average Collection Period | (AR / Credit Sales) × 360 | |
| Inventory Turnover | COGS / Invt'y | |
| Debt to Net Worth | Total Debt / Total Equity | |
| Debt to Total Assets | Total Debt / Total Assets | |
| Times Interest Earned | EBIT / Interest Expense | |
| Gross Profit Margin | Gross Profits / Total Sales | |
| Operating Profit Margin | Operating Profits / Total Sales | |
| Pre-Tax Profit Margin | Pre-Tax Profits / Total Sales | |
| Net Profit Margin | Net Income / Total Sales | |
| Return on Assets | EBIT / Total Assets | |
| Return on Equity | Net Income / Total Equity | |
| Sales to Fixed Asset Turnover | Total Sales / FA | |
| Sales to Total Asset Turnover | Total Sales / Total Assets | |
| Price/Earnings Ratio | Common Share Market Price / Earnings Per Share | |
| Dividend Yield | Common Dividends / Market Price | |
| | Dividends Per Share / Price Per Share | |
| Payout Ratio (PR) | Total Dividends / Net Income | |
| | Dividends Per Share / Earnings Per Share | |
| Retention Rate (RR) | (Net Income – Total Dividends) / Net Income | |
| | 1 - PR | |
| PR + RR | PR + RR | |

## 7.7 Solution for Ratio Analysis Problem

While the mathematical solutions are noted herewith, you should also be able to raise some questions, but not necessarily answers, in connection with the ratios enumerated. Note that we have not indicated anything about the company; thus, your questions will be "generic." For an in-depth illustration of the treatment of Preferred Shares, see the next page.

| Ratio | Solution |
|---|---|
| Current Ratio | $8,000 ÷ 6,500 = 1.23x |
| Quick Ratio | (8,000 – 5,000) ÷ 6,500 = 0.46 |
| Days Sales Outstanding | {[(2,000 + 2,500) ÷ 2] ÷ 30,000} × 360 = 27 |
| Inventory Turnover | 35,000 ÷ [(5,000 + 6,000) ÷ 2] = 6.37x |
| Debt to Net Worth | (8,000 + 6,500) ÷ 3,500 = 4.14:1 |
| Debt to Total Assets | (8,000 + 6,500) ÷ 18,000 = 0.806 |
| Times Interest Earned | 200 ÷ 100 = 2x |
| Gross Profit Margin | 10,000 ÷ 45,000 = 0.2222 |
| Operating Profit Margin | 200 ÷ 45,000 = 0.0044 |
| Pre-Tax Profit Margin | 100 ÷ 45,000 = 0.0022 |
| Net Profit Margin | 70 ÷ 45,000 = 0.0016 |
| Return on Assets | 200 ÷ [18,000 + 20,600) ÷ 2] = 0.0104 |
| Return on (Total) Equity | 70 ÷ [3,500 + 3,450) ÷ 2] = 0.0201 |
| Return on Comm. Equity | (70 - 25) ÷ [(2,500 + 2,450) ÷ 2] = 0.01818 |
| Sales to Fixed Asset Turnover | 45,000 ÷ [10,000 + 10,600) ÷ 2] = 4.37 |
| Sales to Total Asset Turnover | 45,000 ÷ [18,000 + 20,600) ÷ 2] = 2.33 |
| **The following ratios were adjusted for the presence of Preferred Stock** | |
| *Earnings Per (Common) Share* | (70 – 25) ÷ 50 = $0.90 per share |
| Price/Earnings Ratio (Comm.) | $10 ÷ $0.90 = 11.11x |
| *(Common) Dividends Per Share* | 10 ÷ 50 = $0.20 |
| (Common) Dividend Yield | 0.20 ÷ 10 = 0.02 |
| Payout Ratio (PR) – (Comm.) | 10 ÷ (70 – 25) = 0.2222 |
| Retention Rate (RR) | 35 ÷ (70 – 25) = 0.7777 |
| PR + RR | 0.2222 + 0.7777 = 1.00 |

## 7.8 Adjustments to Basic Financial Ratios for Companies That Have Preferred Stock

Most companies do not issue preferred stock, but when a company does have preferred shares on its balance sheet, certain adjustments need to be made to some of the financial ratios, as was presented on the foregoing pages.

For the example just given (Balance Sheet and Income Statement), here are the relevant adjustments. The basic idea is that the preferred shareholders come first, before the common shareholders. Therefore, income “available” to common shareholders must be adjusted. As you will note, this new figure affects other data and ratios. A summary table follows.

(Millions)

| Adjustments | | |
|---|---|---|
| Net Income | $70 | |
| Preferred Dividends | (25) | **Ratios** |
| Net Income Available to Common Shareholders | 45 | |
| Common Dividends (D) | (10) | Payout Ratio = 10 ÷ 45 = 22.2% |
| Addition to Retained Earnings (ARE) | $35 | Retention Rate = 35 ÷ 45 = 77.8% |
| Net Income Available to Common Shareholders | $45 | |
| Number of Common Shares Outstanding (NOSO) | 50 | |
| Earnings Per (Common) Share | 45 ÷ 50 | EPS = $0.90 |
| Market Price Per Share (observed price) | $10 | |
| Price Earnings Ratio | 10 ÷ 0.90 | P/E = 11.11 x |
| Common Dividends Per Share | 10 ÷ 50 | D ÷ NOSO = $0.20 |
| (Common) Dividend Yield | 0.20 ÷ 10 | Common Dividends ÷ Price = 2.0% |

The following page exhibits visually the effect of Preferred Stock on Earnings Retention.

## 7.9 Illustration of Effect of Preferred Stock on Earnings Retention

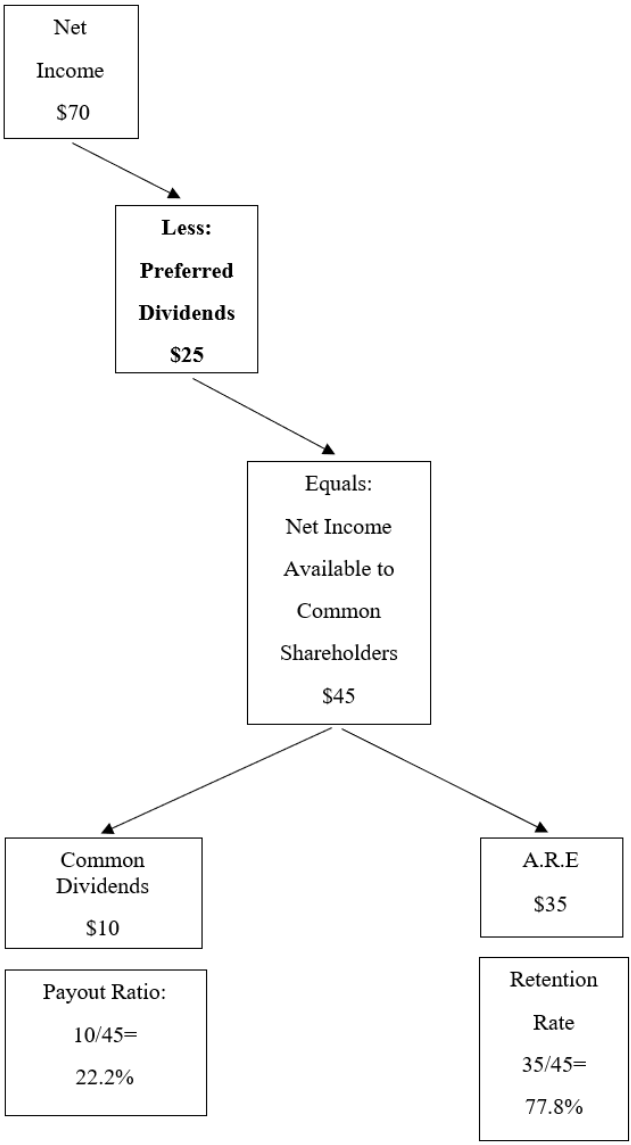



Exhibit of Effect of Preferred Stock on Earnings Retention.

## 7.10 Industry Data Benchmarks

The analyst will want to compare his data and ratios to other companies'. There are numerous sources for such industry data. Here are some useful ones.

*Bloomberg*

*The Risk Management Association* or RMA (once known as Robert Morris Associates). RMA has a database of financial ratios for over 150,000 companies. Data are organized by SIC (industry) codes. Go to www.rmahq.org.

Dun and Bradstreet or "D&B." This consists of over 1 million firms.

ValueLine Investment Service They cover just about 1,700 firms, but also provide investment ratings for their listings.

The Department of Commerce

*Standard and Poor's* or "S&P."

*Moody's*

*Yahoo Finance*

*MSN*

*Money Central*

**Note:**

We know that ratios do not provide answers. Similarly, no single ratio alone can provide an integrative view of the firm. Analysis requires a cross-sectional approach.

Cross-sectional analysis will require combining ratios often across different industry categories in order to get a clearer picture of the goings-on of the firm.

## 7.11 Some Limitations of Financial Ratios

As we have learned, most financial ratios consist of accounting data, which are limited in interpretive usefulness, but may be all we have. The astute analyst is aware of this and makes appropriate adjustments. The principle of *Garbage-in, Garbage-out* always pertains. Ratios are useless if the accounting data inputted are suspect. Here are some issues to look out for.

1. *Accrual Accounting* data management: Garbage-in, Garbage-out.
2. Companies engage in *Real Earnings* "window-dressing" in order to make their statements appear in a certain manner; examples include pulling forward or deferring actual expenses.
3. Accounting policies differ from one firm to another, making cross-sectional analysis difficult; for example, one company uses FIFO while another uses LIFO.
4. Ratios are "static" and do not necessarily reveal future relationships.
5. A ratio can hide problems lying underneath; an example would be a high Quick Ratio hiding a lot of bad accounts receivable.
6. Liabilities are not always disclosed; an example would be *contingent* liabilities due to lawsuit.

Since it may – or may not – happen, the accountant will not disclose it. There has been no transaction. (This may appear in the footnotes only.)

7. Companies are often in multiple lines of business. Therefore, identifying an industry group is virtually impossible, making cross-sectional analysis ineffective.

8. Industry benchmarks (see prior page) are often only approximations, and inaccurate ones at that. Also, there are often data entry errors.

## 7.12 Chapters 5 - 7: Review Questions

1. Calculate as many financial ratios as you can using the company from the last problem set. Average your data where advised.
2. For each ratio, provide some written commentary and analysis.
3. See how many ratios from each of our six categories you know already by rote memory.
    1. Liquidity
    2. Solvency
    3. Profitability
    4. Turnover
    5. Return
    6. Market Ratios
4. How are the Liquidity and Solvency ratio categories different from one another?
5. Why do we use 360 in calculating the Average Collection Period (ACP)? Under what rationale may 365 days be advised?
6. In the ACP, why are Credit Sales, in most cases, larger than Accounts Receivable?
7. Why do we use EBIT, and not Net Income, in calculating the Return-on-Assets?
8. Why don't we utilize the accountant's net worth fig-

ure as a metric for company value?

9. How do Price/Earnings and Price-to-Book ratios illustrate a company's value?
10. Define "Longitudinal" and "Cross-sectional." How can you use these concepts in your company analysis?
11. What is the end-goal of Ratio Analysis?
12. How is it that a strong company, e.g., Walmart, operates with negative Working Capital? What is *your* view?
13. What benefits does the Aging Schedule provide?
14. What is the relationship between a Debt-to-Total Assets ratio and a company's Times Interest Earned ratio?
15. What industries tend to have great amounts of Debt relative to Assets? How do they manage to accomplish this without increasing their default risk?
16. What benefit might there be to using the Debt-to-Assets rather than the more popular Debt-to-Equity ratio?
17. Is a low TIE Ratio always a bad thing? Under what circumstances might a company tolerate a low ratio?
18. Have you figured out yet whether ratios provide answers?
19. What two key pieces of information does the DuPont Model focus on?
20. In what technical, ratio-ways, are "Growth" and "Value" stocks different from one another?
21. What is meant by "signaling"?
22. What is meant by a "Liquidity Premium"?

23. Provide some reasons why two companies, which are identical in all respects, might have radically different Turnover ratios?
24. List and discuss some limitations of financial ratios?
25. After all this, can you “handle the truth”?

# Chapter 8: Cash Flow, Depreciation, and Financial Projections

## 8.1 Chapter Eight: Learning Outcomes

Learning Outcomes

In this chapter, you will:

- **Differentiate** between the tasks and capabilities of internal and external financial analysts
- **Implement** the notion of Incrementalism in forecasting
- **Categorize** corporate investment as incremental Property, Plant, Equipment and inventory
- **Define** "Cost of Capital" and its place in project forecasting
- **Forecast** an Income Statement based on given assumptions
- **Explain** the nature and benefit of the Tax Shield

Note: Review questions for Chapters Eight and Nine will appear at the end of Chapter Nine.

# 8.2 Pro Forma Financial Analysis: The Corporate Environment

## Pro Forma Financial Analysis: The Corporate Environment

Key Terms:

pro forma

**Pro forma** refers to expected/ future financial outcomes using certain assumptions. The financial analyst gathers information from numerous sources. Where he gets his/her information will depend largely on who s/he is. There are two possibilities:

The **internal** corporate analyst, i.e., the analyst who works for and "inside" the corporation, will collect information from the various company departments including marketing, purchasing, administration, economics, and others, each of which will "sign off" on the data that s/he includes in his/her financial projections. This analyst is charged with compiling the data meaningfully and projecting the relative profitability and hence worthiness of alternative, and sometimes competing, possible investment projects. The purpose is to engage incorporate Financial Planning.

For example, the sales projections may come from the mar-

keting department. This department may provide unit sales and pricing data, which the analyst compiles and includes in his/her projections. The analyst may ask that the department "sign off" on this portion of the overall projections and thereby take some responsibility for the data. Alternatively, the analyst may investigate the reasonableness of the data and provide alternative projections.

The **external** (securities) analyst, i.e., one who is not employed by the subject corporation, but perhaps who works instead for a financial institution that has some investment or equity interest in the corporation, will not have access to the same valuable information and data that the internal corporate analyst will have. The methods s/he uses will differ accordingly. The external analyst does not have access to the data regarding corporate investment projects adopted and which are now just coming on-line, or recently came online. S/he therefore will use public financial statements in order to derive inferences about the corporation's growth prospects and future share price.

When we, individuals, think of "investing," many of us think of investing in stocks and bonds. This is not typically what companies invest in. What *do* (non-financial) corporations "invest" in?

Let us remember that, here, we are all financial analysts and hence we take on a "corporate perspective." In this sense, *corporate* investment refers to things that (non-financial) corporations invest in, in order to *maintain* and *grow* their businesses. Such investments include, but are not limited to, the following:

- Property, plant, and equipment – P, P, & E

- Inventory
- Research & Development (R&D) – we will not deal with R&D in our forthcoming discussions.

Some very useful definitions:

- Expense: a reduction (debit) to revenues, reported on the Income Statement. Some expenses involve the movement of cash, e.g., wages; others do not, e.g., depreciation.

- Expenditure: involves the movement of cash – an outlay or outflow of funds, which may either be expensed and/or capitalized. If "capitalized," it will be expensed as depreciation over future years. P, P, & E fit this category. Remember, land ("property") is not depreciable even though it is capitalized. In contrast, (cash-) dividends are expenditures that involve cash outlays; dividends are not expensed because they are returns to the owners.

Below you will find some examples of expenses and expenditures.

| | Expense | Expen |
|---|---|---|
| Wages | Yes | Yes |
| Depreciation | Yes | No |
| Property | No | Yes |

# 8.3 Pro-Forma (Projected) Cash Flow Analysis

Estimating future cash flows: Whether s/he is internal or external, the analyst is involved in projecting future (accounting) profits and often, more importantly, projected net cash flows. In order to gather these data, the internal analyst must involve different departments, thereby playing corporate "politics" in order to complete the job.

For each of the following, indicate from which department the projections may come (e.g., economics, marketing, purchasing, etc.).

Unit Sales

First, the internal analyst may receive unit sales projections from the marketing department.

The analyst must also be aware of the issue of *Cannibalization* – the detrimental effect on other sales of the introduction of the new project (see section #8.4). For example, if Microsoft projects sales of two million units of Windows Vista, of which one million would have otherwise gone to Windows XP sales, the analyst would recognize on his spreadsheet only the one-million-unit increment. This is because the decision to introduce a new product came, in a sense, at the expense of the old product's sales. Analysts are interested only in what the new product con-

tributes incrementally – above and beyond present matters.

## Sales Prices

In order to project future revenues, the analyst will thus possibly need to obtain both unit sales data (i.e., expected quantities sold) and "pricing" information (i.e., expected price per unit).

### Operating Costs

This information may be derived from many different parties, e.g., purchasing, operations, etc. Operating costs may include:

- Cost of Goods Sold: This refers to inventory costs as explained earlier.
- Selling, general, and administrative costs: These costs refer to non-production expenditures, including salaries and wages, rent, advertising, and travel.
- Taxes: Although taxes are not an operating cost per se, we all know what Ben Franklin said about "death and taxes." Since taxes cannot be avoided, they should be deducted in the analysis.

### Non-operating and Capital Costs

In "spreading the numbers," the internal analyst will pay attention only to projected *operating* data, and not to any capital or other non-operating data / costs. S/he is inter-

ested usually only in what the potential project will produce as a business enterprise alone. Capital costs include the cost of paying interest on debt, and the dividend and "growth" costs of equity.

Key Terms:

Cost of Capital

As a heads-up, the **cost of capital** is an issue with which we shall deal later in great depth. Once the analyst has arrived at the yearly operating numbers, usually projected cash flow (rather than accounting profits), the cost of capital will then be brought in as the "discount rate," rather than as a dollar figure. This is the rate at which the future cash flows will be "discounted," or translated, to present values, thereby enabling a level playing field, in terms of time. In other words, by discounting, we are able to equate the value of a dollar paid tomorrow with a dollar today. This way, an investment decision may be made based on profitability and possibly other measures.

Note:

As we continue through these readings, pay careful attention to the words "expense" and "expenditure." They are not the same. "Expense" is an accounting phrase reflected on the Income Statement; expense is a reduction to Net

Income – it may or may not involve cash. “Expenditure” refers to the deployment of funds – it will involve cash but may or may not be expensed. Expenditures may be capitalized or expensed.

## 8.4 Incrementalism

Key Terms:

Incremental

Incrementalism

The analyst's "end-game" is to produce a spreadsheet reflecting future years' respective, **incremental**, or additional, cash flows and/or profits, and possibly other things as well. In general, the principle of **incrementalism** pertains. It is only incremental cash flows, i.e., those which are added due to the project's implementation, which are relevant to decision-making. If the company brought in $100 and now – with the new project – it brings in $125, the increment is $25. Incremental revenues – and costs – did not exist before the project was effected. Therefore, *sunk costs* and the *cannibalized* portions of projections' data are excluded as will be discussed immediately below. It is the incremental data which drive the decision to invest or not. We will learn this better by example.

By contrast, the accountant reports all historical economic events. Later, we will conduct a basic exercise in producing a pro-forma statement, i.e., a projected, income statement. Below are the two types of incrementalism:

Sunk Costs

**Sunk Costs** – these are expenditures which have already been outlaid and should not be therefore "double-counted." For example, suppose a factory has unused, or under-used, equipment, which may be exploited, for a proposed project. The cost of the equipment having already been spent or "sunk," should not be considered again; the cost is not relevant to deciding, for example, whether or not an existing asset should be exploited in a new way. The project merely proposes to exploit unused, or under-utilized, assets – or "sunk costs" – in order to increase sales and profits.

Cannibal-
ization

**Cannibalization** – When a new product is introduced, some of the revenue may have been *cannibalized* from the potential sales of the old product. For example, if the new product is expected to sell 1.5 million units, and the old product had sold 1.0 million, we are interested only in the 0.5 mm increment. That is what the investment decision depends upon. Hence the phrase "cannibalization." A change in pricing would complicate this analysis, i.e., how much of the revenue increase is due to volume and how much is due to a price increase?

# 8.5 Corporate Forecasting and Strategic Planning

A financial analyst will collect reliable financial data from internal numerous sources, if possible, which s/he will cite in his/her forecast, list the assumptions as part of the report, and "spread the numbers" accordingly. Interpretation and strategy then will follow.

The following is an exercise in creating a pro forma (accounting-based) income statement. Later, we will see some analytic techniques, which may be implemented to convert "profits" to "cash flow," if the latter is deemed more desirable. A "strategic plan" involves forecasting financial data for multiple future years.

Pro forma Financial Statements

Complete the pro forma I/S based on the following *assumptions*:

1. Marketing projects sales growth at 10% p.a.
2. "Purchasing" projects inventory costs to rise at a rate of 12% p.a. – due to scarcities.
3. S, G, & A will grow at a rate of 8% p.a.
4. For depreciation, see information below.
5. Interest costs will rise due to acquisition of new property, $10,000 of which will be financed entirely via a 20-year, "non-amortizing" mort-

gage (i.e., interest-only) bond at a rate of 7.5%. The other $1,000 will be financed via retained earnings. The present property is fully paid for.

6. Taxes are charged at a flat 40% rate.
7. Calculate NI.
8. Calculate EPS – There are 1,000s outstanding – no new shares will be issued.

| | Last Year | Year 1 | Year 2 | Year 3 | Year 4 | Year 5 |
|---|---|---|---|---|---|---|
| **Sales** | $10,000 | | | | | |
| **COGS** | (5,000) | | | | | |
| **GP** | 5,000 | | | | | |
| **S, G, & A** | (1,000) | | | | | |
| **Deprec.** | (100) | | | | | |
| **EBIT** | 3,900 | | | | | |
| **Interest** | - | | | | | |
| **EBT** | 3,900 | | | | | |
| **Tax** | (1,560) | | | | | |
| **NI** | $2,340 | | | | | |
| **EPS** | $2.34 | | | | | |

<u>Depreciation</u>

- Next year's ("Year 1″) depreciation expense will be $50. Thereafter the (old) building will be scrapped.
- At the end of Year 1, a new building will be in use (the cost of scrapping the old building will be included in the cost basis of the new building). At that time, a mortgage will be in place. The new building will cost $11,000 and will have a twenty-year life. It will be depreciated on a straight-line basis; a salvage value of $1,000 is assumed.

# 8.6 Forecasting Solution

| | Last Year | Year 1 | Year 2 | Year 3 | Year 4 | Year 5 |
|---|---|---|---|---|---|---|
| **Sales** | $10,000 | $11,000 | $12,100 | $13,310 | $14,641 | $16,105 |
| **COGS** | (5,000) | (5,600) | (6,272) | (7,025) | (7,868) | (8,812) |
| **GP** | 5,000 | 5,400 | 5,828 | 6,285 | 6,773 | 7,293 |
| **S, G, & A** | (1,000) | (1,080) | (1,166) | (1,260) | (1,360) | (1,469) |
| **Deprec.** | (100) | (50) | (500) | (500) | (500) | (500) |
| **EBIT** | 3,900 | 4,270 | 4,162 | 4,525 | 4,913 | 5,324 |
| **Interest** | - | - | (750) | (750) | (750) | (750) |
| **EBT** | 3,900 | 4,270 | 3,412 | 3,775 | 4,163 | 4,574 |
| **Tax** | (1,560) | (1,708) | (1,365) | (1,510) | (1,665) | (1,830) |
| **NI** | $2,340 | $2,562 | $2,047 | $2,265 | $2,498 | $2,744 |
| **EPS** | $2.34 | $2.56 | $2.05 | $2.27 | $2.50 | $2.74 |

Provide some interpretive statements or comments about this company's (or investment project's) prospects. Specifically, calculate the growth rates for each year versus its prior year – for GP, EBIT, and NI.

The Growth Rate is calculated as: (Next Year's Number ÷ Last Year's number) – 1.

For example, how much higher, in percentage terms, is the GP for "Year 1" in comparison to "Last Year"? Do this for each year until you get to Five versus (or "over") Four.

Use four decimal places throughout. What do you find and why?

# 8.7 The Tax Effect of Depreciation

**Depreciation** is a non-cash expense (and NOT an "expenditure"), which serves to recognize the consumption of fixed assets over time, e.g., plant and equipment, but NOT property.

The "consumption," or use, of long-term, fixed assets is reflected as a separate operating, depreciation expense. Although depreciation is a non-cash expense, it provides the corporation with cash because it reduces taxable income and hence the tax liability.

When depreciation can be "imputed" to the manufacturing process, its charges are included in the Cost of Goods Sold (COGS). For example, if the manufacturer of a $1 million piece of equipment warrants that the machine will be used up after manufacturing 1 million units, the accountant may impute (i.e., include) $1 per unit of production to COGS. This could confuse the statements reader. Below, we will *not* assume such "imputation."

Income Statement for
Widget Manufacturing Corp.
Fiscal Year Ended 12.31.XX
($ millions)

| | No Depreciation | With Depreciation | |
|---|---|---|---|
| Gross Profit | $400 | $400 | |
| Selling, General, & Admin. | ($100) | ($100) | |
| Depreciation | 0.0 | (20) | |
| Earnings before Int. & T | $300 | $280.0 | (Interest = $0) |

Illustration of Depreciation Tax Shield (Ignoring Interest Expense):

| | | | Difference |
|---|---|---|---|
| EBIT | $300 | $280 | $20 |
| Tax (@ 40%) | (120) | (112) | $8 (tax shield) |
| NI | $180 | $168 | $12 |

Due to depreciation, a non-cash expense, net income will be lower by only $12 rather than the full $20 of depreciation expensed; the $8 difference represents the tax savings, and hence the "tax shield," due to the deduction of the depreciation expense.

$$\text{Tax Shield} = (D)\ (T) = (\$20)\ (0.40) = \$8$$

The "tax shield" of depreciation, i.e., D (T), is a very real phenomenon in that it actually reduces the tax burden, which is viewed as a provision of cash, albeit depreciation alone provides no cash. The acquisition of a depreciable asset, e.g., a building or equipment, creates a *tax shield* or benefit that provides more cash flow than before the acquisition.

You may think of "EBITDA" (i.e., earnings before interest, tax, depreciation, and amortization) as EBIT plus depreciation (and amortization).

## On Humility, Honor, and Accomplishment

*A life spent making mistakes is not only more honor-*

*able, but more useful than a life spent doing nothing.*

-George Bernard Shaw

*That's why pencils have erasers.*

-Norman Isaac Bigel

(1917 – 2017)

# Chapter 9: Corporate Forecasting Models

# 9.1 Chapter Nine: Learning Outcomes

Learning Outcomes

In this chapter, you will:

- **Define**, in words, both the “Free Cash Flow” and “External Funds Needed” Models
- **Consider**, both mathematically and in words, each element of the two formulae
- **Forecast**, given assumptions, both models for multiple forward years
- **List** some uses of the formulae
- **Differentiate** between Internal and External Funds, providing examples of each
- **Determine** the optimal future Debt-to-Total Assets ratio, given the EFN forecast
- **Explain** why a firm’s financial ratios may change

## 9.2 Free Cash Flow

Why *Cash Flow*? A corporation's value is dependent, in large part, on the income and cash flow it produces. Cash flow is different from income in that "income" is based on "accrual accounting," which will reflect certain non-cash events such as depreciation, and many other "distortions," some of which were discussed earlier. One key distortion had to do with "timing" – the accountant may book a sale, for example when "constructively received," i.e., when legally and economically receivable, but not yet received in actual cash.

Under accrual accounting, the accountant records economic transactions rather than the movement of actual cash received and paid out. (In theory, in the long run such accounting vs. cash differences even out.) Because of these differences, financial analysts, who are more cash-oriented, must make certain adjustments to the accounting data in their own calculations and projections.

In a certain sense, cash flow is more important to corporate valuation than income because dividends are paid out to investors in cash. Further, when engaging in corporate planning, a possible capital investment will be judged attractive dependent upon the cash flow it produces in the future because it is cash that, arguably, fuels growth.

What is *Free Cash Flow*? Capital investments, i.e., "growth" investments, include expenditures for hard assets, as well as for product development, and much more.

"Free cash flow" is funds that an investment project or the corporation as a whole generates beyond its own internal and ongoing needs. We may think of free cash flow as the cash, which is left over from an investment project, after all net operating funds generated by the project are utilized for necessary, non-discretionary purposes, including hard assets' maintenance and replacement, and increases in working capital; this "left-over" amount may be used for other financial purposes – growth or expansion, at the discretion of management.

Working capital must often be increased. The project, or corporation, first generates operating cash flows, some of which are needed to replace or maintain assets, to invest in inventory and receivables, and to maintain the firm's competitive position.

So, think of "free" cash flow (FCF) as *discretionary*, think of it as "*Discretionary Cash Flow*." *Necessary* capital and other non-discretionary expenditures are required in order to maintain the business as is. A leaky roof must be repaired; there is no choice. A worn-out gasket must be replaced.

Analogously, you may recall the parallel notion of "disposable income" in Macroeconomics. (After deducting taxes, and necessities, such as food and rent, from gross income, the consumer has some funds left over for discretionary purposes, which might include investment, or buying that deluxe Apple Watch s/he has coveted for so long.) Similarly, "free cash flow" is the net, operating after-tax cash that the firm may generate from investment in a new building or equipment (i.e., the investment "project") after considering necessary operating and capital expenditures, including maintenance and replacement, but excluding

expansionary, or growth-oriented, capital expenditures as this would be discretionary.

The more FCF the firm generates, the greater the firm's ability to invest in new assets, to use the funds to pay down debt or to pay dividends, and still other discretionary possibilities.

The Mathematics

One formula for use in this process is presented below. We start with operating earnings, which, in this case, is defined as "EBIT" or, alternatively "EBITDA[1]," i.e., earnings before interest, taxes, and amortization[2]. To the financial analyst, the projected income or cash flow statement may look something like the following:

| |
|---|
| Sales |
| (Cost of Goods Sold) |
| (Operating Expenses) |
| "EBITDA" |
| (Depreciation & Amortization) |
| EBIT |
| (Taxes, absent interest) |
| Net Project Income |

Thus the analyst would calculate Free Cash Flow (FCF) to equal:

**[(EBITDA) (1 – T)] + [(Depreciation) (T)] – [Neces-**

1. In some instances, an analyst may choose to use "EBITDA," i.e., EBIT with depreciation and amortization added back. The use of this figure will depend on the data presentation and analyst choice.
2. As we will learn later in this text, amortization has to do with the reduction in the value of an intangible asset over time; in this sense it is like depreciation.

**sary Capital Expenditures] – [Increase in Net Working Capital]**

The elements of the formula

The formula consists of four different parts. Let's look at each of them.

1. EBITDA (1 – T)
2. Add: Depreciation (Tax Rate) = (D) × (T)
3. Less: Necessary Capital Expenditures[3]
4. Less: Increases in Net Working Capital
5. Equals: Free Cash Flow (FCF)

Note that "T" (in "1-T" and in "D x T") stands for the firm's tax bracket, or percent, whereas EBITDA is in dollars, including the "T" there. While at this stage of the corporate planning and investment analysis process, the firm has not yet decided whether it will choose the investment or not and, if so, how it will be financed, and therefore does not know its projected interest payments, it does know its tax bracket, which is based on the firm's meeting a specified lower threshold ("bracket") of earnings.

1. EBITDA:

Take note of the use of EBITDA rather than EBT in the formula. There are several details here, which bear explanation; we will take them one at a time. First, account-

3. Note that "Capital Expenditures" here refers only to *required* expenditures, i.e., maintenance and replacement expenditures, and not *discretionary* expenditures, which may be used for expansion and growth. By subtracting out necessary (i.e., non-discretionary) CE, we are left with Free CF.

ing EBT would include interest expense – as a deduction from EBIT. “Interest expense” is the result of a prior capital financing decision and, as such, should be independent of this analysis, which focuses on the cash-based, operating earnings a firm or a project produces (EBITDA), and not on financial decisions. In our analysis and projections, we should thus exclude interest, which is a discretionary capital structure, solvency-oriented matter for decision.

Accordingly, the “EBITDA (1 – T)” adjustment eliminates the interaction of interest and its effect on tax. To see how this adjustment works, let us compare the results of the formula with the following alternate spreadsheet presentation. Let’s say that the firm produces $100,000 of EBITDA, has interest expense of $20,000, and pays taxes at a 30% rate. By formula we would project EBITDA (1 – T) = $100,000 (1 – .30) = **$70,000**. (For this example, we shall assume, for the moment, that depreciation and amortization are zero.) This is the figure we want, which represents earnings after taxes, in the absence of interest. The “1 – T” multiplier shows what is left after paying taxes. This project produces, or is expected to produce, $70,000 in operating cash flow (next year).

In the following spreadsheet, we would show the same $70,000 result, only by different presentation (in order to prove out the formula) – under the alternate FCF analysis. We also show, side by side, the accounting income reported, which does not suit the purpose of the analysis because it includes interest expense, which is a financial – not an operating – event or cash (out-) flow.

| | With Interest | Alternate FCF Analysis |
|---|---|---|
| EBITDA | $100,000 | $100,000 |
| Interest | (20,000) | --- |
| Taxable Income | 80,000 | $100,000 |
| Taxes at 30% | (24,000) | (30,000) |
| *Accounting income* | $56,000 | |
| *Cash Flow after taxes, absent interest* | | **$70,000** |

2. The Depreciation Tax Shield:

Depreciation is a non-cash expense (as is amortization, which we have been assuming herein to be zero), which is deducted *from* operating earnings *before* income taxes. Therefore, we add back just the tax savings that depreciation provides but ignore depreciation itself. It is assumed that the (internal) analyst knows the firm's tax bracket. Depreciation is tax deductible and, as such provides cash, but otherwise itself directly provides no cash. You found above an in-depth illustration of the "tax effect of depreciation," absent other media. You may wish to view that page again now.

Note:

EBITDA (1-T) + D (T) = EBIT (1-T) + D. (To prove this out, substitute in the equation the following values: EBIT = $100; D = $20; A = $0; and T = .30) The two phrases are mathematically equivalent but appear to have somewhat different connotations. In the latter case, EBIT appears to refer to accounting income, hence our preference for EBITDA, a more cash-oriented notion. In the latter for-

mulation, adding back depreciation, which is a non-cash expense, intuitively appears to contradict the aforesaid idea that depreciation does not provide cash flow, even though it does work out mathematically and conceptually.

3. Capital Expenditures:

There are the three categories of capital expenditures ("Cap Ex"), which we shall need to incorporate in the free cash flow projection. They are:

1. Maintenance
2. Replacement
3. Expansion

Occasionally, the walls must be painted and the premises properly maintained. Roofs may blow off, and machinery gaskets may "blow," requiring replacement. Expansionary investments may involve acquiring additional factory space or equipment, in order to increase production, and hence sales and profits.

Maintenance and replacement expenditures are clearly *necessary*; expansionary expenditures are *discretionary* and are (generally) undertaken in order to provide growth, i.e., increased sales and profits. We would deduct necessary capital expenditures in order to arrive at free cash flow.

Accounting rules do not require separating out necessary from discretionary capital expenditures; any breakdown, if provided, may be found in the footnotes. This information will assist the analyst in gauging the firm's growth prospects. Absent this breakdown, the most common prac-

tice among analysts is to use *all* capital expenditures in the formula, although this is a matter of analyst judgment.

4. Net Working Capital:

"Net working capital" may be defined as current assets less current liabilities (NWC = CA – CL). Any increase in current assets, or in net current assets (i.e., "NWC"), *uses*financial resources and thus *reduces*cash or cash flow. For example, an increase in inventory uses funds.

Similarly, an increase in current liabilities *provides*funds, and may be thought of as a "source" of funds. A decrease in current liabilities uses funds. For example, when a company pays its accounts payable, it uses funds.

If a firm is to grow it will have to increase its Net Working Capital.

The table below summarizes the possibilities and instructs you how to insert the changes into the FCF formula. Some add to FCF; others are subtracted from FCF.

| | | | |
|---|---|---|---|
| Current Assets (excl. cash) | Increase | Uses cash | Reduces FCF |
| | Decrease | Provides cash | Increases FCF |

| | | | |
|---|---|---|---|
| Current Liabilities | Increase | Provides cash | Increases FCF |
| | Decrease | Uses cash | Reduces FCF |

Note that, in calculating Free Cash Flow and the relevant prospective changes in Current Assets, we ignore expected changes in the cash account, focusing instead on changes in the cash flow affecting the cash account. To include changes in the cash account itself in calculating FCF, would be "double-counting."

What can the corporation do with its FCF?

- Purchase more P, P, & E.
- Expand inventory.
- Invest in Mergers and Acquisitions (using shares when high-priced).
- Increase R & D.

- Pay off Debt.
- Pay discretionary (common stock) dividends.
- Buy back common shares (when cheap).

A potential corporate investment project that throws off a lot of FCF is desirable. A firm, as a whole, that throws off lots of FCF may be thought of in a most positive light – as one, among other possibilities, that has a lot of growth potential and makes for a good investment.

# 9.3 Free Cash Flow Exercises

## Free Cash Flow: Exercise #1

That covers the formula. Let us work up a quick, all inclusive, example. You are given the following: solve for "free cash flow." Assume that there is no amortization.

| | |
|---|---|
| EBITDA | $100 million |
| Depreciation and Amortization (included in EBITDA above) | $1 million |
| Tax Rate | 30% |
| Total Cap Ex | $10 million |
| Expansion Cap Ex | $3 million |
| Current Assets (this year-end) | $5 million |
| Current Assets (next year-end) | $6 million |
| Current Liabilities (this year-end) | $3 million |
| Current Liabilities (next year-end) | $2 million |

**[(EBITDA) (1 – T)] + [(Depreciation) (T)] – [Necessary Capital Expenditures] – [Increase in Net Working Capital]**

FCF (Formulation) = $100 (1 – .30) + $1 (.30) – $7 – $2

(Calculation) = $70 + $0.30 – $7 – $2 = **$61.3 million**

Working Capital next year versus this year increased by $2 million: (6-2) – (5-3). Alternatively, one might say that current assets increased by $1 million, a useof funds; current liabilities decreased by $1 million, also a use of funds.

The net use of funds was therefore $2 million, which accordingly reduces free cash flow.

To summarize, *free cash flow* may be thought of as the firm's after-tax cash flows less any spending on either the maintenance or replacement of fixed assets, and in acquiring working capital. The free cash flow left over may be used either to pay down debt, pay dividends, buy back stock, for discretionary growth investments and more. The firm will (should) choose investments that further maximize FCF.

This discussion has, so far, assumed that we, financial analysts, are *perfectly* capable of making accurate, numerical projections about matters that have not yet occurred. In reality, projections are virtually always going to be somewhat incorrect – when all is said and done. Projections "under uncertainty" are beyond this manual's scope.

This example is generic in the sense that it may represent either the view of an external analyst looking at a corporation's most recent financial report, and, based on the report, making an assessment of the corporation's growth prospects and equity investment merits based on its FCF; or it could be a projection that an internal analyst makes for a potential corporate investment project.

## Projecting Free Cash Flow: Exercise #2

We have already projected net income and earnings per share (see above under the heading "Corporate Forecasting and Strategic Planning"). We also understand the implica-

tions of depreciation, a non-cash expense, on income and cash.

Now, we will conduct an exercise, based on the earlier net income and EPS projections, in which we shall create a pro-forma statement – of a sort – of *free cash flows* for multiple future years rather than just one year's net income. The figures for EBIT and Depreciation were calculated and the solutions can be found in Section #8.6; these figures are needed for the FCF problem herewith. This spreadsheet will be useful when, later, we assess such cash flows in *present value* terms in order to make capital investment decisions. In this connection, take note that interest, for example, is excluded from FCF because it is a *capital* cost, as discussed earlier. In order to do this, we need to recall the FCF formula. Here, once again, it is:

**[(EBITDA) (1 – T)] + [(Depreciation) (T)] – [Necessary Capital Expenditures] – [Increase in Net Working Capital]**

In our pro-forma, Five-year income statement, we have many of the elements: EBIT, depreciation, and tax rate (i.e., 40%). Let us assume the following for the missing parts:

1. For EBIT and Depreciation figures, use the data calculated earlier – in Section #8.6.
2. There is no amortization.
3. Replacement and maintenance capital expenditures shall be $1,000 in "Year 1" and grow thereafter at a 5% rate.
4. Current Assets will increase in Year 1 by $500 the first year, and each year thereafter at a

growth rate of 10%.

5. Current Liabilities will increase in Year 1 by $750 and each year thereafter at a growth rate of 5%.
    - (Hint: an increase in current liabilities is a source of funds.)
    - Take note that the differences are incremental.

To make matters simple, a spreadsheet is provided below, which is consistent with the FCF formula. Notice the (horizontal) line breaks, which assist you in separating out the pieces of the formula from their respective sub-parts. Note also that cash outflows should be bracketed, since they are negative. Some of the information in the spreadsheet will be imported from the earlier net income exercise, while some will be derived from the set of four assumptions noted on the prior page.

| | Year 1 | Year 2 | Year 3 | Year 4 | Year 5 |
|---|---|---|---|---|---|
| EBIT | | | | | |
| Depreciation | | | | | |
| EBITDA | | | | | |
| **EBITDA × (1 - .40)** | | | | | |
| ***Plus*** | | | | | |
| Depreciation | | | | | |
| **Depreciation × .40** | | | | | |
| ***Less*** | | | | | |
| **Nec. Cap Ex** | | | | | |
| ***Less*** | | | | | |
| **Incr. in Current Assets** | | | | | |
| **Decr. in Current Liab.** | | | | | |
| ***Equals*** | | | | | |
| **Free Cash Flow** | | | | | |

*Until the day when G-d shall deign to reveal the future to man, all human wisdom is summed up in these two words, 'Wait and hope.'*

-Alexandre Dumas

*The Count of Monte Cristo*

# Projecting Free Cash Flow

# -Solution-

# (Exercise #2)

Here is the solution:

| | Year 1 | Year 2 | Year 3 | Year 4 | Year 5 |
|---|---|---|---|---|---|
| EBIT | 4,270 | 4,162 | 4,525 | 4,913 | 5,324 |
| Depreciation | 50 | 500 | 500 | 500 | 500 |
| EBITDA | 4,320 | 4,662 | 5,025 | 5,413 | 5,824 |
| **EBITDA × (1 - .40)** | 2,592 | 2,797 | 3,015 | 3,248 | 3,494 |

| | | | | | |
|---|---|---|---|---|---|
| Depreciation | 50 | 500 | 500 | 500 | 500 |
| **Depreciation × .40** | 20 | 200 | 200 | 200 | 200 |

| | | | | | |
|---|---|---|---|---|---|
| **Cap Ex** | (1,000) | (1,050) | (1,103) | (1,158) | (1,216) |

| | | | | | |
|---|---|---|---|---|---|
| **Δ Current Assets** | (500) | (550) | (605) | (666) | (732) |
| **Δ Current Liabilities** | 750 | 787 | 827 | 868 | 912 |

| | | | | | |
|---|---|---|---|---|---|
| **Free Cash Flow** | $1,862 | $2,184 | $2,334 | $2,492 | $2,658 |

*A mensch tracht un Gott lacht.*

*Man plans and G-d laughs.*

-Yiddish expression

# 9.4 External Funds Needed Formula (EFN)

A company needs additional "capital" (i.e., financial resources) in order to grow and to maintain its existing plant and equipment, and to acquire additional inventory. It cannot achieve a sales increase ("growth") without adding on productive "capital assets" (not to mention maintaining existing assets) in order to produce more goods for sale.

Key Terms:

Internal Funds

Some further **internal funds** will be generated by accounts payable, also in the normal course of business conduct. Payables are, essentially, free, short-term loans provided by the firm's suppliers for, usually, 30 days. Payables are, financially speaking, (interest-) free sources of funds. We call these funds, including payables and retained earnings, by numerous names: "spontaneous, automatic," or "internal." Some of the company's capital needs will be met "spontaneously," in the normal course of doing business by retaining some, or all, of its earnings. Such funds may be thought of as having been generated "internally."

External Funds

For the balance of its capital requirements, the corporation will need to go "outside" the firm's normal venues – to bank lenders, and/or to bond and stock investors for its **EFN** or **external** requirements. The extent to which the company will use either *debt* or *equity*, and in what proportions, is the subject of the "Capital Structure" topic; this topic will be discussed later. The EFN projection is an essential component to of the corporation's capital planning.

The EFN Formula: The formula, which we shall develop, consists of three parts. The first part tells us how much additional assets the firm will require in order to support a stated sales growth objective. This part is based on the firm's required ratio of assets to sales. It will need more assets in order to produce more sales. We shall refer to this amount as the "gross requirement," for lack of a better phrase.

The next two parts indicate the extent to which first, accounts payable, and second, retained earnings may reduce the gross requirement. After having reduced the gross requirement by the two internally generated or spontaneous sources of funds, we are left with a figure, which tells us how much additional, external financing the firm will have to arrange for itself in order to achieve its preset sales growth objective. External financing will take the form of new debt and/or equity.

EFN Model's Restrictive Assumption:

We shall assume that Financial Ratios remain the

same over time, i.e., we will conduct "static analysis." In practice, projections may not conform to past relationships precisely, for reasons known to, or assumed by, the analyst. Here, for simplicity, we shall dispense with this complication.

In order to focus on the nature of the EFN formula – and not on dynamic financial ratio matters – we will assume, in the exercise to come, **static ratios**. "Static" means unchanging in time.

Static Ratios

## The EFN Formula – The Math

**Terminology:**

Funds – this refers to any and all financial resources that the firm may call upon, including cash, various forms of debt or borrowings, and equity, the latter two of which, like cash, also enable the firm to invest in any proposed corporate investment project.

Internal Funds– these may also be referred to as either "spontaneous" or "automatic." These funds are automatically or spontaneously generated in the normal course of doing business. An example of this is accounts payable. When a producer orders raw materials, a free loan for usually thirty days is created by the supplier. Over the course of the year, and as the thirty days continually "roll over" over the course of the entire year, the producing firm is supplied with a free source of "internal" funds. The company will also internally generate funds as it earns and retains a portion of its earnings – "retained earnings."

External Funds – This refers to any "additional" funds the firm may need to raise beyond any funds that are generated in the normal course of doing business in order to finance a proposed corporate investment project. For external funds the firm will have to go, so to speak, outside the confines of its ordinary business by calling upon bank lenders, debt and equity investors, and possibly others. External funds are required to the extent that internal funds are insufficient.

**The EFN Formula:**

$$\textbf{EFN} = [(A_0/S_0)\ \Delta S] - [(AP_0/S_0)\ \Delta S] - [(M_0)\ (S_1)\ (RR_0)]$$

**OR**

$$[(A_0)\ (S_1/S_0 - 1)] - [(AP_0)\ (S_1/S_0 - 1)] - [(M_0)(S_1)\ (RR_0)]$$

**EFN** = Required increase in assets $[(A_0/S_0)\ \Delta S]$

Less:"spontaneous" increase in Liabilities $[(AP_0/S_0)\ \Delta S]$

Less:"spontaneous" increase in Retained Earnings $[(M_0)(S_1)\ (RR_0)]$

**Key**:

| | |
|---|---|
| $\Delta S$ | The expected amount of growth in sales (i.e., $S_1$ - $S_0$) |
| $S_1$ | Next year's expected sales |
| A | Total Assets (= Total Liabilities + Equity) |
| AP | Spontaneous liabilities (only), especially accounts payable |
| M | The net profit margin (NI ÷ S) |
| RR | The retention rate: = (NI – D) / NI = A.R.E. / NI |
| EFN | The shortfall in finding given the "spontaneous" changes; the external funding requirement |

# 9.5 Internal and External Funds (Summary)

We said above that the firm may reduce its total requirement for acquiring funds through the "spontaneous" or "automatic" generation of internal funds.

Internal Funds:

- Accounts Payable
    - If the firm bought supplies and raw or finished inventory etc. under "cash on delivery, or "COD," terms of sale, it would have to pay for it – with cash! If it has enough cash, it will incur an opportunity costs, because the cash would not be invested. If it had to borrow the money, it would incur an explicit borrowing cost at a rate of interest. However, most firms are provided with 30-day terms of sale from its suppliers, which amounts to a 30-day free loan, during which time it incurs neither an opportunity– nor an explicit–cost. In this sense, credit terms – Accounts Payable – provide cash flow to the buying firm. The corporation receives a free loan, of a sort, from its suppliers.
    - Although the Accounts Payable will

be paid in 30-days, the firm will have on average over the course of the year, 30-days of access to a free source of funds. The firm, more or less simultaneously, pays down the Payables and then re-orders more goods.

- Additions to Retained Earnings
    - This, clearly, provides a stream of internal funds to the corporation as profits are made and retained.

We also said that, to the extent that internal funding is insufficient to satisfy all the investment needs, it may seek additional funding externally.

External Funds:

- Short-term bank lines of credit
- Short-term bank loans – “notes”
- Long-term (bank) loans – “debt”
- Issuance (sale) of corporate notes or bonds
- Issuance (sale) of equity

# 9.6 The EFN Formula Explained

## The EFN Formula Explained

You will observe that the EFN formula has three parts (separated by two minus signs).

$$\textbf{EFN} = [(A_0/S_0)\ \Delta S)] \quad - \quad [(AP_0/S_0)\ \Delta S] \quad - \quad [(M_0)\ (S_1)\ (RR_0)]$$

The first part represents the required increase in total assets $[(A_0/S_0)\ \Delta S)]$ needed to sustain the projected sales increase. Last year, assets equal to $A_0$ were required by the firm to sustain a sales level equal to $S_0$.Hence, we formulate the ratio $A_0/S_0$. Assuming this ratio remains static, next year, the firm will require so much more in assets; this is arrived at by multiplying the ratio by $\Delta S$, the projected sales increase; $\Delta S = S_1 - S_0$.

However, some of this "gross requirement" will be "spontaneously" or "automatically" met by the normal business generation of "internal" funds in the manner of spontaneous liabilities, by which we primarily refer to accounts payable (not notes payable or the current portion of long-term debt payable). Last year, such internal funds represented a certain percentage of sales: $(A_0/S_0)$. If we multiply this dollar figure by the projected sales increase ($\Delta S$), we may see to what extent spontaneous liabilities reduce the original "gross requirement."

Finally, the firm will also – hopefully – generate and retain

some of its earnings, thereby further reducing its "gross requirement." If we take the firm's net profit margin (NI/S = $M_0$) and multiply it by next year's sales ($S_1$), we get next year's projected net profits: ($M_0$) ($S_1$) = $NI_1$. If we the net profit margin (i.e., $M_0$= NI/S) times next year's sales, we get next year's net profits. If we further multiply this by the firm's retention rate ($RR_0$), we get the firm's projected retained earnings.

After all is said and done, we have a figure – the dollar amount of EFN – that enables the firm to plan for next year's acquisition of external financing, and hence increased asset levels in support of the planned sales increase. Interestingly, this formula does not instruct us relative to the extent to which the external requirement should be met by either debt or equity, and in what Debt-to-Equity proportion.

Note:

For simplicity of presentation only, below we will ignore the "rule" of using an average balance sheet datum when concocting mixed ratios (i.e., those that include both balance sheet and income statement data). This simplification is in addition to the static analysis already assumed.

<u>Some useful formulae</u>:

- $\Delta S = S_1 - S_0$
- $M_0 = NI / S$
- $RR = (NI - D) / NI = A.R.E. / NI = 1 - PR$

# 9.7 EFN Application

## EFN Application

You are given the following income statement and balance sheet. Solve the question listed at the bottom of this page.If Sales are projected to grow to $2,500, what is the additional amount of funds needed ("EFN") to finance the growth in sales?

**Balance Sheet for "TC Corp"**

As of Fiscal Year Ending 12.31.20XX

($ Millions)

| Assets | |
|---|---|
| Cash & Equivalents | 20 |
| Accounts Receivable | 240 |
| Inventory | 240 |
| Current Assets | 500 |
| Property, Plant, & Equipment | 400 |
| Other Long-term Assets | 100 |
| Total Assets | **1,000** |

| Liabilities & Equity | |
|---|---|
| Accounts Payable | 100 |
| Notes Payable | 100 |
| Current Liabilities | 200 |
| Long-Term Debt | 100 |
| Common Stock | 500 |
| Retained Earnings | 200 |
| Liabilities & Equity | **1,000** |

**Income Statement for "TC Corp"**

For Year Ending 12.31.20XX

($Millions)

| | |
|---|---|
| Sales | $2,000.00 |
| Variable Costs (60%) | (1,200.00) |
| Fixed Costs | (700.00) |
| Earnings before Interest & Taxes | 100.00 |
| Interest Expense | (16.00) |
| Earnings before Taxes | 84.00 |
| Taxes (40%) | (33.60) |
| Net Income | 50.40 |

| | |
|---|---|
| Dividend Paid | (15.12) |
| Addition to R/E | 35.28 |

## 9.8 EFN Solution

**EFN** = $[(A_0/S_0)\ \Delta S] - [(AP_0/S_0)\ \Delta S] - [(M_0)\ (S_1)\ (RR_0)]$

$= [(1{,}000/2{,}000)\ (500)] - [(100/2{,}000)\ (500)] - [(50.40/2{,}000)\ (2{,}500)\ (0.70)]$

= \$250 – 25 – 44.10

= **\$180.90**

Question #1: What does this number mean?

Answer: It means that, if my company wishes to grow its sales next year by \$500, it will need to add on \$250 worth of assets, of which \$180.90 will be funded externally.

Remember: If it wishes to increase assets by \$250 in order to achieve its sales objective, it will have to increase the other side of the balance sheet by the same amount. In this case, \$25 will be provided internally by accounts payable, \$44.10 will be provided internally through retained earnings, and \$180.90 will be provided externally by some mix of debt and equity.

| | Source |
|---|---|
| $180.90 | External (Debt & Equity) |
| 69.10 | Internal (AP and ARE) |
| $250.00 | Added Assets Requirement |

Note 1:

1. Use *spontaneous* changes only, that is, AP for liabilities, exclude Notes Payable (NP) and Long-term debt (LTD).
2. This was based on a static ratio analysis – our restrictive assumption.
3. This has been an *incremental* analysis; we were only interested in the *additional* amount of funds needed, and that's what we got!

Question #2: How much of the $180.90 will be externally funded by debt and how much by equity?

Question #2, Answer 1:

The present debt/equity ratio is 3:7, i.e., 30% debt and 70% equity. Total capital is $1,000, with $300 of debt and $700 of equity (D ÷ TA = 30%). Assuming static analysis, 30% of the $180.90 will be

financed by debt and 70% by equity. This will maintain the capital ratios in the same proportions as prior to the new external funding.

Question #2, Answer 2:

Another, perhaps better, way of calculating the debt ratio, for this purpose, would be by excluding internal capital from the figures. In this way, we would be establishing only how much external debt and external equity should be raised, an approach, which would be more consistent with the purpose of the EFN formula. We agreed that the firm needs $180.90 of external funds.

Thus *presently*, external debt ÷ external equity = 200 ÷ 500; that is 28.5% (2/7) in debt as compared to the total of external capital. We had raised $2 of external debt for every $5 of external equity. Total external capital was $700 (5/7) (not the $1,000 in total capital used in the prior calculation). In this alternate calculation, we have ignored *internal* accounts payable ($100) and retained earnings ($200). Incremental internal funds will be provided over the coming year as in the past.

Below we illustrate both answers.

| **Answer #1** | |
|---|---|
| **(30% Debt Ratio)** | |
| Debt = | $54.30 |
| Equity = | 126.60 |
| ***Total*** | $180.90 |
| ***D/TA*** | 3/10 = 30% |

| **Answer #2** | |
|---|---|
| **(28.5% Debt Ratio)** | |
| Debt = | $51.69 |
| Equity = | $129.21 |
| ***Total*** | $180.90 |
| ***D/TA*** | 2/7 = 28.5% |

Note 2:

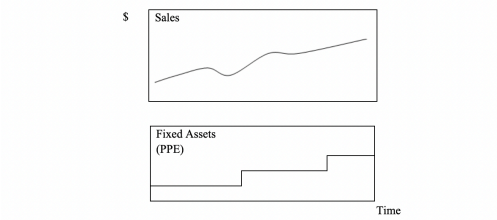
In the first two expressions in the EFN model, i.e., $[(A_0/S_0)\ \Delta S]$ and $[(AP_0/S_0)\ \Delta S]$, we utilize the incremental, projected sales increase (i.e., $\Delta S$) whereas the third portion $[(M_0)\ (S_1)\ (1 - PR_0)]$, we utilize the entire projected sales amount ($S_1$).

## Why, in fact, may Financial Ratios change?

In the foregoing analysis, we assumed that financial ratios do not change over time. In fact, ratios are dynamic. Here are some reasons why, in fact, ratios will change.

1. **Economies of scale** – as companies grow larger and produce more, they benefit, according to Microeconomic theory, from lower production costs per unit produced, even though total (production) costs are rising. The phenomenon will continue until the firm reaches full capacity and needs to add on assets in order to continue growing. The Asset/Sales and Profit ratios will be affected. Assets will remain flat as sales grow – until full capacity is reached. With unit costs down, gross profits will increase.
2. **"Lumpy Assets"** i.e., firms may reach a level where they have to acquire more (expensive) assets (assets don't grow smoothly or in the same proportion as the sales increase, which is likely smoother). For example, the factory may be at only 60% of capac-

ity, which affords substantial growth in production and sales without adding on capacity or expanding to an additional facility. Should sales continue to grow beyond present capacity, it would be incumbent upon the firm to add to its fixed assets at some point. Such assets would suddenly jump in size. Sales, by contrast, may grow relatively smoothly over time. The Sales/ Total Asset ratio will provide some insight.




3. **Capital Costs** (i.e., the *economic* costs to the corporation of borrowing money and issuing stock) will likely change over time – interest rates will go up and down, causing changes in the cost of debt, borrowed money. If the cost of debt *decreases*, after-tax income may *increase*. It will then be cheaper to finance new fixed assets, so the firm may then decide to add on more P, P, & E. The T.I.E. Ratio (EBIT / Interest Expense) may provide some added insight here. Also, stock investors will demand that the company produce dividends and increased profits – to be retained if not

distributed as dividends. Additions to Retained Earnings will increase the company's value and hence its share price. (Capital Costs should not be confused with Capital Assets Costs.) In short, what investors require or demand, the corporation must provide; return to the investor is an *economic cost* to the corporation.

## 9.9 Summary: The Fundamentals of Accounting and Financial Analysis

This might be a good place to review what we have learned thus far. First, we highlighted some difficulties in reading financial statements for financial analysts. We did this by focusing on examples of current (inventory) and long-term asset accounting (depreciation) respectively. These highlights assisted us in listing and understanding the four primary issues related to accounting data interpretation namely, historical bias, the arbitrary use of cost method, and problems having to do with estimates and reserves. For us, the statements may not be what they seem, with consequent reduced usefulness. If we are going to use accounting data as inputs for ratio analysis, we must first and foremost be cognizant of this, and (later) learn how to adjust the numbers, a skill, which you would acquire in a *Financial Statements Analysis* course.

Between highlighting accounting problems and presenting ratio analysis, we discussed the creation of pro forma financial statements and created a projected income statement. We also looked at a model for projecting free cash flow, a very important predictor of a company's ongoing performance, and a metric by which possible investment projects are evaluated. Finally, we discussed the *External Funds Needed* model, an important tool in capital planning in order to accommodate growth.

# 9.10 Chapters Eight & Nine: Review Questions

Chapters 8 – 9: Review Questions

1. Define each of the following terms: Incrementalism, Sunk Costs, and Cannibalization.
2. In words, explain what is meant by Free Cash Flow.
3. Why is FCF important? Give two reasons.
    - How do we use this model – for individual projects, for the entire corporation, or both? Explain.
    - What options does the company have regarding how it may choose to utilize its Free Cash Flow?
4. Create a *Free Cash Flow*template and spread the forecasted numbers based on the following assumptions:
    - Last year's sales were $15.5 million

and are expected to grow for the next two years at 15% per year, followed by three years of 8% growth.

- Cost of Goods Sold last year were $12.6 million and are expected to grow at a 7% rate per year indefinitely.
- Depreciation is $550,000 per year at a straight-line rate; in the fifth year, the building will have been fully depreciated. This company has no depreciable equipment.
- There is no amortization.
- Selling and General Administrative expenses last year were $200,000 and will grow modestly at an annual 2% rate.
- This company is in the 30% tax bracket, including Federal and State. There are no local taxes.
- The company expects to spend $2 million each year on "CapEx," all of which will be necessary.
- Last year's Current Assets, excluding Cash, were $2.5 million, and is expected to grow at a 5% rate per year indefinitely.
- Last year's Current Liabilities were $2 million and are expected to grow at a 3% rate for at least five years.

5. How does the analyst handle depreciation in

the FCF Model? Why does s/he handle it that way? Note that depreciation occurs twice in the formula.

6. Can you list all four capital items, which are included in the Balance Sheet?
7. Why don't we include capital costs in the FCF Model?
8. An increase in Current Assets *provides for/uses* funds. Which is it? Why?
9. On what basis do we distinguish between "internal" and "external" funds?
10. List some internal and external funds.
11. Calculate the *External Funds Needed* formula for the LCM Company (below), based on the following assumptions.
    - Last year's sales were $5,000 million.
    - Next year's objective is to increase sales by 30%.
    - Variable costs will be 70% of sales. (Variable costs change with sales volume.)
    - Fixed Costs are expected to run 30% of P, P, & E (Fixed costs do not change and are unrelated to sales volume.)
    - Interest Expense is 5% of Notes Payable and 7% of Long-term debt.
    - Taxes are 40%.

- The company expects to maintain its Payout Ratio at 20% of income.

What is the company's EFN if it is to meet its growth objective?

| Assets | |
|---|---|
| Cash & Equivalents | 40 |
| Accounts Receivable | 420 |
| Inventory | 480 |
| Current Assets | 940 |
| Property, Plant, & Equipment | 1,500 |
| Other Long-term Assets | 350 |
| Total Assets | **2,790** |

| Liabilities & Equity | |
|---|---|
| Accounts Payable | 375 |
| Notes Payable | 100 |
| Current Liabilities | 475 |
| Long-Term Debt | 900 |
| Common Stock | 800 |
| Retained Earnings | 615 |
| Liabilities & Equity | **2,790** |

**Income Statement for "LCM Corp"**
For Year Ending 12.31.XX
($ Thousands)

| | |
|---|---|
| Sales | |
| Variable Costs | |
| Fixed Costs | |
| Earnings before Interest & Taxes | |
| Interest Expense | |
| Earnings before Taxes | |
| Taxes | |
| Net Income | |

| | |
|---|---|
| Dividend Paid | |
| Addition to R/E | |

1. What do this year's three Solvency ratios look like?
2. Why will the company's financial ratios change next year?

## Selected Answers To Chapters 8 and 9

**FCF Table**

- You are ill-advised to do this by XL. Do it by hand. Place it in a Word table.
- One needs to figure EBIT by adding in the Income Statement data to the template in the chapter.
- "Last Year's" numbers are not illustrated in this Spread Sheet.
- "Year 1's" numbers follow "Last Year's." For example, Last Year's Sales were $15.5 Million. "This Year's" sales increased by 15%. Therefore: (15.5) (1.15) = $17.825.
- Be careful about the Current Assets and Current Liabilities numbers. We first calculate increases or decreases, not the gross numbers. Which data *add* to FCF?

**($ Millions)**

| | Year 1 | Year 2 | Year 3 | Year 4 | Year 5 |
|---|---|---|---|---|---|
| Sales | 17.825 | 20.499 | 22.139 | 23.910 | 25.823 |
| COGS | (13.482) | (14.426) | (15.436) | (16.516) | (17.672) |
| Gross Profits | 4.343 | 6.073 | 6.703 | 7.394 | 8.151 |
| S, G, & A | (0.204) | (0.208) | (0.212) | (0.216) | (0.221) |
| Depreciation | (0.550) | (0.550) | (0.550) | (0.550) | 0 |
| EBIT | 3.589 | 5.315 | 5.941 | 6.628 | 7.93 |
| Add Back: Depreciation | 0.550 | 0.550 | 0.550 | 0.550 | 0 |
| EBITDA | 4.139 | 5.865 | 6.491 | 7.178 | 7.93 |
| **EBITDA × (1 - .30)** | **2.897** | **4.106** | **4.544** | **5.025** | **5.551** |
| ***Plus*** | | | | | |
| Depreciation | 0.550 | 0.550 | 0.550 | 0.550 | 0 |
| **Depreciation × .30** | **0.165** | **0.165** | **0.165** | **0.165** | **0** |
| ***Less*** | | | | | |
| **Nec. Cap Ex** | **(2.0)** | **(2.0)** | **(2.0)** | **(2.0)** | **(2.0)** |
| ***Less*** | | | | | |
| Current Assets | 2.625 | 2.756 | 2.894 | 3.039 | 3.191 |
| **Incr. in Current Assets** | **(0.125)** | **(0.131)** | **(0.138)** | **(0.145)** | **(0.152)** |
| Current Liabilities | 2.060 | 2.122 | 2.185 | 2.251 | 2.319 |
| **Decr. in Current Liab.** | **0.060** | **0.062** | **0.063** | **0.066** | **0.068** |
| ***Equals FCF*** | **0.997** | **2.202** | **2.634** | **3.111** | **3.467** |

## EFN Formula

**($ Millions)**

I have assumed, in the question itself, that the B/S and I/S data are stated in Thousands, rather than Millions. This is more palpable. Note that here, the numbers are simplified to Millions; it's shorter. Assume that PR = 20%. Beware the differences!

| | | |
|---|---|---|
| Sales | 5.0 | 5.000 |
| (VC) | (.5) (0.70) = | (3.500) |
| (FC) | (1.5) (0.30) = | (0.450) |
| EBIT | 5 – 3.5 – 0.45 = | 1.050 |
| (Interest) | (100) (0.05) + (900) (0.07) = | (0.068) |
| EBT | 1.05 – 0.068 = | 0.982 |
| (Tax) | (0.982) (0.40) = | (0.393) |
| NI | EBT – T = | 0.589 |

| | | |
|---|---|---|
| Dividend | (0.20) (0.589) = | 0.1178 |
| ARE | (0.80) (0.589) = | 0.4712 |

- D/E = (900 + 475) / 1,415 = 0.97
- D / TA = (900 + 475) / (900 + 475 + 1,415) = 0.4828
- TIE = EBIT / I = 1.050 / 0.068 = 15.44x

- **EFN** = $[(A_0/S_0)\ \Delta S] – [(AP_0/S_0)\ \Delta S] – [(M_0)\ (S_1)\ (RR_0)]$
- We will assume "Static Analysis."

**($ Millions)**

| | | |
|---|---|---|
| $(A_0/S_0)$ | 2.790 / 5.000 = | 0.558 |
| $\Delta S$ | [(5.0) (1.3)] – (5.0) = | $1.5 |
| **$[(A_0/S_0)\ \Delta S]$** | | **$ 0.837** |
| *Less:* | | |
| $AP_0/S_0$ | [(0.375) / 5.00)] = | 0.075 |
| **$[(AP_0/S_0)\ \Delta S]$** | | **$ 0.113** |
| *Less:* | | |
| $M_0$ | 0.589 / 5.00 = | 0.1178 |
| $RR_0$ | | 0.80 |
| **$[(M_0)\ (RR_0)\ (S_1)]$** | **(0.1178) (0.8) (6.500) =** | **$ 0.6126** |
| *EFN Equals:* | | **$ 0.1114** |

- Sales and VC will change next year, but FC will remain the same. Therefore, we should see a change in the net profit margin, and changes in the dividend paid and retention rates – assuming no change in payout (percent of earnings) policy.

# Part III: The Time Value of Money

Part III: The Time Value of Money

Chapter 10: The Time Value of Money: Simple Future- and Present-Values

Chapter 11: The Time Value of Money: Annuities, Perpetuities, and Mortgages

# Chapter 10: The Time Value of Money: Simple Present- and Future-Values

# 10.1 Chapter Ten: Learning Outcomes

Learning Outcomes

In this chapter, you will:

- **Calculate** Simple Future- and Present-Values both mathematically and with a simple calculator, and by using an Interests Rates Tables.
- **Apply** the *Three Commandments* of the Time Value of Money (TVM).
- **Differentiate** between simple interest and interest-on-interest.
- **Consider** the curvilinear nature of compound interest and TVM.
- **Compare** the relative volatilities of short-term versus long-term cash flows.

Note: Review questions for Chapters Ten and Eleven will appear at the end of Chapter Eleven.

## 10.2 The Time Value of Money and Interest

Key Terms:

Future Value

For each of the following questions, assume you have $1 and that interest on it will be paid in full, at the END of the stated period. What are the **future values** (FVs) given each of the following questions? In other words, how much money will you have at the relevant future points in time? (If you had more than $1, the answers would be the appropriate multiple thereof.)

Present Value

As we go through the questions and calculations, observe how the outcomes, or solutions, change. Try to explain the reasons for the differences in the outcomes. Also, observe that the seemingly small differences in outcomes are really not as trivial as may seem at first glance. We are illustrating Future Values, in each question, of just one dollar of money that we have now – of **Present Value**. Suppose we were instead dealing with millions of dollars?

As we go through each question, we will, methodically and

painstakingly, create a general symbolic formula, which may be employed for any similar problem. Insert the appropriate values into the formulae to solve the problems numerically. (Solutions follow.)

1. You will earn 5% interest, paid once a year, at the END of the year, for one year.

You have $1 now of Present Value (PV). In one year, you will receive your "principal" of $1 back *plus* interest at an annual rate of return (R) of 5%. A general Future Value (FV) formula will therefore be:

$$\mathbf{FV = PV\ (1 + R)}$$

Insert the relevant data into the formula in order to solve for the Future Value.

As we go through this analysis, you will need to learn and remember the symbols or abbreviations.

2. Same as question #1, but R = 10%.

Here we will use the same formula as above, but you will insert a different rate for R. What is the Future Value? Why is the outcome different?

3. Same as question #2, i.e., R = 10%, but for two years (n years), rather than just one.

We will now have two years of compound interest; n = 2. Therefore, we apply the FV formula, slightly modified, a second time:

$$\mathbf{FV = PV\ (1 + R)\ (1 + R)}$$

$$\mathbf{FV = PV\ (1 + R)^n}$$

Here, the exponent, "n," stands for the number of years in which the money compounds.Once again, in this case, n = 2. What is the Future Value? Why is the outcome different than in the prior question?

## 4. 10% interest, twice a year, for one year.

Interest is *always* quoted as an <u>annual</u> rate, unless explicitly noted otherwise.

Our annual rate is still 10%, but we will receive half of it, i.e., R ÷ p = 0.10 ÷ 2 = 0.05, at the *end* of each half-year. The letter, "p" stands for the number of compounding *periods* per year; here *p* = 2.

$$\mathbf{FV = PV\ (1 + R/p)(1 + R/p)}$$

Notice that, while *n* = 2, there are now two compounding periods per year, so the exponent must reflect that. Our exponent is therefore now: "*n* × *p*."

Whenever p ≠ 1, we must make *two* adjustments to the formula: "R/p" in the rate part of the formula and "n × p" in the exponent. While in theory P can take on any integer value, it will actually be equal to 1, 2, 4, 12, or 365 for annually, semi-annually, quarterly, monthly, or daily respectively. (In this example, n = 1; when p = 1, we tend to leave it out as the power of one is implicit.) Our general formula now becomes:

$$\mathbf{FV = PV\ (1 + R/p)^{n \times p}}$$

Our Future Value formula is now complete. What Future Value do you get? Why is the outcome different than in question #2?

Note that, when solving for Future Value, we multiply the Present Value by a "factor" of $\mathbf{(1 + R/p)^{n \times p}}$. A **factor** is simply a **multiplier**. This multiplicative and **exponential** process is referred to as **compounding**.

Factor

Multiplier

Exponential

Compounding

5. Same as question #4, but for two years.

We can employ the formula in the prior question. Remember, "p" occurs twice in our formula. Here, n = 2, and p = 2. What is your Future Value? Why is the outcome different than in question #3?

6. What happens to future values as interest rates ("R"), the number of years ("n"), and compounding frequency ("p") increase? In answer to this question, we present, in summary, the "Three Commandments" of the Time Value of Money.

**The Three Commandments of TVM**

| If R, N, and/or P Rise | Future Value | Present Value |
|---|---|---|
| Annual Interest Rates (R) | Rises | Declines |
| Number of Years (N) | | |
| Compounding Periods (P) | | |

Of course, the opposite will occur if R, N, and/or P decline. Below, we explain *Present Values*.

7. For each of the above questions, what would be the *present value*of $1 to be received at the end of the stated periods? In other words, where in questions 1 though 5, we assumed $1 of PV and had to calculate its FV, we will now assume $1 of FV and calculated its PV using the data in each of the first 5 questions. Here, we will re-employ our Future Value formula, but transpose the FV and PV numbers so that we can solve for PV, rather than for FV.

$$\mathbf{PV = FV \div (1 + R/p)^{n \times p}}$$

Discounting

Reciprocal

Thus, when solving for Present Value we *divide* the Future Value by the **(1 + R/p) n × p** factor. This division process is referred to as **discounting**.

Rather than dividing, we can also multiply the Future Value by the **reciprocal** of the divisor.

$$\mathbf{PV = FV \times [1 \div (1 + R/p)^{n \times p}]}$$

<u>Definition</u>: A *reciprocal* is the inverse of a number, which is arrived at by turning the number upside down! So, 1/2 or 0.5 is the reciprocal of 2. The reciprocal of 5 is 1/5 – or 20%. So, $(1 + R/p)^{n \times p}$ and $[1 \div (1 + R/p)^{n \times p}]$ are reciprocals of one another.

In using Interest Rate Tables (soon), which display ready-made, already-calculated factors, you will note that all the factors' values are stated as multipli-

ers, including the Present Value Factors (PVFs). We will use tables in order to cut down on the number of calculations that we must make and to thereby reduce errors.

8. Is it a realistic question to ask what the PV is? How might this actually occur?

Of course it is! We will often know the future payments and wish to figure out the PVs! For example, you may wish to put aside some money for the down payment on a house in "*n*" years. Assuming you know the "discount rate," how much must you set aside today in order to fund that amount? How much will you need to set aside today to fund your newborn child's college tuition? A bond pays interest every six months in a known amount. How much should you pay today in order to receive those future amounts?

Solutions and Explanations:

The $1 in the question is referred to as Present Value (PV). The amount of money we will have in the future is referred to as Future Value (FV). Remember, we are, so far, assuming that the interest payments are made at the *end* of each relevant payment period.

| Question | Calculation | What we learn from this |
|---|---|---|
| 1 | $1 (1.05) = $1.05 | $1 times one (representing 100% of your principal) plus the interest rate. |
| 2 | $1 (1.10) = $1.10 | As *interest rates* increase, so too do FVs. |
| 3 | $1 (1.10) (1.10) = $1 $(1.10)^2$ = $1.21 | As *time* (n) increases, so too do FVs. |
| 4 | $1 (1.05) (1.05) = $1 $(1.05)^2$ = $1.1025 | Rates are always quoted in annualized terms, unless otherwise indicated. You must make the necessary adjustments. Here we see that as the number of compounding periods (p) per year increases, so too does the FV. |
| 5 | $1 $(1.05)^4$ = $1.2155 | We have two adjustments here: n = 2 *and* p = 2. |
| 6 | | See the "Three Commandments." |
| 7.1 | $1 ÷ 1.05 = $0.9524 | Whereas solving for FV involves multiplication (i.e., compounding), solving for PV involves division or discounting. |
| 7.2 | $1 ÷ 1.10 = $0.9091 | As interest rates increase, PVs decrease. |
| 7.3 | $1 ÷ $(1.10)^2$ = $0.8264 | As time increases, PVs decrease. |
| 7.4 | $1 ÷ $(1.05)^2$ = $0.9070 | As discounting frequency increases, PV decreases. |
| 7.5 | $1 ÷ $(1.05)^4$ = $0.8227 | The present value is lower because the exponent is higher than in 7.4. |
| 8. | Yes! | This is how bonds and many financial things are figured. Can you provide (more) examples? |

*An investment in knowledge pays the best interest.*

-Benjamin Franklin

## 10.3 Interest-on-the-Interest: The Nature of Compound Interest

After one year (as noted in the example above), the investor will have earned $0.10 for every dollar invested at 10%. This was represented by the formula: $1 (1.10) = $1.10.

We noted that if the investor had invested for two years (again at 10% per year) he would have $1.21. This was represented by: $1 (1.10)2 = $1.21. In other words, in the first year, s/he earned $0.10 in interest, while in the second year, he earned $0.11. Why does he earn more interest in the second year if the interest rate – 10% – remains the same?

In the second year, he once again receives $0.10 interest on his principal investment of $1. However, since he has already earned $0.10 of interest from the prior year, he will also earn 10% on that dime! That is equal to another penny of interest earned: ($0.10) (10%) = $0.01. Thus, in the second year he will have earned another $0.10 plus $0.11 or a total of $0.21. The following summarizes this notion:

| Year | Math | FV | | Interest Earned | |
|---|---|---|---|---|---|
| 1 | $1 (1.10) | = $1.10 | Interest | ($1) (0.10) = | **$0.10** |
| 2 | $1 $(1.10)^2$ | = $1.21 | Interest | ($1) (0.10) = | $0.10 |
| | | | I-on-I | ($0.10) (0.10) | $0.01 |
| | | | Total Interest: Year 2 | | $0.11 |
| | | | Plus, Prior Year's Interest | | $0.10 |
| | | | Total: Two Years | | **$0.21** |
| 3 | $1 $(1.10)^3$ | = $1.3310 | Interest | ($1) (0.10) = | $0.100 |
| | | | I-on-I | ($0.21) (0.10) | $0.021 |
| | | | Total Interest: Year 3 | | $0.121 |
| | | | Plus, Prior Years' Interest | | $0.210 |
| | | | Total Interest: Three Years | | **$0.331** |

This is the nature of interest-on-the-interest: *Compound Interest*! Each year, the interest-on-the-interest will continue to compound.

## 10.4 Some More Simple TVM Problems

What is my *Future Value* under the following scenarios? (After each solution, write out the equivalent mathematical, symbolic notation.)

Scenario 1: Assume I have $1 and I invest for one year, receiving an interest payment of .12 at the year's end.

$$\$1\ (1.12) = _____$$

Scenario 2: What if I invest for two years, receiving an interest payment of .12 at the end of each year?

$$\$1\ (1.12)\ (1.12) = \$1\ (1.12)^2\ _____$$

Scenario 3: What if Scenario 1 is changed to account for semi-annual compounding?

$$\$1\ (1 + .12/2)^{1 \times 2} = _____$$

Scenario 4: What if Scenario 2 is changed to account for semi-annual compounding?

$$\$1\ (1 + .12/2)^{2 \times 2} = _____$$

- If I know the Future Value of $1, how do I calculate the PV? Solve for each.

Note:

Asking "what is the present value of some money to be received in the future," is equivalent to asking how much money is needed today in order to have a certain amount later, assuming a given investment rate. In other words, for a person to have one dollar five years from now, i.e., FV, how much will s/he have to invest today at x%?

## Some More Simple TVM Problems (Solutions)

1. $1 (1.12) = $1.12
2. $1 (1.12) (1.12) = $1 $(1.12)^2$ = $1.2544
3. $1 $(1 + .12/2)^{2 \times 1}$ = $1 $(1.06)^2$ = $1.1236
4. $1 $(1 + .12/2)^{2 \times 2}$ = $1 $(1.06)^4$ = $1.2625

Notice how the above solutions display the fundamental principles of the *Time Value of Money* about which we already spoke. Namely, as interest rates, the number of compounding periods per year, and time increase, the Future Value increases and the Present Value decreases.

It is very easy to make mistakes in doing these calculations. For example, remember that the compounding frequency adjustment, "p," occurs twice in the basic TVM formula; don't forget to make the relevant adjustments here. Be methodical and go slowly.

In the end, “eyeball” your solution. If it does not look right in terms of the TVM rules that we already know, it probably is not! It will look correct if it seems to be consistent with the above-cited characteristics of TVM.

So far, in all the foregoing examples, we have used $1 as present value. This makes it easy to learn and allows one to focus on the manner in which present and future values multiply out. In reality of course, present values would be other, greater numbers, such as $1,237,874.32. All one need do is substitute in the relevant number where heretofore we had the lonely $1 value.

*Time is money.*

-Benjamin Franklin

# 10.5 Simple Future and Present Values (Formulas)

Having done the foregoing work, it is plain to see that we can symbolically represent the mathematics using the following "language."

| | |
|---|---|
| Simple Future Values | $FV = PV \times (1 + R/p)^{n \times p}$ |

| | |
|---|---|
| Simple Present Values | $PV = FV \div (1 + R/p)^{n \times p}$<br>or<br>$PV = FV \times [1 \div (1 + R/p)^{n \times p}]$ |

<u>Key</u>:

| | |
|---|---|
| FV | Future Value (in dollars) |
| PV | Present Value (in dollars) |
| R | (Annual) *Rate* of Interest |
| n | *Number* of Years |
| p | Number of Compounding/Discounting *periods* per year |

The expression, or "factor," $(1 + R/p)^{n \times p}$, may be used as a "multiplier" when *compounding* from present to future values and, in its reciprocal form, as a multiplier again when *discounting* from future to present values. You will find the factors, calculated out, in interest rate tables, truncated versions of which you will find on the pages following.

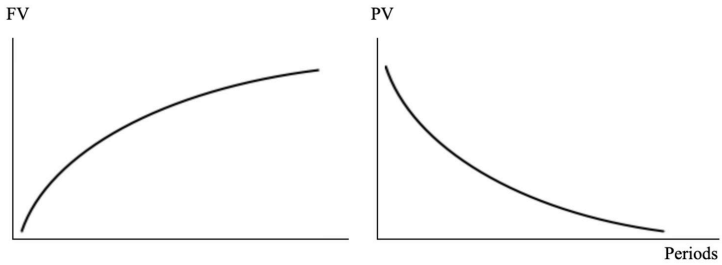
"R/p" means that if the annual interest rate (R) is 12% and the number of compounding periods (p) is 12 (i.e., monthly compounding) the periodic compound rate is .12 ÷ 12 = .01. After one year, the FV would be $1 $(1.01)^{12}$ = $1.1268. (Notice that this compares with once–a–year compounding at 12%: $1 (1.12) = $1.12. The difference in Future Values is *not*trivial.

# 10.6 Compounding Frequency Assumption

Let's examine the effect of changing the compounding (or discounting) frequency on both the Present- and Future-Values. Assume that we earn 10% for five years (R = 0.10; *n*= 5). Assume that we are given $1 of Present- and Future-Values respectively.

| Compounding Frequency Assumption | *P* | Future Value of $1 | Present Value of $1 |
|---|---|---|---|
| Annually | 1 | $1.6105 | $0.6209 |
| Semi-Annually | 2 | $1.6289 | $0.6139 |
| Quarterly | 4 | $1.6386 | $0.6103 |
| Monthly | 12 | $1.6453 | $0.6078 |
| Daily | 365 | $1.6486 | $0.6066 |
| Continuously | | $1.6487 | $0.6065 |

Notice how, as "P" increases, FVs increase, and PVs decrease – both at decreasing rates!




The mathematics for continuous compounding and discounting follow on the next page. You will note that the difference between daily and continuous compounding and discounting is very small. Today, we don't often – if ever – see instruments that exhibit continuous compounding.

# 10.7 Simple Future and Present Values: Continuous Compounding (Supplemental)

In order to solve for continuous compounding, we must engage the "rule of limits" or otherwise utilize the "natural log." The natural logarithm is the logarithm to the base *e*, where *e*is equivalent to the irrational number 2.71828. The following presents an exemplary solution for continuous compounding.

$$FV = PV\ (e^{Rn})$$

and

$$PV = FV\ (e^{-Rn})$$

Where, e = 2.71828

R = interest rate

Note:

P is omitted since the compounding is continuous rather

than periodic.

<u>Example</u>: PV = $1

R = .09

N = 10 years

FV = ?

<u>Solution</u>: FV = ($1) (2.71828 $^{(.09)\,(10)}$ )

= $2.4596

## 10.8 Characteristics of the Time Value of Money: FV and PV

The following summarizes and reviews some key characteristics of the *Time Value of Money* about which we already learned.

- As the interest rate increases, *Future* Value also increases.
- As the number of years (n), periods per year (p), or total periods (n × p) increases, the *Future* Value increases.
- As the compounding frequency per year increases, the *Future* Value increases.
- Restate each of the above statements for *Present* Values.
- FV and PV factors – or multipliers – are reciprocals of one another.

You will find below a partial interest rate table. If you use such tables properly, you will be able to locate the correct multipliers – or "factors" – for a given situation. You will note that the PV factors are expressed as the reciprocals of their corresponding FVs. In order to arrive at the FV or PV of a specified dollar amount, one need only choose the correct cell and multiply the specific dollar amount by the factor.

For example, the FV of $1 at 5% for ten years is $1 ×

1.6289. The PV for 10% compounded semi-annually for five years is $1 × .6139.

In cases where we have more than one compounding or discounting period per year, we would need to make the same adjustments that were made mathematically earlier. For example, the multiplier for 10% and 5 years semi-annually would be found under the 5% column and 10-period row. “Period” in the tables would be equivalent to “n × p” in our, more mathematical, nomenclature.

| | **Future Value Factors** | | **Present Value Factors** | |
|---|---|---|---|---|
| **Period** | **5%** | **10%** | **5%** | **10%** |
| 1 | 1.0500 | 1.1000 | .9524 | .9091 |
| 2 | 1.1025 | 1.2100 | .9070 | .8264 |
| 5 | 1.2763 | 1.6105 | .7835 | .6209 |
| 10 | 1.6289 | 2.5937 | .6139 | .3855 |
| 20 | 2.6533 | 6.7275 | .3769 | .1486 |
| 30 | 4.3219 | 17.449 | .2314 | .0573 |
| 50 | 11.467 | 117.39 | .0872 | .0085 |

# 10.9 Future and Present Value Factors (Multipliers)

Here is another look at a somewhat less abbreviated interest rate table. Assume that we are given $1 of Present- and Future-Values respectively. (Fill in the empty 15% column by hand and compare your answers to the factors in the published tables; see the link to Interest Rate Tables below.) Note that, in using tables, "Periods" = n × p.

**Future Value Factors** Formula: $FV = PV (1 + R/p)^{n \times p}$

| Period $n \times p$ | .05 | .10 | .12 | .15 | .20 |
|---|---|---|---|---|---|
| **1** | 1.0500 | 1.1000 | 1.1200 | | 1.2000 |
| **5** | 1.2763 | 1.6105 | 1.7623 | | 2.4883 |
| **10** | 1.6289 | 2.5937 | 3.1058 | | 6.1917 |
| **15** | 2.0789 | 4.1772 | 5.4736 | | 15.407 |
| **20** | 2.6533 | 6.7275 | 9.6463 | | 38.338 |
| **30** | 4.3219 | 17.4490 | 29.960 | | 237.38 |
| **50** | 11.467 | 117.390 | 289.00 | | 9,100.4 |

**Present Value Factors** Formula: $PV = FV \div (1 + R/p)^{n \times p}$

| **Period** $n \times p$ | **.05** | **.10** | **.12** | **.15** | **.20** |
|---|---|---|---|---|---|
| **1** | .9524 | .9091 | .8929 | | .8333 |
| **5** | .7835 | .6209 | .5674 | | .4019 |
| **10** | .6139 | .3855 | .3220 | | .1615 |
| **15** | .4810 | .2394 | .1827 | | .0649 |
| **20** | .3769 | .1486 | .1037 | | .0261 |
| **30** | .2314 | .0573 | .0334 | | .0042 |
| **50** | .0872 | .0085 | .0035 | | .0001 |

Here are some interest rate tables for you to use:

http://www.retailinvestor.org/pdf/futurevaluetables.pdf

# 10.10 A Word on Compounding Frequency and Annual Equivalent Rates

Questions:

1. What is the difference between 10% annual rate, compounded annually versus a 10% annual rate, compounded quarter- in terms of FV of $1?
2. How may the two rates be equated? In other words, how may I define the *annual* frequency at a rate equivalent to the *quarterly* frequency?
3. What happens as the compounding frequency increases?

Solution to Question #2:

In general, $(1 + R/p)^{np} = FV$

Annually, $(1.10)^{1} = \mathbf{1.10}$

versus

Quarterly, $(1 + .10/4)^{1 \times 4} = (1.025)^{4} =$
**1.1038**

That is, 10% quarterly is equivalent to 10.38% annually!

In other words, the Annual Percentage Rate (APR) for 10% compounded quarterly, is 10.38%!

## 10.11 Interpolation

What if we have a *fractional* compound or discount rate – such as 9.5% – which is not to be found in published tables? Can we still use the tables? Or do we need to solve the problem mathematically (or by financial calculator)?

One way to get around this is by *estimating* the multiplier by means of averaging the two whole multipliers that bracket the fractional one in question. We call this process "interpolation" because we *insert* the average number in between the relevant values or factors, which are stated in the table. Let's use the future value table below to illustrate this.

**Future Value Factors**

| | .09 | .10 |
|---|---|---|
| **10 Years** | 2.3674 | 2.5937 |

What is the appropriate, estimated future value multiplier for 10 years at 9.5%? This can be approximated by taking the simple average of the multipliers for 9% and 10%:

$$(2.3674 + 2.5937) \div 2 = 2.4806$$

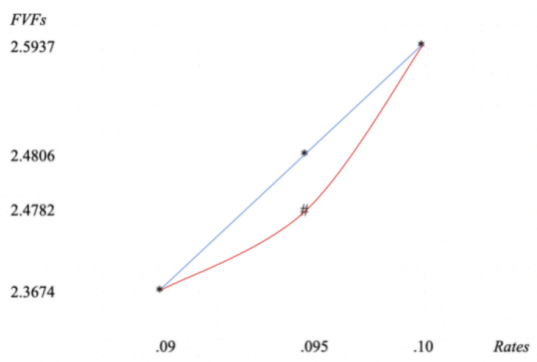
This is only an estimate. Remember that time value factors increase (in this case – or decrease in the case of present values) at an *exponential* rate. The true mathematical future value for 9.5% for 10 years is:

$$1.095^{10} = 2.4782$$

This differs from the simple average estimate, of 2.4806 – 2.4782 = 0.002. We first note that the simple average produces a higher number than the true, exponential figure; this is notable. This is as expected because, in dealing with future value factors, the numbers grow exponentially or more quickly, and thus we would start off with a lower number. Put differently, we do not grow from 9% to 10% at a linear rate, but at an increasing, curvilinear rate.

## 10.12 Interpolation Illustrated

Interpolation may be useful in order to estimate future (and present-) values when one does not wish, or have the ability, to calculate more precise measures. The reason for the error has to do with the curvilinear relationship between (discount- and) compound interest rates and their related multipliers. This is best seen by illustration.




One may readily see that, by connecting the asterisks, the interpolated value of 2.4806 resides on a straight line between the correctly calculated future value multipliers for 9% and 10% respectively.

However, the time value of money is not linear. Any time an exponent is involved, you will not get a linear relationship, but a curvilinear outcome of some sort. Hence, the correct multiplier for 9.5% is 2.4782, which is lower

than the interpolated arithmetic average of 2.4806. If one joins the asterisks for 9% and 10% to the mathematically calculated middle value of 2.4782 (represented by “#”), one readily observes the curvilinear relationship between compound interest rates and their respective multipliers. In short, as rates increase, future values increase in non-linear fashion. Correspondingly, present values would decrease non-linearly. The non-linear nature of these curves will soon be discussed in greater depth when we get to “Volatility.” Basic mathematical examples will be presented.

# 10.13 Some TVM Practice Questions

You will need to solve all these problems by hand. You will not be able to use the tables.

1. You are given the following. Investment = $2,800, Rate = 0.54%, P = quarterly, N = 8 years. What is the Future Value?
2. You are given two choices: 1. invest at an annual rate of 10% compounded monthly, or 2. at 10.1% compounded semi-annually. Which will you prefer?
3. Bonus question: You will receive $24,000, $29,500, $58,000 and $87,000 each year consecutively for the next four years. What are both the Present- and Future-Values of this *uneven*income stream? Assume an annual rate of 4.6%. (We will learn how to do *Uneven Cash Flows*after we do *Annuities*– below.)

**Solutions**

1. ($2,800) $(1 + 0.0054/4)^{8 \times 4}$= **$2,923.5256**

2. First Choice: $(1 + .10/12)^{1 \times 12}$ = **1.1047** *This is the one.*

Second Choice: $(1 + .101/2)^{1 \times 2}$ = 1.1036

3. **Present Value =**

($24,000 ÷ $1.046^1$) + ($29,500 ÷ $1.046^2$) + ($58,000 ÷ $1.046^3$) + ($87,000 ÷ $1.046^4$) =

$22,944.55 + $26,962.41 + $50,679.57 + $72,676.25 = **$173,262.78**

**Future Value =**

($24,000 × $1.046^3$) + ($29,500 × $1.046^2$) + ($58,000 × $1.046^1$) + ($87,000 × $1.046^0$) =

$27,466.69 + $32,276.42 + $60,668 + $87,000 = **$207,411.11**

Notice the nature of the exponents in the Future Value calculation; the exponents decrease as we near the *horizon*. Interestingly, it is also true that $173,262.78 ×1.046 (to the fourth) = $207,411.11. If you had already calculated the Present Value of this uneven series of cash flows, you would not have had to go through the long calculation of the Future Value.

(If you have trouble with this, it's OK. We will get to *Uneven Cash Flow* series soon. You can come back to it later.)

## 10.14 The Volatility of the Time Value of Money

Discrepancies in TVM factors will widen as time increases, as one observes the relative factors between interest rate columns.

For example, a five-year IOU with a future value of $1,000, using the tables, would have a present value of $1,000 (0.7835) = $783.50 – at a discount rate of 5%. The IOU could be purchased or sold for that amount, or price. Think of present value as an item's dollar price. If the discount rate instead were 10%, the present value would be only: $1,000 (0.6209) = $620.90. In percentage terms, the present value of $1,000 to be received five years from now, discounted at a rate of 5% is greater than at 10% by a difference of (783.5 ÷ 620.9) – 1 = **26.2%.**

If however the IOU had a 30-year term, the difference in present value would itself compound. At 5%, the present value would equal $1,000 (0.2314) = $231.40. At 10%, the PV would equal $1,000 (0.0573) = $57.30. In percentage terms, the present value of $1,000 to be received thirty years from now, discounted at a rate of 5% is greater than at 10% by a difference of (231.4 ÷ 57.30) – 1 = **303.8%**.

| | 5% | 10% | Percentage Difference<br>5% over 10% |
|---|---|---|---|
| **5 Years** | 0.7835 | 0.6209 | (783.5 ÷ 620.9) – 1 = **26%** |
| **30 Years** | 0.2314 | 0.0573 | (231.4 ÷ 57.30) – 1 = **303%** |

Key Terms:

Geometry

This demonstrates the volatility and geometry of TVM! By **geometry** here we refer to its non-linear and exponential nature. Whenever there is an exponent in a formula, we get some kind of curve. As time increases, differences in present- and future-values for a given number of years themselves increase non-linearly.

If you had purchased a thirty-year IOU as an investment, any changes in interest rates (i.e., due to market conditions) would have a far greater impact on the value of your IOU investment than if you had, instead, purchased a five-year obligation. For a given change in discount rates of interest, the impact on the multipliers is greater the greater the time is. The impact on price, which is present value, is greater, the greater the time–period. “Price volatility,” so to speak, increases as the future payment grows more distant.

Again, this is because, the time value of money is non-linear; it is exponential. We are dealing, quite literally, with compound interest, i.e., interest on the interest. Holders of long-term fixed obligations, such as bonds, may experience

greater price, or market value fluctuations, when discount rates for their bonds suddenly change.

Bonus Question: In the example above, we examined the increase in the Present Value Factor when interest rates drop from 10% to 5%. What would be the percentage change in the Factors if rates increased from 5% to 10%? Would it be same percentage change?

**Rates of Change in TVM Factors**

| **Years** | **Discount Rate** | | **Percentage Difference In Present Value:**<br><br>**5% over 10%**<br><br>**(Rate of Change)** |
|---|---|---|---|
| | **5%** | **10%** | **First Derivative** |
| 5 | 0.7835 | 0.6209 | (783.5 ÷ 620.9) – 1 = **26%** |
| 10 | 0.6139 | 0.3855 | (0.6139 ÷ 0.3855) – 1 = **59%** |
| 15 | 0.4810 | 0.2394 | (0.4810 ÷ 0.2394) – 1 = **101%** |
| 20 | 0.3769 | 0.1486 | (376.9 ÷ 148.6) – 1 = **154%** |
| 25 | 0.2953 | 0.0923 | (0.2953 ÷ 0.0923) – 1= **220%** |
| 30 | 0.2314 | 0.0573 | (231.4 ÷ 57.30) – 1 = **303%** |

The virtually instantaneous changes in present values, when going from a discount rate of 10% to 5%, increases("first derivative") at a decreasing rate("second derivative"). The table displays the extent to which 5% discounted present values exceed 10% discounted values.

When going from 10% to 5%, a five-year payment will

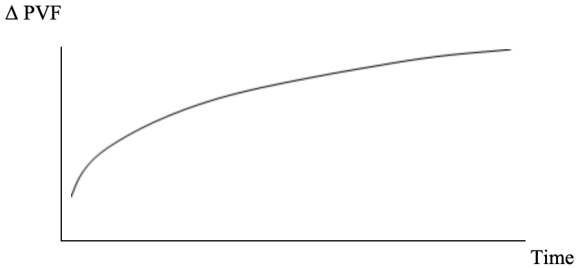
increase in value about 26%, while a thirty-year payment by over 300%! Imagine if you could buy an IOU at 10% and immediately ("instantaneously") turn around and sell it at 5%! Your profit would be much greater had you invested in the thirty-year obligation. While this case is exaggerated, the bond market works in similar fashion. Bond prices can, at times, be quite volatile due to changes in market rates although large changes do not occur instantaneously) except in the case of a disaster). Remember: prices are the present values of a bond's future payments!

These relationships can also be illustrated using Differential Calculus, which would give you a more "continuous," rather than a "discrete," view of the progress of the numbers.

The Take-away: If interest rates change, (bond) prices could change dramatically!!

## 10.15 The First and Second Derivatives Illustrated

- We have just demonstrated that changes in longer-term discount rates affect present value (i.e., prices) more than in the case of short-term rates and cash flows. In our example, the rate of changes in present value factors increased in time, e.g., from 26%, to 101%, to 303%.
- However, the rate of increase is decreasing (i.e., "deceleration" or second derivative), e.g., from 126%, to 71%, to 38%.
- In Calculus, we would say here that the first derivative is positive while the second derivative is negative. If we think of the price of the cash flows as distance covered as a function of time ("speed"), then we can think of the "First Derivative." We can think of the "Second Derivative" here as the change in the rate of speed ("deceleration") Were the second derivative positive, we would be speaking of "acceleration." We observe these characteristics in this diagram:




Imagine a payment due in the future; its present value would be determined by its rate – a short-term rate (and PVF!) for a short-term payment and so on. The rate of change in present value was noted in the first derivative column, while the "rate of change in the rate of change" in the present value was noted in the second derivative column.

In other words, as time increases, the differences in present values – for 5% versus 10% – increase, but the rate of increase decreases! Remember: the price of a financial asset today is its present value!

The Take-away (again): If rates change, prices (i.e., present values) could change dramatically!!

*Interest* (lit. "bite"). Exodus, 22:24: "Bite" means "interest," since it is like the bite of a snake, which bites a small wound on one's foot, which he does not feel, but suddenly it swells and blows up as far

as his head. So, with “interest,” one does not feel (it) and it is not noticeable until the interest increases and causes him to lose much money.

-Rashi’s commentary on the Bible.

(Translated by Ben Isaiah and Sharfman, Brooklyn, NY: S. S. & R. Publishing Company, 1949)

## On Work

*Success in business requires training and discipline and hard work. But if you’re not frightened by these things, the opportunities are just as great today as they ever were.*

-David Rockefeller

Economist and Banker

*If you train hard, you'll not only be hard, you'll be hard to beat.*

-Herschel Walker

American professional football player

and Politician

*What doesn't kill me makes me stronger.*

-Friedrich Nietzsche

German Philosopher

*Whenever heaven is about to confer a great responsibility on any man, it will exercise his mind with suffering, subject his sinews and bones to hard work, expose his body to hunger, put him in poverty, place obstacles in the path of his deeds, so as to stimulate his mind, harden his nature, and improve wherever he is incompetent.*

-Meng Tzu, Third Century B.C.E.

Quoted in The Coddled Mind, Chapter One, by Jonathan Haidt

*Ben Heh-Heh used to say: "According to the effort is the reward."*

-Mishnah, Avoth, 5:26

*No pain, no gain.*

-Somebody

*When the goin' gets tough, the tough get goin'.*

-Somebody Else

# Chapter 11: The Time Value of Money: Annuities, Perpetuities, and Mortgages

# 11.1 Chapter Eleven: Learning Outcomes

Learning Outcomes

In this chapter, you will:

- **Derive** the additive nature of Annuities.
- **Calculate** both Future- and Present-Value Annuity dollar values, using a simple calculator and Annuity Tables.
- **Adjust** Ordinary Annuity factors to Annuities Due.
- **Relate** the Law of Limits to Perpetuities.
- **Provide** the numerical analysis of No-Growth and Constant Growth Perpetuities, and Mortgages.
- **Determine** the total amount of interest paid on a mortgage over time in comparison to the principal originally borrowed.

## 11.2 Annuities

An *annuity* is a series of cash flows that must satisfy both of the following conditions, in order to qualify as an annuity, namely it:

- Consists of equal dollar amounts.
- Arrives (or leaves) in regular intervals.

If a series of cash flows may be defined as an annuity, we will employ an *Annuity Table* to figure the series' FV/PV.

In investments, most (if not all) annuities may be qualified as "Ordinary Annuities," since their cash flows occur at the end of the relevant periods.

Other annuities are called "Annuities Due," meaning that the CFs occur at the start of the relevant periods.

If, as in most cases, the cash flows do not qualify as an annuity, then their FV/PVs may be derived only by calculating the FV/PV of each discrete CF and then aggregating.

This is the same process by which we shall derivethe "short-cut" Annuity TVM factors.

## 11.3 The Derivation of (Ordinary) Annuity Factors

You are given the following information. Column by column, complete the table by filling in the appropriate future value factors (FVF), the future values of each respective cash flow (FVCF), as well as the same for the present value factors and cash flows (PVF and PVCF). Once completed, add up the columns at the bottom.

Note that here we are dealing with "ordinary" annuities, which means that all the cash flows in the series are received at the *end* of the relevant period. Soon, we will examine another convention. Use the timeline below to properly place each of the three cash flows temporarily (see the timeline below). Placement will determine the proper exponents and hence periods.

Given:

3-year annuity

$100 received per year.

Annual Discounting/Compounding Factor = R = .10

| Period | Cash Flow | FVF | FVCF | PVF | PVCF |
|---|---|---|---|---|---|
| 1 | $100 | | | | |
| 2 | $100 | | | | |
| 3 | $100 | | | | |
| Annuity Factors | | | | | |
| Dollar Values | | | | | |

| Timeline | | $CF_1$ | $CF_2$ | $CF_3$ | |
|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | Period |

Code:

FVF = Future Value Factor

FVCF = Future Value of the Cash Flow

PVF = Present Value Factor

PVCF = Present Value of the Cash Flow

CF1 = First Cash Flow

CF2 = Second

CF3 = Third

# 11.4 The Derivation of Annuity Factors (Solution)

Below you will find the solution to the problem on the prior page. Note that if you had an interest rate table for annuities, you would be able to multiply the annuity cash flow (in this case, $100) by the appropriate factor. You would then arrive at the future- or present-values of the cash flows, in one step. Such annuity interest tables exist; a link is provided at the bottom of this page.

| Period | Cash Flow | FVF | FVCF | PVF | PVCF |
|---|---|---|---|---|---|
| 1 | $100 | *1.2100* | *$121.00* | *.9091* | *$90.91* |
| 2 | $100 | *1.1000* | *$110.00* | *.8264* | *$82.64* |
| 3 | $100 | *1.0000* | *$100.00* | *.7513* | *$75.13* |
| Annuity Factors | | *3.3100* | | *2.4868* | |
| Dollar Values | *$300* | | *$331.00* | | *$248.68* |

What may we observe from this table? Future value annuity factors are always greater than the number of periods. Here the FVF was 3.31, or greater than $n \cdot p = 3$ periods. This is because the annuity multiplier is the sum of each respective yearly factor, each of which is greater than 1.0 (except the last one, which = 1.0) since they are all multiples of $(1 + R)^n$. (This assumes that $R > 0$.)

Contrarily, each PVF is less than the number of periods because the respective factors are all less than 1.0, as each factor is the reciprocal of $(1 + R)^n$. (Again, this assumes that $R > 0$.)

When utilizing a table, it is always a good idea to “eyeball”

the factors you are using to make sure you didn't lift the figure from the wrong table or make some other error. Use your head at all times. Do not be a robot!

The present value of the annuity is $248.68. If you, alternatively, had had a single sum in the amount of $248.68, and had invested it for three years at 10%, you would have $331 at the horizon:

$$(\$248.68)\ (1.10)^3 = \$331$$

and

$$\$331 \div (1.10)^3 = \$248.68$$

Here are some more simple- and annuity- interest rate tables for you to use:

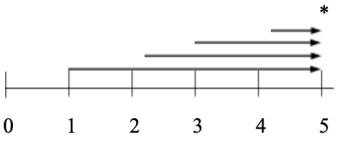
http://www.cengage.com/resource_uploads/downloads/0324406088_41656.pdf

# 11.5 Future and Present Annuity Values: The Nature of Their Cash Flows

Note the differences in the nature of compounding versus discounting of annuity cash flows. Let's assume that the Cash Flows are to be received at the end of the respective periods. In the examples following, there are five cash flows.

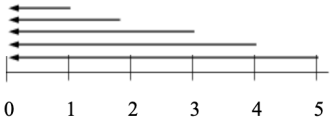
Compounding:

There are, altogether, four compounding "periods"; the last cash flow to be received is not compounded because it is received at the "horizon" of the deal. Indeed, the last CF does have a zero-exponent attached to it: $[1 + R]^0 = 1$. The exponents are zero through four.




Discounting:

In contrast, there are five discounting periods. The exponents are one through five. Note that the arrows go in the opposite direction from before as we are now discounting to present values rather than compounding to future values.




While Simple Future and Present Values factors (as observed by the relevant tables earlier) are reciprocals, or mirror images of one another, annuities are not.

# 11.6 Future and Present Annuity Factors: Mathematical Formulas

Let's "put on the table" the formal mathematical formulas for ordinary annuities' factors. Remember: a "factor" is a multiplier (for the given cash flows). These formulae will be useful when your tables do not have a particular interest rate that you need, and especially when you need to calculate a fractional rate, e.g., 10.23%.

Key: PVAF – Present Value Annuity Factor. FVAF – Future Value Annuity Factor.

**Present Annuity Factors**:

$$\textbf{PVAF} = [(1) \div (R/p)] - [(1) \div (R/p)\,(1 + R/p)^{n \times p}] \quad \textbf{Or}$$

$$\textbf{PVAF} = [(1) \div (R/p)] - \frac{1}{(R/p)\,(1 + R/p)^{n \times p}}$$

Example 1: R = 0.10; N = 5; P = 2

Solution 1: $[(1) \div (0.10/2)] – [(1) \div (0.10/2)\,(1 + 0.10/2)^{5 \times 2}]$ = 7.72173493

*This multiplier should be the same as in your Present Value Annuity Table.

Example 2: R = 0.095; N = 5; P = 2

Solution 2: $[(1) \div (0.095/2)] – [(1) \div (0.095/2)\,(1 + 0.095/2)^{5 \times 2}]$ = Fill in your answer

*This multiplier is not in your Present Value Annuity Table. Compare the two solutions.

**Future Annuity Factors**:

**FVAF** = $[(1 + R/p)^{n \times p} - 1] \div R/p$

Example 3: R = 0.10; N = 5; P = 2

Solution 3: $[(1 + 0.10/2)^{5 \times 2} - 1] \div 0.10/2$ = 12.57789253554883

*This multiplier should be the same as in your Future Value Annuity Table.

Example 4: R = 0.1012; N = 5; P = 2

Solution 4: $[(1 + 0.1012/2)^{5 \times 2} - 1] \div 0.1012/2$ = Fill in your answer

*This multiplier is not in your Future Value Annuity Table. Compare the two solutions.

## 11.7 Characteristics of Annuity Factors: A Review

Let's review the basics of annuities:

- The number of periods counted in discounting versus compounding are different.
- The Future Value Annuity Factor (FVAF) must always be _____ than the number of compounding periods.
- The Present Value Annuity Factor (PVAF) must always be _____ than the number of discounting periods.
- FVAFs and PVAFs, unlike simple TVM factors, are <u>not</u> reciprocals of one another for the following reasons:

1. The counting of the time periods is different for each, i.e., the timelines and "arrows' directions" are different, therefore so are the simple factors' respective exponents!
2. The annuity factors are themselves the result of an additive (aggregating) process for which reciprocals do *not*apply, e.g., $1 \div (1 + 2 + 3) \neq 1/1 + ½ + 1/3$. The reciprocal of a sum is not equal to the sum of reciprocals.

| | Annuity Factors | |
|---|---|---|
| | **FVAF @ 10%** | **PVAF @ 10%** |
| 1 Year | 1.0000 | 0.9091 |
| 5 Years | 6.1051 | 3.7908 |
| 10 Years | 15.937 | 6.1446 |
| 20 Years | 57.275 | 8.5136 |
| 30 Years | 164.49 | 9.4269 |

The answers to the fill-ins above are:

- The FVAF must always be – *greater* – than the number of compounding periods.
- The PVAF must always be – *less* – than the number of discounting periods.

Notes:

1. Over 30 years, an investor who has invested $100 per year at 10% will have put down $100 ×30 = $3,000 in nominal terms. The FV of the $100 annuity at 10% in comparison will be $100 ×164.49 = $16,449
2. Notice how quickly both the present and future value annuity factors increase. That is because we are constantly adding additional cash flows each year. That is also why the PVAFs increase, in contrast to the simple PV factors, which, of course, may only decrease – as per our three commandments – as time increases.

Key:

PVAF = Present Value Annuity Factor

FVAF = Future Value Annuity Factor

# 11.8 Annuities: Practice Problems

For each of the following problems, solve for both the present- and future values of the given annuity – at the given rate and for the stated number of years. Try not to look at the solutions in the table below.

1. $2000 each year for 5 years @ 5% = _____
2. $1,000 each year for 10 years @ 5% = _____
3. $1,000 each year for 10 years @ 10% = _____
4. $500 every six months (semiannually) for 10 years @ 10% = _____

| Problem | FV/ PV | Dollars | R ÷ P = | N × P = | Factor | Solution |
|---|---|---|---|---|---|---|
| 1 | FV | $2,000 | 5% | 5 | 5.5256 | $11,051.20 |
| | PV | | | | 4.3295 | $8,659.00 |
| 2 | FV | $1,000 | 5% | 10 | 12.578 | $12,578.00 |
| | PV | | | | 7.7217 | $7,721.70 |
| 3 | FV | $1,000 | 10% | 10 | 15.937 | $15,937.00 |
| | PV | | | | 6.1446 | $6,144.60 |
| 4 | FV | $500 | 5% | 20 | 33.006 | $16,503.00 |
| | PV | | | | 12.4622 | $6,231.00 |

# 11.9 Annuities Due

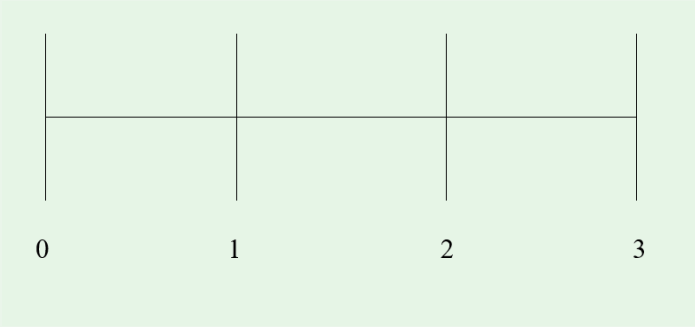
## Annuities Due

An "annuity due" is a type of annuity whose cash flows occur at the *start* of each period. To illustrate, we will use the example – and chart – from above. Since the timing of the cash flows is different than in an ordinary annuity, the factors (and exponents) are also different. As you fill in the factors and the dollar amounts, draw the appropriate time-line.

Question: What would be the PV and FV for the $100 three-year annuity at 10%?

| Period | Cash Flow | FVF | FVCF | PVF | PVCF |
|---|---|---|---|---|---|
| 1 | $100 | | | | |
| 2 | $100 | | | | |
| 3 | $100 | | | | |
| Annuity Factors | | | | | |
| Dollar Values | | | | | |

Timeline




Questions:

1. Notice that both PV and FV annuity due factors (and dollars therefore) are larger than the respective ordinary annuity factors.
2. The PV of an annuity due is equal to the PV of an ordinary annuity multiplied by one plus the *discount* rate $(1 + R/P)^1$ for one period.
3. The FV of an annuity due is equal to the FV of an ordinary annuity multiplied by the *compound* rate for one period. Show this mathematically.

# 11.10 Annuities Due (Solutions)

Question: What would be the PV and FV for a $100 three-year *annuity due* at 10%?

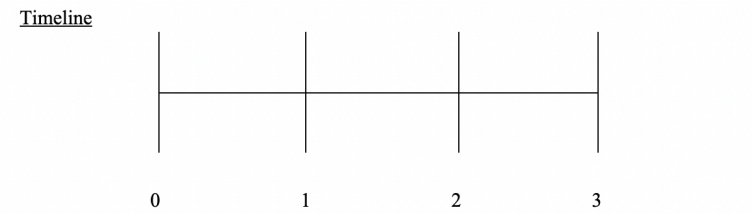
| Period | Cash Flow | FVF | FVCF | PVF | PVCF |
|---|---|---|---|---|---|
| 1 | $100 | *1.3310* | *133.10* | *1.0000* | *100.00* |
| 2 | $100 | *1.2100* | *121.00* | *0.9091* | *90.91* |
| 3 | $100 | *1.1000* | *110.00* | *0.8264* | *82.64* |
| Annuity-Due Factors | | *3.6410* | | *2.7355* | |
| Dollar Values | *$300* | | *$364.10* | *÷ 1.1³ =* | *$273.55* |

Note:

The PV and FV for an *ordinary* annuity with the same term (time), rate of interest, and dollar amounts, were calculated at $248.68 and $331 respectively. Since an *annuity due*provides each of its cash flows one period earlier than the ordinary annuity, the PV and FV of the annuity are both equal to the ordinary annuity multiplied by $(1 + R/p)^1$. In this case, that would be:

$$\$248.68\ (1.10)^1 = \$273.55$$

$$\$331.00\ (1.10)^1 = \$364.10$$




Due to the fact that the cash flows come in sooner there are both more compounding periods and fewer discounting periods. The fewer number of periods leads to more compounding and less discounting, hence greater future- and present-values.

This is handy to know because most interest rate tables provide ordinary annuities factors, but not annuities due.

## 11.11 Adjustment from Ordinary Annuity to Annuity Due

The PV and FV for an ordinary annuity with the same term, rate of interest, and dollar amounts, were calculated earlier at $248.68 and $331 respectively. (See the table at the bottom of this page for a key summary of the differences in exponents.) Since an annuity due provides each of its cash flows one period earlier than the ordinary annuity, the PV and FV of the annuity is equal to the respective ordinary annuity factors multiplied by $(1 + R/p)^1$. (Note that in all instances the exponent is one because the cash flows are pulled ahead just one period.) In this case that would be:

$$\$248.68\ (1.10)^1 = \$273.55$$

$$\$331.00\ (1.10)^1 = \$364.10$$

The fact that the cash flows are received (or paid) sooner in the example of an annuity due has an interesting implication (as noted in the adjustment formula above). In the case of the PV, there will be fewer discounting periods than with an ordinary annuity, so the PV will be higher. In the case of the FV, there will be more compounding periods, hence the FV is also higher. Again, in both instances, the ordinary annuity factor is adjusted by a multiple of $(1 + R/p)^1$. Note that even when $p \neq 1$, the exponent will always be one, representing just the one period (even if part of a year) in which the series is "pulled ahead."

<u>Question</u>: How would the adjustment in our example be

made if the compounding frequency instead were semi-annual?

Answer: In this case, one would multiply by $(1 + R/p)^1 = (1 + 0.10 / 2)^1 = (1.05)^1$.

| Summary of Exponents | | | |
|---|---|---|---|
| Future Value | | Present Value | |
| Ordinary Annuity | Annuity Due | Ordinary Annuity | Annuity Due |
| 2 | 3 | 1 | 0 |
| 1 | 2 | 2 | 1 |
| 0 | 1 | 3 | 2 |

# 11.12 Uneven Cash Flows

Not all cash flows series are as neat as annuities. Using the TVM Tables, calculate both the PV and FV for the series of Uneven Cash Flows presented below. Assume a periodic discount/compound rate of 6%.

The method by which this exercise will be done is the same as that which was done for deriving ordinary annuity factors earlier – except that the cash flows here are uneven (or unequal) rather than all the same. While we have already done a similar exercise earlier, well, you know, practice makes perfect!

| Period | Cash Flow | PVF | PVCF | FVF | FVCF |
|---|---|---|---|---|---|
| 1 | 100 | | | | |
| 2 | 150 | | | | |
| 3 | 175 | | | | |
| 4 | 200 | | | | |
| 5 | 225 | | | | |
| 6 | 250 | | | | |
| 7 | 230 | | | | |
| 8 | 210 | | | | |
| 9 | 190 | | | | |
| 10 | 175 | | | | |
| | | | | | |

# 11.13 Uneven Cash Flows (Solutions)

| Period | Cash Flow | PVF | PVCF | FVF | FVCF |
|---|---|---|---|---|---|
| 1 | 100 | .9434 | 94.34 | 1.6895 | 168.95 |
| 2 | 150 | .8900 | 133.50 | 1.5938 | 239.07 |
| 3 | 175 | .8396 | 146.93 | 1.5036 | 263.13 |
| 4 | 200 | .7921 | 158.42 | 1.4185 | 283.70 |
| 5 | 225 | .7473 | 168.14 | 1.3382 | 301.10 |
| 6 | 250 | .7050 | 176.25 | 1.2625 | 315.63 |
| 7 | 230 | .6651 | 152.97 | 1.1910 | 273.93 |
| 8 | 210 | .6274 | 131.75 | 1.1236 | 235.96 |
| 9 | 190 | .5919 | 112.46 | 1.0600 | 201.40 |
| 10 | 175 | .5584 | 97.72 | 1.0000 | 175.00 |
| | **$1,905** | | **1,372.48** | $\times 1.06^{10} =$ | **2,457.87** |

Solution Guidelines: First, fill-in the first or uppermost cell in the PVF column by dividing 1 by 1.06; then fill in each subsequent lower cell by once again dividing by 1.06 iteratively – or by copying the rates from the tables. Next, multiply each cash flow horizontally by the appropriate PVF. Aggregate.

In order to complete the FVF column, start at the bottommost cell and fill in 1.00; in the next cell up, fill in 1.06. As you go up, continue multiplying by 1.06. Remember, the arrows in the FV timeline are pointed in the other direction.

Take note of the fact that the PV of the series is less than its nominal value and that the FV is greater. You will also

note that, if you know the PV of the uneven series, you can simply multiply it by $(1 + R^n)$ in order to arrive at the FV of the series. Wow!

(If you were unable to solve question #3 on **Some TVM Practice Questions**, you can go back to it now.)

# 11.14 Uneven Cash Flows (Practice Problem)

For the following series, assume a discount/compound factor of 7%.

| Period | Cash Flow | PVF | PVCF | FVF | FVCF |
|---|---|---|---|---|---|
| 1 | 125 | | | | |
| 2 | 175 | | | | |
| 3 | 210 | | | | |
| 4 | 250 | | | | |
| 5 | 300 | | | | |
| 6 | 265 | | | | |
| 7 | 235 | | | | |
| 8 | 200 | | | | |
| 9 | 160 | | | | |
| 10 | 140 | | | | |
| 11 | 120 | | | | |
| 12 | 110 | | | | |
| 13 | 700 | | | | |
| | | | | | |

# 11.15 Uneven Cash Flows (Practice Problem Solutions)

| Period | Cash Flow | PVF | PVCF | FVF | FVCF |
|---|---|---|---|---|---|
| 1 | 125 | .9346 | 116.83 | 2.2522 | 281.53 |
| 2 | 175 | .8734 | 152.85 | 2.1049 | 368.36 |
| 3 | 210 | .8163 | 171.42 | 1.9672 | 413.11 |
| 4 | 250 | .7629 | 190.73 | 1.8385 | 459.63 |
| 5 | 300 | .7130 | 213.90 | 1.7182 | 515.46 |
| 6 | 265 | .6663 | 176.57 | 1.6058 | 425.54 |
| 7 | 235 | .6227 | 146.33 | 1.5007 | 352.66 |
| 8 | 200 | .5820 | 116.40 | 1.4026 | 280.52 |
| 9 | 160 | .5439 | 87.02 | 1.3108 | 209.73 |
| 10 | 140 | .5083 | 71.16 | 1.2250 | 171.50 |
| 11 | 120 | .4751 | 57.01 | 1.1449 | 137.39 |
| 12 | 110 | .4440 | 48.84 | 1.0700 | 117.70 |
| 13 | 700 | .4150 | 290.50 | 1.0000 | 700.00 |
| | 2,990 | | 1,839.56 | $\times 1.07^{13} =$ | 4,433.13 |

## 11.16 Uneven Cash Flows: Another Self-Test Practice Problem

| Period | Cash Flow | PVF | PVCF | FVF | FVCF |
|---|---|---|---|---|---|
| 1 | $75 | | | | |
| 2 | 80 | | | | |
| 3 | 100 | | | | |
| 4 | 400 | | | | |
| "bottom row" | | | | | |

- Fill in the empty cells. Use 8% as your compound and discount rate.
- What if the order of the cash flows were exactly reversed? Answer each of the following three questions below.

1. Would the nominal value of the cash flow (i.e., the bottom cell at left) be changed? Answer "Yes" or "No."
2. Would the total PVCF increase/decrease/not change?
3. Would the total FVCF increase/decrease/not

change?

# 11.17 Solution to Another Uneven Cash Flow Practice Problem

| Period | Cash Flow | PVF | PVCF | FVF | FVCF |
|---|---|---|---|---|---|
| 1 | 75 | 0.9259 | 69.44 | 1.2597 | 94.48 |
| 2 | 80 | 0.8573 | 68.58 | 1.1664 | 93.31 |
| 3 | 100 | 0.7938 | 79.38 | 1.08 | 108.00 |
| 4 | 400 | 0.7350 | 294.00 | 1.0 | 400.00 |
| | 655 | | 511.41 | | $695.79 |

Answers to questions:

1. No change. When dealing with nominal figures, the order of the cash flows is irrelevant.
2. Higher – when larger cash flows come in sooner there is less discounting of the greater numbers – PVs increase!
3. Higher – when larger cash flows come in sooner there is more compounding of the greater numbers – FVs increase!

# 11.18 Perpetuities: No-Growth Perpetuities

A **perpetuity** is a series of equal cash flows that arrive in equal intervals and are never-ending, a kind of forever annuity. We cannot evaluate its FV since it would be **infinite**, as would be its **nominal** value. However, we *can* figure the PV.

Key Terms:

Perpetuity

Infinite

Nominal

The PV of a perpetuity is simply the (fixed) payment divided by the interest rate. For example, if the cash flows are $100 and the discount rate is 10%, the PV would be:

$$PV = \$100 \div .10 = \$1{,}000$$

The general formula would thus be:

$$PV = CF \div i$$

This works because of the mathematical "law of limits." Simply put, as the cash flows grow more distant, the respective PVs of these cash flows approach zero. Adding increasingly distant cash flows will have an infinitesimalimpact on the outcome, the present value. Thus, the

sum can be calculated as mentioned. Let us examine this further.

For example, a $100 annuity for five years at 10% has a PV of $379.08. A ten-year annuity has a PV of $614.46. An annuity of 25 years has a PV of $907.70 and a 50-year annuity would be $991.48. As time goes on, the PV of each additional year's cash flow becomes very, very small (i.e., time increases, PV decreases) so that adding such *infinitesimal* amounts provides no material addition to the aggregate. In the end, you will note that the above present values, when aggregated, will eventually approach, and theoretically equal $1,000.

This formula is applicable to preferred stock whose dividends ad infinitum are fixed.

## 11.19 The "Law of Limits" and Perpetuities

We have already seen that present value of a perpetuity can be described by the formula: $P_0 = CF \div i$. We have also provided both the algebraic derivation of this formula and a discussion as to how the present value of a perpetuity approaches (but never quite reaches) the price described by the formula.

Still, many students find the idea of limits – and of the infinitesimal – difficult to grasp. Our intent here is to further elucidate the notion that present values decrease over time with the result that distant cash flows become so small, so infinitesimal – they have so many decimal places – that they add virtually nothing to the aggregate present value. In making this observation, you will also note that, as a result, the cumulative value of the perpetuity's present value approaches the theoretical price described by the formula, as posited by mathematicians. This is as it should be.

Let us say that a perpetuity's cash flows are \$1 and the interest rate at which we shall discount the series is 10%. The present value therefore would be: $P_0 = \$1 \div .10 = \$10$. This is confirmed in the table below.

| Period | PV of $1 | Cumulative PV |
|---|---|---|
| 1 | .9091 | $0.9091 |
| 2 | .8264 | 1.7355 |
| 3 | .7513 | 2.4868 |
| 4 | .6830 | 3.1698 |
| 5 | .6209 | 3.7907 |
| 6 | .5645 | 4.3552 |
| 7 | .5132 | 4.8684 |
| 8 | .4665 | 5.3349 |
| 9 | .4241 | 5.7590 |
| 10 | .3855 | 6.1445 |
| 11 | .3505 | 6.4950 |
| 12 | .3186 | 6.8136 |
| 13 | .2897 | 7.1033 |
| 14 | .2633 | 7.3666 |
| 15 | .2394 | 7.6060 |
| 16 | .2176 | 7.8236 |
| 17 | .1978 | 8.0214 |
| 18 | .1799 | 8.2013 |
| 19 | .1635 | 8.3548 |
| 20 | .1486 | 8.5134 |
| 21 | .1351 | 8.6485 |
| 22 | .1228 | 8.7713 |
| 23 | .1117 | 8.8830 |
| 24 | .1015 | 8.9845 |
| 25 | .0923 | 9.0768 |

| Period | PV of $1 | Cumulative PV |
|---|---|---|
| 26 | .0839 | $9.1607 |
| 27 | .0763 | 9.2370 |
| 28 | .0693 | 9.3063 |
| 29 | .0640 | 9.3703 |
| 30 | .0573 | 9.4276 |
| 31 | .0521 | 9.4797 |
| 32 | .0474 | 9.5271 |
| 33 | .0431 | 9.5702 |
| 34 | .0391 | 9.6093 |
| 35 | .0356 | 9.6449 |
| 36 | .0324 | 9.6773 |
| 37 | .0294 | 9.7067 |
| 38 | .0267 | 9.7334 |
| 39 | .0243 | 9.7577 |
| 40 | .0221 | 9.7798 |
| 41 | .0201 | 9.7999 |
| 42 | .0183 | 9.8182 |
| 43 | .0166 | 9.8348 |
| 44 | .0151 | 9.8499 |
| 45 | .0137 | 9.8636 |
| 46 | .0125 | 9.8761 |
| 47 | .0113 | 9.8874 |
| 48 | .0103 | 9.8977 |
| 49 | .0094 | 9.9071 |
| 50 | .0085 | 9.9156 |

After 25 years, we reach 90% of the aforecited aggregate present value, and by the 50thyear, we exceed 99% of the $10 value. With the passage of more time, we shall get closer and closer to $10, but only reach $10 in infinite space.

## Perpetuities: Pondering the Infinite

*He counts the number of the stars; to all of them, He assigns names.*

-Psalms 147:4

*Lift high your eyes and see: Who created these? He who sends out their host by count, who calls them each by name: Because of His great might and vast power, not one fails to appear.*

-Isaiah 40:26

*If you always put limits on everything you do, physical or anything else, it will spread into your work and into your life. There are no limits. There are only plateaus, and you must not stay there, you must go beyond them.*

-Bruce Lee

# 11.20 Growth Perpetuities

A "growth" perpetuity is a perpetual cash flow stream (CF) that grows at a *constant* rate of growth, which we shall call "*g*." It is a special case of perpetuity. If, in the last period, we received a cash flow of $100 and its growth rate is 5%, the next cash flow would follow this formula: (Last Cash flow: $CF_0$) (1 + g) = Cash flow in next period ($CF_1$). "G" represents a constant rate of growth in the cash flow over time.

$$CF_1 = CF_0 (1 + g)$$

$$\$105 = \$100 (1.05)$$

The formula for a growth-perpetuity is: (Cash flow $_{\text{next period}}$) ÷ (Discount rate – Growth rate). Symbolically, this may be expressed as:

$$PV = CF_1 \div (r - g)$$

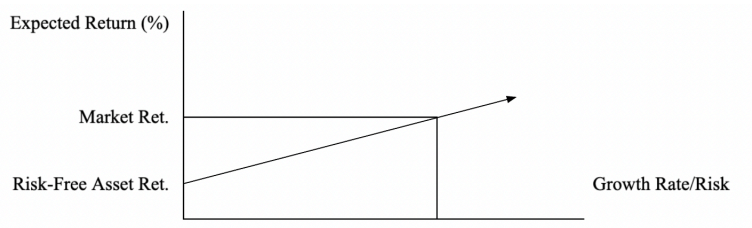
In the above example (where r = 0.10), if the growth rate had been 5%, the present value would be (assuming here that the *next* CF is $105, that is, [$100] [1.05] = $105):

$$\$105 \div (.10 - .05) = \$2{,}100$$

Notice that for this formula to work, "g" cannot equal or exceed "r." Mathematically, if g exceeds r we would get a negative denominator, resulting in a negative present value, which makes no sense. A mathematical rationale, while

necessary, is however not sufficient to justify this relationship. There must be a financial explanation.

One, theoretically, earns some positive return – "r" – for investing in a no-risk asset[1]; otherwise, no one would invest at all. Even with no risk (and no growth!), there is a positive return. Therefore r > g. As growth rates increase (in linear manner), so too does risk; higher growth rates are more difficult to achieve and hence are riskier. R ("r") should exceed g, as a practical, non-mathematical matter. This is depicted in the diagram below.




This formula may also come in handy for cases of **negative growth**. Since "g" would be negative, in this case, the formula would require that one add the growth rate to the interest rate in order to determine the present value.

Key Terms:

Negative Growth

Suppose the last cash flow ($CF_0$) was $100, there is a nominal cash flow growth rate of negative 5%, and a discount rate of 10%. What would the present value be?

1. As this is being written (2021), we observe an unusual environment wherein interest rates in many parts of the world are negative.

$$[(\$100)\ (1 - (0.05)] \div [.10 - (-.05)] =$$

$$\$95 \div .15 = \mathbf{\$633.34}$$

Of course, the PV in the case of positive growth far exceeds the outcome we observed with negative growth. Is that not as it should be?

This formula is applicable to common stock, whose dividends may grow – positively or negatively.

Note:

Looking at this graph alone one may conclude that, at some point, r < g. This is because the line is steeper than at a horizontal plane, thus, there would be less "rise" than "run," leading eventually to growth exceeding the discount rate, and producing a negative present value. While this may be true if one limits the analysis to the graph alone, it cannot be correct, in fact, that dividend growth is not reflected in return – which is also the discount rate, and part of the denominator. As dividends rise, so too must return. (Remember: discounting and compounding are the same; just the arrows go in different directions.) This anomalous and paradoxical case will be discussed in depth when we discuss the "Dividend Discount Model."

## 11.21 Fractional Time Periods

So far, we have dealt only with whole periods and, therefore, whole exponents. True, you may argue that we have also broken whole periods into halves (or other pieces), such as (whole period) half years, so that, for example, an annual rate of 10% compounded / discounted semi-annually became 5% per period; exponents were still whole numbers. But let's not stop here!

Occasionally, cash flows may occur before the end of a period. Suppose you earn 10% per year, compounded daily. What would be the FV after ¾ of a year?

If we assume a 360-day year[13], ¾ ×360 = 270 days. (In short-term financial analysis, we often deal with, or otherwise assume, twelve 30-day months to a year.) Further, 10% ÷360 = .0002777. Therefore, after 270 days, you will have earned:

$$\$1\ (1 + .10/360)^{270} =$$

$$\$1\ (1.0002777)^{270} = 1.077850$$

By comparison, if the compounding period is three months or a quarter of a year, then:

$$1\ (1 + .10/4)^{3} = 1.0769$$

If, in fact,the compounding period is a whole year, then we may employ a fractional exponent:

$$1\ (1.10)^{3/4} = 1.0741$$

All of the foregoing choices are arithmetically correct. The compounding convention you use matters. In practice, you will have to know the proper convention commonly used in each situation.

Here are some more examples. Notice that if we calculate the FV of $1 at 10% annually, after half a year we would have arrived at an FV of $1.05 ($1 $(1 + .10/2)^1$) – as we may have assumed until now. However, if we use fractional exponents, we get a slightly different result:

$$\$1\ (1.10)^{1/2} = \$1.048809$$

$$\$1\ (1 + .10/2)^1\ P \neq \$1\ (1.10)^{1/2}$$

It is therefore very important to understand the context of the problem and the use of the appropriate compounding/ discounting convention.

<u>The Take-away</u>: Compounding assumptions matter. Be sure you use the correct assumptions.

## 11.22 Loans: The Conventional Mortgage

Mortgages are different from ordinary loans. With most loans, interest is paid over the term, or life, of the loan, and the entire principal is paid in one fell swoop at the loan's term, or maturity. In contrast, mortgages are self-amortizing, which means that all payments include portions of both interest and principal, resulting in decreasing principal balances over time until, at maturity, the entire loan will have been paid off. Let us see, by way of (an unrealistic) example how this may work.

| | | | | |
|---|---|---|---|---|
| Given: | Principal: | $100,000 | Rate: | 9 % |
| | Term: | 10 Years | Period: | Yearly |

Mathematical Rationale:

The loan proceeds, i.e., $100,000 in this case, represent the present value. The "periodic payment" represents the annuity payments to be made over future years and will include both Interest and Amortization. Amortization goes toward the reduction of the loan or principal balance. The present value of the annuity payments should equal the loan principal:

Principal = Periodic Payment x Present Value Annuity Factor

Calculation:

Payment = Principal ÷ PV Annuity Factor = $ 100,000 ÷ 6.42 = $ 15,576.32

Interest = Opening Balance × Rate
Amortization = Annuity Payment less Interest

Balance = Opening Balance less Amortization

Payment and Amortization Schedule:

| | Cash Payments | | | |
|---|---|---|---|---|
| Year | Payments | Interest | Amortization | Balance |
| 0 | | | | $100,000 |
| 1 | $15,576 | $9,000 | $6,576 | $93,424 |
| 2 | 15,576 | 8,408 | 7,168 | 86,256 |
| 3 | 15,576 | 7,763 | 7,813 | 78,443 |
| 4 | 15,576 | 7,060 | 8,516 | 69,927 |
| 5 | 15,576 | 6,293 | 9,283 | 60,644 |
| 6 | 15,576 | 5,458 | 10,118 | 50,526 |
| 7 | 15,576 | 4,547 | 11,029 | 39,497 |
| 8 | 15,576 | 3,555 | 12,021 | 27,476 |
| 9 | 15,576 | 2,473 | 13,103 | 14,373 |
| 10 | 15,576 | 1,294 | 14,282 | 0 |
| Totals | $155,763 | $55,763 | $100,000 | |

In the first year, the interest portion of the payment is 9% of $100,000 or $9,000. This leaves $15,576 less $9,000 = $6,576 going toward *amortization*. The new, *amortized balance*is hence $100,000 less $6,576 = $93,424. Each year the annuity installment is first used to pay the interest on the loan and the balance is used to reduce the capital outstanding. This continues for each period until maturity. In the maturity year the last annuity installment is sufficient to cover the interest still owed and the remaining balance. (You will note a rounded number in the last year.)

Note:

Mortgage payments are usually made in MONTHLY installments, and often with greater maturities. Mortgages in the USA today are at least 15 years in term and usually up to 30 years. Rates as of this writing are also substantially lower than illustrated. This has been simplified for illustration purposes, so that the reader may easily refer to standard interest rate tables.

## 11.23 A Few Thoughts about Mortgages

There are a few key points regarding mortgages, which require summary and notice.

1. Interest payments are tax deductible. Tax deductibility is important simply because it reduces the after-tax cost of the mortgage. Funds that would otherwise have gone to pay tax instead go to debt service. This benefit is reduced as the interest portion of the mortgage payment is reduced with time.
2. Partway through the loan, half the loan will have been paid off. A mortgage has a kind of "half-life." In the foregoing example, half the loan will have been paid off after six years. This half-life will be greater than half the length of the mortgage because initially the annuity payments are used mainly to pay interest on the loan!
3. Total interest paid may be much greater than and in longer-term cases, a multiple of the principal – this can be calculated by evaluating the total payments minus the principal.

In the foregoing example, over the ten years the borrower will have paid a total of 10 annual payments of $15,576.32 for a total of $155,763.20. If you subtract from this figure the loan principal of $100,000, you are left with a figure of

$55,763.20, which represents the total amount of interest paid over the life of the mortgage (unadjusted for time value). In other words, interest paid represents an additional 55% approximately of the principal borrowed. (We take note again that most mortgages require monthly payments and that, in today's marketplace, most mortgages are 15 to 30 years.)

Let us compare 15- and 30-year mortgages in terms of the ratio of interest payments made relative to the principal. We remind ourselves that in the above instance (10 years and 9%) we had 55% interest payments relative to the loan principal. Let us use $100,000 of principal again, and, this time, 6% in interest. We shall again employ the formula:

Principal ÷ PV Annuity Factor = Periodic Payment

15 years: ($100,000) ÷ (9.7122) = $10,296.33

Over 15 years total payments will equal (15) ($10,296.33) = $154,444.95. In this case, interest payments will equal 54.44% of the principal.

30 years: ($100,000) ÷ (13.7648) = $7,264.91

Over 30 years, total payments will equal (30) ($7,264.91) = $217,947.30. In this case, interest payments will equal 117.95% of the principal.

Even though the annual payments are less in the 30-year case, we see that interest payments multiply enormously over time.

# 11.24 Summary Comparison of 15- and 30-Year Mortgages

The following table presents a comparison of the $100,000 annual pay mortgage (above) at 6% interest for 15- and 30-years.

| Borrow $100,000 @ 6% | 15 Years | 30 Years |
|---|---|---|
| Annuity Payments (Principal & Interest) | $10,296.33 | $7,264.91 |
| Total Payments over life | $154,444.95 | $217,947.30 |
| Interest Payments over life | $54,444.95 | $117,947.30 |
| Total Interest ÷ Principal | 54.44% | 117.95% |

Notes and Questions:

- The shorter-term mortgage presents a higher periodic payment requirement but entails less overall interest payments over time.
- The longer-term mortgage presents a much higher total interest payment requirement but requires a lower periodic outlay.
- Question: Under what conditions does it pay to take out the shorter-term mortgage?
- Answer: It pays if the mortgagee has sufficient cash flow, and wishes to minimize total pay-

ments, especially interest, over time.

- Question: Under what conditions does it pay to take out the longer-term mortgage?
- Answer: It pays if one does not have sufficient cash flow, is unwilling to settle for a less-costly home, and is relatively unconcerned about the long-term, larger amount to be paid; perhaps he does not intend to stay for the full thirty years.
- In most circumstances, a 15-year mortgage will bear a lower rate than a 30-year mortgage – unlike this illustration. Here we focused on a single variable – time – which greatly impacts the scenario depicted relative to the minimal impact that a small premium interest rate would have for the increased term to maturity.
- Taxation will also have an effect. Recall that interest payments on mortgages are, under current law, tax-deductible.

The mortgage formula is important to master as it will be used again in three additional contexts: 1. Leasing; 2. Bond Accounting; and 3. Capital Budgeting: The Annual Annuity Approach.

# Mortgages in the New Millennium

*Innovation has brought about a multitude of new products, such as sub-prime loans and niche credit programs for immigrants… Unquestionably, innovation and deregulation have vastly expanded credit availability to virtually all income classes. Access to credit has enabled families to purchase homes, deal with emergencies, and obtain goods and services. Home ownership is at a record high, and the number of home mortgage loans to low- and moderate-income and minority families has risen rapidly over the past five years.*

-Dr. Alan Greenspan (2005)

Chairman of the Federal Reserve Bank

http://www.federalreserve.gov/BOARDDOCS/Speeches/2005/20050408/default.htm

*Following an extended boom in construction driven in large part by overly loose mortgage lending standards and unrealistic expectations for future home price increases, the housing market collapsed – sales and prices plunged and mortgage credit was sharply curtailed. Tight mortgage credit conditions*

*are continuing to make it difficult for many families to buy homes, despite record-low mortgage interest rates that have helped make housing very afford-able… the contribution of housing investment to overall economic activity remains considerably below the average seen in past recoveries.*

-Dr. Janet Yellin (2013)

Chairman of the Federal Reserve

http://www.federalreserve.gov/newsevents/speech/yellen20130211a.htm

# 11.25 Personal Financial Planning Problem

You are given the following:

1. This year, Abraham will start graduate school. The annual cost is $30,000 per year for each of two years, payable at the start of the year.
2. The tuition will increase by 3% in the second year, due to inflation.
3. Abraham currently owes $25,000 from his undergraduate student loans.
4. When he finishes his M.B.A. in two years, his parents will give him a $50,000 gift.
5. Upon graduation, Abraham plans to pay off his loans fully in ten years. How much will he have to pay annually in order to achieve his goal?
6. Assume throughout an 8% cost of funds rate, compounded quarterly, except for the annuity payoff payments, which will be at an 8% *annual* rate.

Solution Plan:

- First lay down the given data, in nominal terms, in their proper places in a timeline; then, import the numbers into a spreadsheet.
- Calculate the future value of the costs at the end

of year 2, using the cost of funds rate given. Note the gift as money in.

- Use the mortgage formula to calculate the annuity payment required to pay off the accumulated debts in the last 10 years.

Solution:

| Steps | 0 | 1 | 2 | … | 12 |
|---|---|---|---|---|---|
| 1 | ($30,000) | | ($35,149.78) | | |
| 2 | | ($30,900) | ($33,447.15) | | |
| 3 | ($25,000) | | ($29,291.48) | | |
| 4 | | | $50,000.00 | | |
| 5 | | | (47,888.41) | | 0 |

Calculations:

Step 1: $(\$30,000)\ (1 + .08/4)^{2 \times 4} = \$35,149.78$

Step 2: $\$30,000 \times 1.03 = \$30,900$

$$(\$30,900)\ (1 + .08/4)^{1 \times 4} = \$33,447.15$$

Step 3: $\$25,000 \times (1 + .08/4)^{2 \times 4} = \$29,291.48$

Step 4: $50,000 gift

Step 5: Sum of Steps 1-4

Step 6: Calculate the annual annuity payments.

(47,888.41) = (x) (PVAF 0.08; 10)

(47,888.41) (6.7101)

x = **7,136.77**

# 11.26 Summary: The Time Value of Money

Virtually everything one does in the finance discipline involves, at some level, the time value of money. It is central to all of financial analysis and must be mastered. It is a basic tool.

In the prior two chapters, we developed a rationale for assessing "simple" future- and present-values. We defined the two qualifications for a cash flow series' being categorized as an annuity, and then defined an ordinary annuity as the sum of the series' present- and future-values. We further defined "annuities due" and outlined a means for converting an ordinary annuity into an annuity due. Additionally, we determined how interest rate tables may be used as a shortcut tool for calculating the time value of money.

Then we assessed uneven cash flow series that do not qualify as annuities. We examined two cases of perpetuities (no-growth and growth) as a special case of annuity. We examined fractional time periods and discovered the arbitrary nature of certain time value calculations, which have to do with the varying conventions regarding the definition of "period" and the use of interest rates in this regard. Finally, an illustration of a mortgage was presented and evaluated.

"Before one continues further in this text, it is imperative

that the student master fully the critical concept of time value of money in all its variety.”

# 11.27 Chapters 10 - 11: Review Questions

1. You are given $2.30 in the present. It will compound quarterly at annual rate of 12% for ten years. What is its Future Value?
2. What if you *will*have $2.30 in ten years – in the prior question. What is its Present Value?
3. Define "Annuity."
4. Why are simple Present- and Future-Value factors reciprocals of one another while annuity factors are not?
5. How are Ordinary Annuities and Annuities Due different?
6. How does one adjust an Ordinary Annuity in order to make it an Annuity Due?
7. Give real world examples of Annuities.
8. How are annuities and perpetuities different from one another?
9. An annuity due pays $138.55 every quarter for seven years at a rate of 4.375%. Calculate both its present- and future-values. (Hint: use the mathematical formula for calculating annuity factors and also use the annuity adjustment

multiplier.)

10. What is a “Growth Perpetuity”?
11. Explain the “Law of Limits.” How does it apply to Perpetuities? (Search Law of Limits online if it helps.)
12. Simple Future Factors grow at a(n) *increasing/decreasing*rate. Which is it? Why?
13. The rate of change in Future Value factors is *increasing/decreasing*. Which is it? Why?
14. A mortgage is self-amortizing. Explain.
15. Over time, interest expense on a mortgage is *increasing/decreasing*. Which is it? Why?
16. Over time, a mortgage’s amortization *increases* or*decreases*. Which is it? Why?
17. You are given an 8% annual rate on a bank Certificate of Deposit, which pays quarterly. What is its Annual Percentage Equivalent Yield?
18. A mortgage charges 5% interest payable annually for thirty years. How much interest and amortization will there be in the second year? Assume a loan of $1 million.
19. Over the life of this mortgage, how much interest will there have been – above and beyond the principal payments?
20. An investor will receive a $400, 4% annual annuity for the next ten years, payable semi-annually; that is $200 every six months. What are the present- and future values of the annuity?
21. What if this were an Annuity Due?

22. In the case of a Perpetuity, why is Present Value unaffected by discounting frequencies?
23. A semi-annual, “constant-growth” cash flow series last paid, $5.80. Payments will be made every six months and will grow at an annual rate of10% per year. Assume a four-year horizon. What is the Present Value of the cash flow series?Utilize a 12% discount rate.
24. In the prior question, what if “G” were negative 5% (annually)?
25. A Perpetuity last paid $1.50. It will be discounted at an annual rate of 16% and its cash flow will grow at an annual rate of 8% to be paid in quarterly installments. What is its Present Value? Be sure to adjust for frequencies.
26. What would the future values be in each of the prior two questions?
27. Besides for the mathematical necessity, why must “R” exceed “G,” in the Perpetuity model? We are assuming here, “normal” economic circumstances.
28. The Present Value Annuity Factor must have a value greateror lessthan “n × p.” Which is it and why? What about the Future Value Annuity Factor?

# Part IV Interest Rates, Valuation, and Return

Part IV: Interest Rates, Valuation, and Return

Chapter 12: Fixed Income Valuation
Chapter 13: Interest Rates
Chapter 14: Equity Valuation and Return

# Chapter 12: Fixed Income Valuation

## 12.1 Chapter Twelve: Learning Outcomes

Learning Outcomes

In this chapter you will:

- **Define** three forms of Income.
- **Introduce** the "Valuation Premise."
- **Calculate** both the *Holding Period Return*and the Dollar Price of a Bond.
- **Distinguish** the characteristics of Discount, Par, and Premium bonds.
- **Explore** how Creditability and Maturity affect Market Yields, i.e., the Yields-to-Maturity.

Note: Review questions for Chapters Twelve through Fourteen will appear at the end of Chapter Fourteen.

# 12.2 Security Return: The Holding Pattern Return (Raw Calculation)

In order to figure the return on an asset (a percent), it is useful to first outline the various components of the return on the asset – and then to calculate the return. The two principal components of return are income and profit, i.e., sales or "exit" price less investment cost. Income is common to many assets including bonds (interest), stocks (dividends), and real estate (rent). The exit price may be equal to, greater or less than the asset's original investment cost, hence "profit" is included in return.

| Varieties of Income on Financial Assets | | |
|---|---|---|
| **Bonds** | **Stock** | **Real Estate** |
| Interest | Dividends | Rent |

Example:

- Stock/bond/real estate cost or investment (C) = $1,000. This is an "outflow."
- Income (I) received over the course of the holding period = $100.
- Sales (exit) price (P) = $1,050.
- The income and sales price are "inflows."

Holding Period Return (HPR):

HPR = inflows ÷ outflows -1 = **[(I + P) / C] – 1**

HPR = [(100 + 1,050) / 1,000] – 1 = + 15%

Alternatively, we could have calculated the HPR by focusing only on the profit ("*Π*"), in which case, we would have left out the "-1" expression at the end. Note that the profit = $1.050 – $1,000 = $50. The result is the same this way as depicted below.

HPR = [(I + *Π*) / C]

HPR = [(100 + 50) / 1,000] = + 15%

Note:

In either application, this is a "raw" calculation in that it does not account for the *time value of money*. This approach ignores the length of the holding period and the timing of the cash flows. Clearly one would rather earn the raw 15% return over a shorter period than longer. Would you rather 15% over one year or ten? A dollar earned today is worth more than a dollar earned tomorrow.

While the HPR has severe conceptual limits, it provides key information upon which a better model may be constructed. In this section, we will present the calculation for dollar price and return of a bond.

*When you make a sale to your fellow…do not victimize one another.*

-Leviticus 25:1

*And when you transact a purchase to your friend, or acquire from the hand of your friend, you shall not defraud one another.*

-Leviticus 25:14

## 12.3 Valuation Premise

So, the HPR is not acceptable because it does not include the critical notion of time value of money. In general, we will find that the (dollar-) valuation of any asset will adhere to the following rule:

***The value of an (financial) asset is equal to***

***the future cash flows the asset is expected to produce,***

***each of which cash flow is discounted to its present value***

***and then all such present values are aggregated to***

***a single value.***

This says that in valuing an asset, we must first identify its future cash flows, and then discount each of those cash flows at an appropriate discount rate, and aggregate the figures into a sum, which shall be the asset's valuation or price.

We will also find that, in a certain sense, the dollar value and the discount rate are interchangeable; in fact, we will note that the discount rate, in a large sense, is the "true" price of the asset.

These premises are not universally applicable nor accepted

by all analysts and researchers, but are nonetheless essential to finance.

## 12.4 Fixed Income Securities: Bond Components and Valuation Formula

Since the coupon cash flow of a bond constitutes an annuity, the calculation of a bond's dollar price involves a simple solution:

$$P_{Bond} = \sum_{i=1}^{N} \frac{C_i}{(1+\frac{YTM}{P})^i} + \frac{Face\ Value}{(1+\frac{YTM}{P})^{np}}$$

The above formula says that every (annuity) coupon payment plus the one-time face value payment are discounted and aggregated to present value, which is the bond's price. This is based on our "valuation premise." Here are some important terms to know:

<u>Face Value = Maturity Value = Par Value = Principal</u>: These phrases are synonymous and interchangeable. Maturity value is the amount of money the investor, or bondholder/lender, gets back when the bond matures. This represents, in most instances (and herewith) a one-time cash inflow to the investor.

<u>Coupon Rate (C) = Interest Rate (I)</u>: This is the amount of interest that the bond pays. (An exception would be a variable rate, but we do not deal with that here.) The dollar amount paid is this rate times the face value. Thus, if the

rate is 10% and the Face Value is $1,000, it will pay $100 per year. If this bond pays semi-annually, the investor will receive two $50 payments per year, one every six months. This represents an annuity series of cash flows for the life of the bond; this is the bond's second set of cash flows.

Yield-to-Maturity (YTM) = Market Rate = Discount Rate:This is the market-determined rate at which the bond's cash flows – both the Face Value and the coupon payments – are "priced," or discounted, to present value. Do not confuse coupon and market rates; they are separate and mathematically distinct. The market yield will constantly change. Coupons are (generally) fixed.

We may think of Face Valueand the bond's Dollar Pricein terms of $100s or $1,000s or any multiple thereof. Since the bond's price is expressed in terms of 100% of the bond's "Par" value, it doesn't matter for calculation purposes, how many zeros we add on in an illustration or exercise. This will be demonstrated immediately below.

The bond trader will quote the bond in terms of a percent of par and will then ask the buyer or seller what the "size" of the trade is? In other words, how much they are dealing with. Of course, in reality, buying or selling a thousand, or a million, dollars' worth of bonds matters a great deal. An example follows.

*Creditors have better memories than debtors.*

-Benjamin Franklin

# 12.5 Fixed Income Securities: Dollar Price and Yield-to-Maturity

## Fixed Income Securities: Dollar Price and Yield-to-Maturity Calculation

Since the coupon cash flow is an annuity, a simple valuation solution would utilize the following formula. Take note, that it does not matter how many zeros you use, although, we shall always figure a face value in this text of $1,000 for consistency and convenience, unless otherwise noted:

Example:

| | |
|---|---|
| Coupon | .10 (Semi-annual) |
| Term-to-Maturity | 5 years |
| Market/Discount Rates (Y-T-M) | .08, .10, and .12 |

You'll need calculators and interest rate tables for this!

## Discount Rates

Here you shall need to insert the appropriate present value annuity factor (PVAF) for the coupon and the present value factor (PVF) for the Maturity Value. Do not forget to make the proper adjustments for semi-annual discounting.

| | .08 | .10 | .12 |
|---|---|---|---|
| Coupon (PVAF) | | | |
| Par Value (PVF) | | | |

## Dollar Values

Here you shall need to multiply the above factors by the dollar amounts of the coupon/annuity and face value respectively.

| | | | |
|---|---|---|---|
| Coupon ($50) | | | |
| Par ($1,000) | | | |
| Total/Price | | | |
| Price as Percent of Par | | | |
| Price Characteristics (Discount/Par/Premium) | | | |

The “true” price of the bond may, in fact, be considered the YTM. It is conceivable that there exist two bonds with the same maturity (or even issued by the same company) but due to having been issued at different times, the two bonds carry different coupons. Thus, the YTM will be the same for each, but the dollar prices will be different. Therefore, YTM is the true market price, at which bonds’ values are assessed.

Question: What would the prices be if the above bond’s coupon were 0%?

Answer: The PV of just the Face Value at the appropriate YTM / Discount Rate.

## Fixed Income Securities: Dollar Price and Yield-to-Maturity (Solution)

It is important not only to solve problems, but to interpret them as well.

Discount Rates

| | .08 | .10 | .12 |
|---|---|---|---|
| Coupon (PVAF) | 8.1109 | 7.7217 | 7.3601 |
| Par Value (PVF) | 0.6756 | 0.6139 | 0.5584 |

Dollar Values

| | | | |
|---|---|---|---|
| Coupon ($50) | 405.55 | 386.10 | 368.01 |
| Par ($1,000) | 675.60 | 613.90 | 558.40 |
| Total/Price | **$1,081.15** | **$1,000.00** | **$926.41** |
| Quoted Price | **108.115%** | **100% or "Par"** | **92.641%** |

| | **Premium** | **Par** | **Discount** |
|---|---|---|---|
| | **Cpn > YTM** | **Cpn = YTM** | **Cpn < YTM** |

- Note that this illustration has used ordinary annuity factors, rather than only simple present value factors.
- The "Quoted Price" is stated as a Percent of

Par.

- Premium/(Discount):
    - You get more (less), you pay more (less)!
    - You get more (less) coupon than the current market yields, you pay more (less)!

It is interesting to note, for instance, using the discount bond above, that the future value of the bond's price equals the future value of its cash flows. 926.41 (1.7908) = $50 (13.181) + 1,000.

## 12.6 Bond Dollar Prices: Discount, Par, and Premium

In the case of investment grade bonds (but not in the case of "junk bonds"), corporate issuers usually endeavor to set the coupon so that the initial offering price for the new bond at the time of issuance is Par (100% of the bond's face value). The coupon and other important terms of the underwriting are written into a legal document called the **indenture**.

Once the indenture is completed, prospective bonds investors are provided with a "prospectus," which lays out the terms of the bond, the financial condition of the company, and other important facts. Investors have some time until the bond is issued in which to decide whether they wish to purchase the bond or not. In this short time, market yields are subject to fluctuation. As a result, at the ultimate time of issue, the coupon rate and the contemporary market yield (Yield-To-Maturity or "YTM") will likely have diverged.

Therefore, it is not terribly common to see new bonds being issued at a price of precisely Par; this of course results in new issue prices being at either **discounts** or **premia** to Par. There are these two possibilities. Moreover, over the life of the bond, market yields may diverge still further from the coupons.

For example, when an investor purchases an "old" bond, which may have been issued when market yields – and hence coupons – were lower, it may be looked at as inferior

to newer bonds in that its coupon pays less than bonds issued today with the same maturity and credit rating. This is why – mathematically – such lower paying bonds trade at a discount to par. ***You get less*** (i.e., a lower coupon rate of interest than the current YTM), ***so you pay less***.Of course, this is also true in reverse.

Similarly, for premium bonds, you pay more, because you get more (coupon than the current YTM). The time value of money discount rate (i.e., YTM) is the great equalizer. You may take two bonds that are similar in all respects except the coupon; they will trade at the same market yield, as they should, by definition, but their dollar prices will differ. Therefore, the "true" price of a bond is the YTM!

While many people believe that bond prices are stable, any volatility can increase investment risk relative to bond portfolios. Volatility may stem from default risks and macroeconomic causes and will be reflected as changes in YTM.

***You get more (Coupon than Market Yield) you pay more!***

## 12.7 The True Price of a Bond

Once again, it may be said that the “true price” of a bond is its yield-to-maturity. YTM is the means by which the bond’s cash flows are discounted or “priced.”

Two bonds may have the same credit rating and maturity but may have been issued at different times and thus will have different coupon rates, and therefore will be valued at different dollar prices even though, the market yields today are the same for both.

To illustrate this, solve the following problem for two bonds. You will note that the YTMs and TTMs for each of the two bonds in the example are the same, but the coupons differ, causing the dollar prices to differ. We assume equal creditability for each.

YTM = 4%

N = 10

P = 2

Cpn. $_{\text{Bond \#1}}$ = 4%

Cpn. $_{\text{Bond \#2}}$ = 0%

Solve this without glancing at the solutions.

Price $_{\text{Bond #1}}$ = ($20) (16.3514) + ($1,000) (0.6730) = 100%

You should have known that the price would be Par- without having to calculate.

Price $_{\text{Bond #2}}$= ($00) (16.3514) + ($1,000) (0.6730) = 67.30%

We now also see, that as the coupon rises, so too does the dollar price.

Note:

Review questions for Chapters Twelve through Fourteen will appear at the end of Chapter Fourteen.

# Chapter 13: Interest Rates

# 13.1 Chapter Thirteen: Learning Outcomes

Learning Outcomes

In this chapter you will:

- **Distinguish** between the return on an investment and the firm's Cost of Capital.
- **Define** various qualitative bond risks.
- **Contrast** Price or Interest Rate Risk versus Reinvestment Rate Risk.
- **Compare** Investment Grade to High-Yield Bonds, and their different Credit Ratings.
- **Discuss** Liquidity Preference Theory and its impact on the slope of the Yield Curve.
- **Define** each of four Yield Curve Theories.
- **Calculate** the Spot Curve.
- **Explore** Credit Spreads
- **Consider** the Macroeconomic circumstance under which Credit Spreads will narrow and widen.
- **Utilize** Credit Spreads in a predictive manner.

## 13.2 Interest Rates: Returns to Investors; Cost to the Corporation

Interest rates, and, hence securities' returns, are a function of economic circumstances that are manifest in the financial markets. *Return* to investors represents cost to the corporations that issue the securities; they are the two sides to the same coin. As in the case of "return on investment," the phrases "return" and "cost of capital" to a corporation are described in percentage terms, i.e., as a rate, and not as a dollar amount. Cost of capital refers to the weighted-average cost of the corporation's debt and equity.

Both debt and equity provide the corporation with funds with which to acquire assets, so that the corporation may grow. Investors who provide these funds to the corporation expect a return on their investment; this return represents an "economic cost" to the corporation. The money is not free. "Economic Cost" is a financial term, not an accounting term such as "expense," and should be understood differently.

- The **cost of debt capital** to the corporation is the after–tax cost of interest paid on the debt.

Key Terms:

Cost of Debt Capital

Cost of Equity Capital

- The **cost of equity capital** includes the dividends paid to investors plus their expectations of capital gains resulting from the growth in earnings. Remember, shareholders may expect to receive dividends and to see additions to retained earnings. Should the investors' expectations, which we may also view as their minimum required return, not be met, they may sell the security.

Investors expect that retained earnings and other capital sources be productively employed in the growth of the company so that their shares' value increases – due to increased earnings and growth expectations. Corporations therefore must provide the assurance of price appreciation, or shareholders will sell, and/or hire new managers and directors. This prospective price appreciation is also part of the firm's capital costs, as viewed from the economist's eye.

Required Return

Risk-free

In general, the **required return** (R) whether for stocks or bonds, consists of two parts: one, a return associated with the "risk-free" instrument and the **risk-free rate of return** (RF); the other, a premium, or extra, return for incremental risk above zero-risk. That is, the required return equals the risk-free

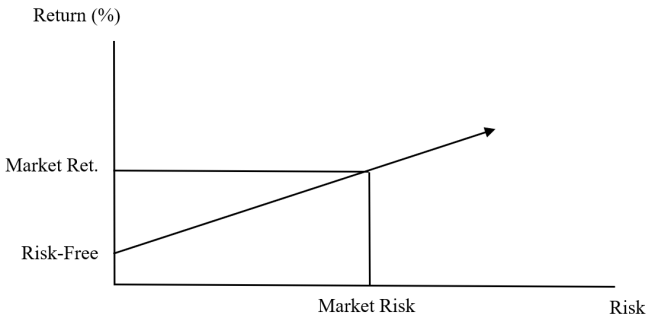
rate of return plus a "market risk premium" (MRP).

Rate of Return

$$R = R_F + MRP$$

$$MRP = R_M - R_F$$

Here, you will find a graph that depicts these concepts.




- The phrase "risk premium" (referring to $R_M - R_F$) is a bit of a misnomer, as the term "risk" draws your attention toward the horizontal axis rather than the vertical axis where the premium (-return for incremental risk) is observed.
- The "Market Risk Premium" (MRP) = Market Return ($R_M$) less Risk-free Rate-of-Return ($R_F$).

- In the graph above, we have risk (a quantifiable measure) and return, which is measured in percentage terms. We also have the overall or average "market risk" and its corresponding market

return.

There is also a theoretical zero-risk investment and its corresponding risk-free rate of return. Even though the zero-risk investment has no risk (excuse the redundancy), it still provides a positive rate of return. If it provided no return, no one would invest in it. There is no agreement about the real-world proxy for the risk-free instrument; it is usually taken as either the three-month Treasury Bill, or the ten-year Treasury Note. More precisely, the risk-free instrument's expected and required returns will be the same, and the standard deviation of its return will be zero.

Market Risk Premium (MRP)

Again, the **Market Risk Premium** (MRP) will be the difference between the risk-free and market rates of return. In other words, the market return provides a premium or additional return to the investor for taking on a level of risk greater than zero. Any particular investment, or portfolio of investments, may provide more, or less, risk and return than the overall market.

In such cases, the investor's portfolio may contain a level of risk greater than or less than the market's risk level, i.e., to the right or left of the Market Risk on the horizontal axis. Accordingly, the investor may expect to earn more than the market return, or less. The investor's expected return will then be either above or below the Market Return

on the vertical axis. Can you draw this? What would be the investor's **Portfolio Risk Premium**?

Take note, once again, of the odd use of the term "risk premium" relative to additional return. The more risk one takes on, the proportionally greater should the return be. This is due to the notion of **Rational Expectations**. One might say that to accept more risk for no added return is irrational, and no one would do that. That is why the slope of the diagonal is positive. The diagonal market line in the diagram reflects this positive risk/reward relationship. The more the risk, the more the (expected) return.

Rational Expectations

Portfolio Risk Premium

- Many researchers will use the 3-month Treasury Bill as the proxy for the theoretical risk-free instrument. This choice has to do with three explanations.
    - First, the United States Treasury is considered the least risky of all debtors (although that may no longer be true).
    - Second, because the 3-month term is so short, market price fluctuation is not at issue; in a very short while, the bills will mature and pay their full, face value.

    - Third, Liquidity is also enhanced by the sheer size of the Treasury market itself; size comes with greater trading volume and hence more liquidity.

- Not all agree about the risk-free proxy. Some argue for the use of the 10-Year Treasury Note as the real–world equivalent of the risk-free instrument. This is because Equities tend to be longer-term investments and, more importantly, because Credit Spreads (to be discussed in a later chapter) are usually figured in a ten-year timeframe.

*The sages of Yavne used to say: I am a creature of G-d and my neighbor is also;*

*my work is in the city and his is in the field;*

*I rise early to work and he rises early to his.*

*As he cannot excel in my work, I cannot excel in his.*

*But you may be tempted to say,*

*'I do great things and he small things!'*

*We have learned that it matters not whether one does much or little,*

*if only he directs his heart to heaven.*

-Babylonian Talmud

Tractate Berachoth, 17a

## 13.3 Inside the Banker's Brain

It is important to know both what you are getting into when dealing with a financial adviser and who may be some of your co-workers if you choose to work on "the street." What insights do you derive from the table below?[1]

| ***Top Five Financial Industry Mental Models*** | |
|---|---|
| *Complexity Bias* | Complex solutions are not just necessary, but also a sign of high intelligence. |
| *Desire for Financial Success* | Money is the best measure of success; it earns respect and I want people to recognize my financial achievement. |
| *Self-Interest* | I prioritize my own interests, ahead of those of my clients and the company, and disregard rules that restrict my freedom. |
| *Recognition of Intelligence* | I am smarter than my customers and the average person, and it is important that others recognize my intelligence. |
| *Short-term Outlook* | I am expected to maximize revenue in every transaction, and I want to reap the fruits of my labor quickly. |

1. From: Inside the Banker's Brain: Mental Models in the Financial Services Industry and Implications for Consumers, Practitioners, and Regulators, by Susan M. Ochs (November 2015). Initiative on Financial Security, The Aspen Institute, Washington, D.C. ifsinfo@aspeninstitute.org.

## 13.4 Fixed Income Risks

Risk in theoretical finance is defined mathematically usually as the chance or extent to which the actual return may differ from the required or expected return.

"Differ" allows for the actual or "realized" return to be either less or greater than the expected return.

From the investor's perspective, bond risks may be divided into the following qualitative components:

**Liquidity Risk** – the risk that the security cannot be converted to cash at its reasonable intrinsic or "fair market value" due, possibly, to a lack of buyers.

**Credit- or Default Risk** – the possibility that a corporation may, in the worst case, go bankrupt, or, in a lesser case, not honor its interest and other related payments timely and in full.

- If the firm is in violation of any of the technical terms ("covenants") of its loan agreement ("indenture"), it may also be considered in "technical-" default even if it is timely in making all its payments.
- Note that interest must be paid before any dividend payments on preferred and common stocks may

be made.

- If a borrowing company defaults on its debt, it may thus become insolvent which would lead to bankruptcy.

**Inflation Risk**– the risk that the return, i.e., the interest payments, will be eroded over time by inflation, i.e., a reduction in the purchasing power of the interest payments.

- Note that, in most cases, interest payments are fixed.
- Many governments sell inflation-linked, or inflation-indexed bonds that have payouts linked to the inflation rate. The United States Treasury sells Treasury Inflation-Protected Securities (TIPS).

**Interest Rate (or "Price") Risk**– the negative effect on market values, or prices, due to rising levels of general interest rates.

- Interest rates and prices are inversely related. If rates go up, ceteris paribus, bond market prices go down.
- Think of it this way: If you already own a bond and interest rates, in general, go up since the time of purchase, then new bonds will be issued at higher rates than your bond. All else equal, that will make your bond relatively less attractive

and thus its market value will go down.

- Possibly more than any other risk, this is tied into macroeconomic factors, including Federal Reserve policies.

**Reinvestment Rate Risk**– the risk that cash interest payments received during the life of a bond will be reinvested at less than the rate originally expected, thereby reducing the overall holding period return.

- When the investor purchases a bond, s/he has some expectation to either spend the money or to reinvest the interest payments at some anticipated future rate. Should rates go down, the reinvestment rate will be less than expected (or less than it was at the time of purchase) and the investor's savings at the bond's maturity (i.e., the bond's "future value") will have accumulated to less than that which was originally anticipated.
- Note that interest rate and reinvestment risks are inversely related to (i.e., the opposite of) one another.

**Countryor Sovereign Risk**– for example, war, changes in administration, etc.

- When there is war, people nervously sell stock, as occurred in

December 1990 when the Gulf War broke out. When Donald Trump was elected U.S. President in November 2016, the stock market rose sharply.

**Foreign Currency Risk**– possible negative effect of repatriation of funds

- In general, most U.S. companies have substantial assets domiciled or revenues generated overseas. These funds may be repatriated or brought home at an unfavorable exchange rate. Thus, even American companies bear some foreign currency risks even though their stocks are dollar denominated.

Notes:

- "Nominal" rates (or prices) are not adjusted (lower) for inflation.
- "Real" means "corrected" for inflation, i.e., adjusted. The real rate will be lower than the nominal rate, given some inflation.
- The following phrases all usually mean *percent*:

return, rate, yield, margin, bracket.

## 13.5 Interest Rate and Reinvestment Rate Risks

An additional, clarifying word about interest rate and reinvestment rate risks is warranted at this time.

If in general, interest rates go up, market prices for existing, "older" bonds will go down – and vice versa. General interest rates and bond prices are inversely related. In an environment where market rates have gone up, bond investors holding "old" bonds will receive less interest income than if they had purchased "new" bonds that were issued more recently – at higher (coupon) rates. This will cause market values for existing bonds to go down; they are simply worth less competitively. Rising rates are "bad" news for (existing) bondholders in terms of interest rate risk.

However, bondholders will be able to reinvest their interest proceeds at higher rates than before if rates rise. Investors receive regular, periodic interest payments over the life of the bond, which upon receipt, is assumed to be reinvested into other alternatives that pay higher rates than before. Thus, rising rates are "good" news for bondholders – in terms of reinvestment risk.

In short, interest rate and reinvestment risks are inversely related to one another. The table below summarizes the relationships vis á vis the bond investor.

| Risk | Rising rates | Declining rates |
|---|---|---|
| Interest Rate / Price Risk | Bad | Good |
| Reinvestment | Good | Bad |

## 13.6 Credit Ratings

Corporations are rated by credit-rating agencies that assess a corporation's ability to service its debt (i.e., to pay interest and principal in full and on time) and, thus, to avoid a default and to ultimately keep bankruptcy at bay. This is a risk, which may also be referred to as "default risk."

We have already examined three solvency ratios which attempt to provide some insight into this risk. Clearly, there are many more tools and considerations that enter into the process of credit analysis and rating. Companies must pay for this service, but not all do so, as some bond issues are too small to justify the expense, while others may be sold directly to institutional investors and thus do not require a rating.

Today, there are two major rating agencies: Standard & Poor's (S&P) and Moody's. Fitch is still a third agency, but it does not enjoy the market presence of the others. In addition, there are numerous nationally recognized statistical rating organizations or "NRSRO's" that perform similar functions (see: https://www.sec.gov/ocr/ocr-current-nrsros.html). The agencies rate bonds on the scale illustrated below. As you will note, there are some differences in the rating scales, and, although not visible in the ratings themselves, the manners in which the agencies conduct their respective analyses and what they consider important also differ from one agency to the other. As a result, the agencies may not rate the same bond issuer the same.

| | Moody's | Standard & Poor's |
|---|---|---|
| **Investment Grade** | | |
| Highest Quality | Aaa | AAA |
| High Quality | Aa | AA |
| Upper Medium | A | A |
| Medium | Baa | BBB |
| **High Yield ("Junk")** | | |
| Speculative | Ba | BB |
| Highly Speculative | B | B |
| | Caa | CCC, CC |
| Default | Ca | D |
| | C | |

Within each rating category, the agencies may append additional notation, such as "A-" in the case of S&P. This provides some further refinement to the ratings.

Note:

The "S&P 500" is a stock index and does not apply to bonds.

Ratings will clearly affect the bond's yield – in inverse relation to the rating, with lower ratings generally bringing higher yields. That is to say that **the lower the credit rating, the higher the default risk. A greater default risk means a lower dollar price, which, in turn means a higher yield.**

The better the bond's initial rating, the lower the bond's coupon rate of interest, and, of course, the lower the issuer's cost of debt capital; this is also

true for the bond's subsequent secondary market yield, (a.k.a. yield-to-maturity; YTM). Do not confuse coupon and market yields; they are separate. The coupon determines how much the interest bond pays, whereas the YTM is the ever-changing market discount rate which is used to price the bond.

The rating agencies do not collude or necessarily agree with one another; in some cases, the agencies may rate the same company somewhat differently. Interestingly, the bond market itself seems to understand what the true yield for a bond should be; indeed, yields often adjust long before a rating change (i.e., either an upgrade or a downgrade) announcement. Further, a bond's rating is not necessarily "correct."

In the case of municipal bonds, the agencies conduct similar analyses and provide similar ratings. The interpretation of municipal ratings, however, is somewhat different in practice, with municipal bonds less likely to default than their ratings might imply. In some instances, municipal bonds may be insured by private insurance companies, in most of which cases the bonds will therefore bear a "AAA" rating regardless of its intrinsic creditability. Most insurers are considered to be in the best financial health.

United States Treasury and Agency bonds are highly rated but are no longer perceived as being completely risk free. In 2011, Standard & Poor's

lowered its rating for Treasury Securities from AAA to AA+. The agency felt that the congressional deadlock in arriving at a budget agreement warranted the downgrade. Since then, the agency has not upgraded the rating back to AAA. The other rating agencies have kept the Treasury's rating at AAA.

"Agencies" are considered slightly lower in quality than "Treasuries" and will therefore trade at very slightly higher yields. The markets "know" how to price their securities based on YTM and its associated default risk.

While the rating agencies do not provide the details of the manner in which they conduct their ratings, it is accepted that their formulae consist of a combination of the examination of the issuer's financial health (based on its financial statements), its economic environment, and an examination of the bond's covenants.

Key Terms

Covenants

Indenture

**Covenants** are agreed-upon terms to which the issuer assures it will adhere; in a way it is a restriction imposed by the lender in order to secure or strengthen his position. For instance, the issuer may assure the investors that its financial ratios will remain within certain parameters, or that it will not issue any further debt, which shall be senior to the issue at hand. In general, such *negative covenants*will also prohibit the firm from paying too much in dividends, selling or pledging assets to other lenders, maintaining collateral in good condition, or adding on more debt. Timely interest

and principal payments would certainly be included in this category. These assurances are spelled out in the bond **indenture**, a legal document in which all the terms of the bond issue are spelled out, including the bond's maturity and coupon interest rate.

In Default

Technical Default

Although a bond issuer, or borrower, may pay its bond interest in full and on time, which is to say it is not **indefault**, any violation of any of the bond indenture's covenants may be interpreted as a **technical default** and result in a downgrading of the bond's credit rating. The bond covenants are aimed at providing investors with reasonable assurances of the issuer's fealty to its obligations under the bond, but not so much as to disenable it from running its business effectively. Failure to abide by the covenants is, thus, a negative circumstance.

The restrictiveness of the bond's covenants will vary. Secured, or asset-backed, securities will have less restrictive covenants than unsecured issues. Furthermore, lower rated, non-investment grade, bonds are likely to be more restrictive and more complex, and hence will require a more careful reading prior to purchase and investment. Evaluation of these riskier securities' covenants is relatively more important than in the case of investment grade bonds. Moody's places some emphasis in the determination of its ratings for high-yield bonds on the issue's covenants. High yield bonds tend to attract more sophisticated investors who are capable of studying the technical terms of the covenants; moreover, many such issues are placed privately among institutional investors.

Investment grade bonds will typically contain three standard covenants. First, there may be a covenant regarding "liens," which assure the investor that no other party may obtain a "senior" (prior) claim to the asset (or collateral) subsequent to the bond's issuance. Under the law, a **lien** is a claim by a person upon another person's property for purposes of securing payment of a debt or the fulfillment of an obligation. If there is a prior lien on a property, no other person can *subsequently*make a claim on the same property. By manner of another example, when you buy a house, your attorney will conduct a "lien search," to be sure that no one else owns the house except the seller with whom you are dealing.

Next, a covenant will protect against any reduction in seniority of the bond in case of a merger of the issuer with another corporation. This covenant will ensure that the bond is paid off first in any case.

Finally, another covenant may address the sale of an underlying asset – in the case of a secured bond. Obviously, should an asset underlying a collateralized bond be sold, the bond's financial status will change markedly. Such assets ought not to be sold unless the bond is also redeemed.

There is no doubt that credit ratings are critical to pricing primary and secondary market issues. Ratings influence both the coupon rates of interest in the primary market, and market yields, or yields-to-maturity in the secondary markets.

## 13.7 The Yield Curve

Key Terms:

Yield Curve

Yield-to-Maturity

Term-to-Maturity

The **Yield Curve** depicts the relationship between **Yield-to-Maturity** (YTM) and **Term** (or Time)-**to-Maturity** (TTM) for a given class of bond, such as Treasury Notes and Bonds, Municipal Bonds, or Corporate Bonds. When one speaks of the Yield Curve without specifying the instrument, it is assumed to refer to the market for Treasury securities. The yield usually is stated in nominal terms, i.e., not in real terms, which would adjust for inflation. The horizontal axis is the term, or **Time-to-Maturity** (TTM), and the vertical is **Yield-to-Maturity** (YTM) or market yield. The slope of the yield curve can be flat, positive (upward sloping) or negative. It may, at times, be kinked. As will be shown later, YTM will determine the dollar-price for a given bond.

Under "normal" circumstances, the yield curve will reflect an upward, but not necessarily linear, slope; that is, as maturities get longer, yields will also rise. The theory of **Liquidity Preference** attempts to explain this. Investors prefer being liquid rather than illiquid. When liquid, investors have the entire range of options available to them; they may either spend

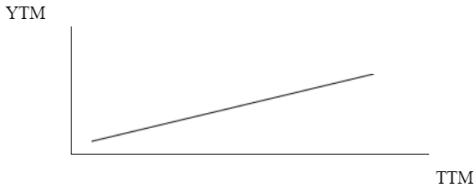
or invest their money. When illiquid, choices and alternatives are reduced or eliminated. The longer one ties up his money, the greater his illiquidity. As one's illiquidity increases, s/he will accordingly demand more and more return as compensation for the illiquidity, hence a positively sloped yield curve.

Liquidity Preference

The Yield Curve need not always be positively sloped, however. For instance, in times of tight Federal Reserve monetary activities, short-term rates will be driven up. As markets have learned to interpret tight money policy as effective inflation-reducing medicine, which takes effect over future periods, the longer end of the yield curve tends to decline, leading to a negatively sloped, or **"inverted," yield curve**. As Fed tightening usually does not last long (the 2004-2007 period being an obvious exception to this), inversions are similarly short-lived.

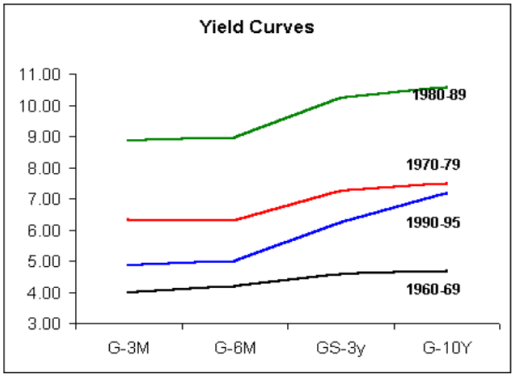
Inverted Yield Curve

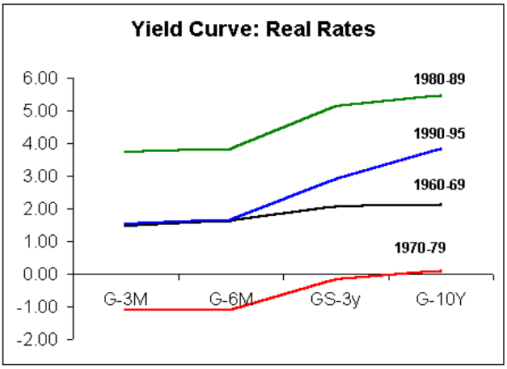
The Yield Curve *normally*is positively sloped due to *Liquidity Preference* – although it does not have to be a straight line.




Below, we observe average yields for three-month to ten-

year government (Treasury) bonds for the decades noted. Yields are first stated in nominal (unadjusted) terms, followed by real (adjusted for inflation) terms. In general, the yield curve is stated in nominal terms, because that is how bond yields are quoted in the market.




Note:

The yield curve, typically, is presented in nominal terms.

The following link provides some insight into the "historical yield curve," which is to say, how the yield curve has changed over the last few decades.

http://fixedincome.fidelity.com/fi/FIHistoricalYield

## 13.8 The Term Structure of Interest Rates: Four Yield Curve Theories

Here we shall present four theories that attempt to explain why the Yield Curve may take on one or another slope – upward (positive), flat, or downward (negative). We cannot say that any one theory is more correct than the other, nor can we necessarily reconcile one theory in terms of another. Still, the following theories are eminently informative.

**1. Pure Expectations** – The Market Yield reflects the average of future short-term rates.

First, we assume that investors think about the future and, specifically about the future direction of short-term interest rates. In this notion, we say that the observed Yield Curve is, in a sense, secondary to what market players believe – in their minds – future short-term yields will be. If, as in the mathematical example below, market participants believe that future short-term yields will go higher and higher, then the observed yield curve will reflect this collective belief and be positive in slope – and vice versa. In other words, the Yield Curve reflects market participants' *a priori* beliefs.

What makes this interesting is that while we can directly observe the Yield Curve – after all, it is quoted in the media, and by traders and brokers – it is first the unexpressed thoughts and beliefs about the direction of future short-term rates that determine the Yield Curve's openly

expressed slope. We know what is in the public's minds regarding the future by observing the effect of their collective thought on the slope and shape of the "Yield Curve." The Yield Curve expresses what people think about the future!

Similarly, the observed Yield Curve – mathematically – will express the average of market participants' expectations about the course of future short-term rates. For example, standing here today, in the here and now, and assuming first that the investor's horizon is two years, the investor is faced with two choices: (1) Buy a two-year bond, or (2) Buy a one-year bond and "roll it over" for another year when it comes due at the end of the first year.

Given this line of thinking, if his horizon is three years, an investor can buy a three-year bond, or choose to consecutively roll it over twice. If we assume initially that the two choices in each case should be and are equivalent, we can *extrapolate* the investor's beliefs about the "Spot Curve," i.e., the market's collective belief about future short-term rates, from the Yield Curve, working backwards. Let's clarify by mathematical illustration.

Here is an example of how we may calculate "Spot Rates," or the "Spot Curve" given the Yield Curve ("Market Y-T-M").

| Given: | Maturity (Years) | Market Y-T-M | Implied Spot Rate |
|---|---|---|---|
| | 1 | 7.0% | 7.0% |
| | 2 | 8.0% | **x** |
| | 3 | 8.5% | **y** |

Question:

"x" and "y" represent the future short-term rates that market players anticipate. Specifically, "x" represents the rate for the second period. Likewise, "y" represents the rate for the third period. Working backwards from the observed Yield Curve, what are the values for the two unknowns? Again, we must assume that the two alternatives are equivalent. This mathematical process, by the way, is referred to as "Boot Strapping."

Solution:

$(1.08)^2 = (1 + .07)(1 + x)$

**x** = .09009 where, x = $_2\mathbf{r_1}$ (i.e., the one-year rate in the second year)

The two alternatives – that of buying (or lending) a two-year instrument or buying (or lending) a one-year instrument and rolling it over at the end of the first year – must be viewed as equivalent alternatives if this idea were to work. And it does because should one alternative be superior, rational, smart market players would go for that

one, and the market's efficient self-correcting mechanism would drive the alternatives together.

This is known as the **Law of One Price**. This "law" says that if two equal alternatives are present, they must offer the same price or, in this case, yield. If one of the choices were more attractive, investors would choose that one, driving up the price and lowering the yield. They would sell the other, which would have, in the end, an equal and opposite effect. While we have used here the term "investors," this argument refers to the activities of both borrowers and lenders, in fact.

You should be able to draw these two curves (i.e., both the YTM Yield Curve and Spot Curve, given the calculated values of "x" and "y") on a chart, with the yield on the vertical and the years-to-maturity on the horizontal. Note that after the first year, the two curves diverge, with the Spot Curve, in this example, rising above the Yield Curve, pulling it upward. Investors expect future, short-term rates to increase! Of course, if the Yield Curve is inverted (negatively sloped), the spot curve would be lower that it and we would conclude that future, short-term rate expectations are decreasing.

As for the nomenclature noted above, and more formally speaking, "$_2r_1$" means the "spot rate" starting at the beginning of the second period for the length of one period, while "$_3r_1$"starts at the beginning of the third period for one period's length.

Again, and in summary, why does this math work? This is based on two notions: first, that market players are "rational," and make optimal decisions that maximize their wealth; second, that markets react "efficiently" to a per-

ceived mispricing should the two opportunities not be equivalent. Together, these two concepts (i.e., Rational Expectations and Efficient Markets) act in unison and cause the alternatives to be equivalent at present. What will actually happen tomorrow is another story.

Under this Pure Expectations Theory, we say that the Yield Curve has no *a priori* upward (positive) or downward (negative; inverted) bias. The slope of the Yield Curve simply reflects whether people think rates will be going up or down and will acquire its slope accordingly. The observed Yield Curve's slope thus is a consequence of Pure Expectations.

**2. Liquidity Preference** – Investors prefer Liquidity to illiquidity.

Investors prefer to be liquid at all times in order to have the freedom to choose whether to spend or invest their funds. Don't we all want to be free? Should investors choose to tie up their money in an investment, they would demand to be compensated for the illiquidity that comes with investment. Should they tie up their money longer and longer, they would demand that they be increasingly compensated, in terms of higher yield, for their increasing illiquidity. Under this theory, therefore, we conclude that the Yield Curve would have a notable upward bias.

The theories of Pure Expectations and Liquidity Preference are said to work hand-in-hand. When we get both theories working in the same direction, that is, when the Spot Curve is positive, we get a positively sloped Yield Curve. However, if Pure Expectations are such that market participants believe that future short-term rates will decline, the slope will be determined by which force is greater than the other

– whether Pure Expectation outweighs Liquidity Preference or not. The following formula expresses this theoretical notion:

$$(1.07)\ (1 + x + L) = (1.08)^2$$

While we can measure "x," i.e., the spot rates, and have just done so, the Liquidity Preferences (L) of the market are not measurable.

There are two more theories to go.

**3. Market Segmentation** – different segments of the yield curve attract different issuers and investors and are thus subject to varying supply/demand conditions respectively. These conditions will determine the independent slopes of each segment.

In Market Segmentation Theory, we say that participants generally invest or borrow in limited portions or "segments" of the market. Commercial Banks invest very little in the long-term whereas Pension Funds are heavily invested there. Different players tend to "reside" in different "segments."

Given this segmentation, rates within it would be a function of the supply and demand characteristics of each individual segment, separately and alone. Any changes in a particular maturity's yield would not affect any other segment, or rate, for any other maturity. There would, thus, be no ex-ante bias whatsoever for the slope of the Yield Curve. In fact, the Yield Curve could conceivably have multiple kinks.

**4. Preferred Habitats** – Market Segmentation may be

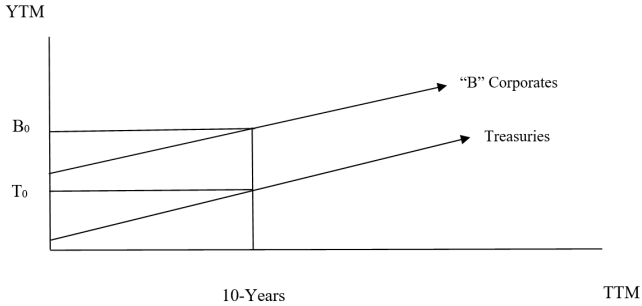
altered by yield incentives whereby investors and borrowers may be lured away from their Preferred Habitats.

Preferred Habitats Theory comes to the rescue! Maybe. This theory says that yes, players indeed have their preferred segments, or habitats, but they (i.e., both lenders and borrowers) can be lured away from their preferred habitats if interest rates are attractive enough – low enough for borrowers and high enough for investors.

YOU decide!

# 13.9 Credit Spreads

Credit Spreads represent the incremental yield provided to compensate for additional default risk above a no-risk alternative (Treasury Notes), and for a given maturity – usually 10-years. We have drawn in the yield curves for Treasuries and "B"-rated Corporates. The Credit Spread below is represented by the difference in yields between points "B" and "T." Remember that **the lower the credit rating, the higher the default risk. A greater default risk means a lower dollar price, which, in turn, means a higher yield.**




Remember that that **the lower the credit rating, the higher the default risk. A greater default risk means a lower dollar price, which, in turn means a higher yield.**

Questions:

Which companies issue credit ratings?
What are the important credit ratings?

What do “Investment Grade” and “Junk” or “High-Yield” Bonds mean?

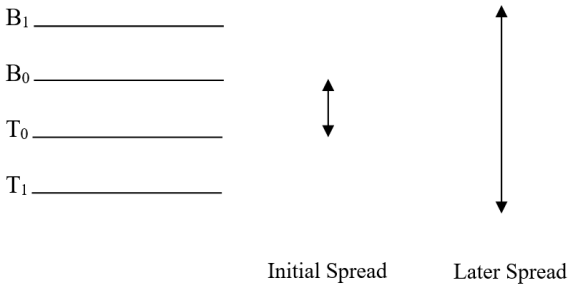
Reality Check: Credit spreads are not constant. During weak economic periods, credit spreads widen, in response to fear – but not necessarily the reality – of the *perceived* risks associated with lower-rated bonds. In fact, default rates themselves may not worsen, but still, people get nervous. In such times, people demand more premium return versus Treasuries than before.

How exactly does this happen? Many will sell their lower-rated bonds and buy safer Treasuries with the sale proceeds. Lower-rated bonds’ prices therefore fall and their yields rise; Treasury prices will thus rise, and yields will go down. Credit spreads widen. Remember that price and yield (i.e., discount rates) are inversely related. This does not say that actual credits, i.e., default risks, have worsened; that may or may not eventually happen. A bond may be considered a “credit.” This phenomenon – whereby credit spreads widen – is referred to as a *flight to quality*.

Below we illustrate the flight to quality.

The Flight to Quality

Yields




Above, in anticipation of weak times ahead, Credit Spreads widen from $B_0$ less $T_0$ to $B_1$ less $T_1$. That does not imply that default rates have worsened in reality, although, in time, as conditions worsen, that could happen as well.

When rates increased between 1978 and 1982, the spread between (Ten-Year) Treasuries and Baa Corporate Bonds widened. Baa and BBB bondholders lost money in part due to increased rates and, in addition, due to a widening of the spread (such bondholders would have lost less in Treasuries).

Credit spreads were wide when the technology "bubble" burst in 2000-2001. Greater credit spreads reduce corporations' ability to borrow and thus serve to dampen economic growth.

Just prior to the Banking Crisis of 2007-2008, Credit Spreads were very narrow by historical standards. To some prognosticators, the optimism implicit in such narrow

spreads boded ill for near-term economic conditions. They were right.

Many things can happen that may interact with credit spreads, including moves up or down in the entire yield curve. What is important to recognize, at the base level, is that lower grade bonds – and their yields – are (usually) priced relative to Treasuries, i.e., at some premium to Treasury yields. The lower the credit rating, the greater the impact of "nervousness" on yield.

For a graph of historical credit spreads, click on: Bond Yield Credit Spreads – 150 Year Chart | Longtermtrends

## 13.10 High Yield Securities: Junk Bonds and Other Speculative Securities

The phrase "junk bonds" is a bit unfair because no rational person would throw out something of value, as s/he might do with something that is truly "junk." Even bonds in default may have some monetary value greater than zero. High yield securities merely have lower credit ratings and therefore higher yields.

In 1978, there was less than $10 billion worth of junk bonds outstanding – in terms of aggregate face value. In 2006, this figure exceeded $1 trillion! Whereas in the past, most issues consisted of "fallen angels," or securities issued by corporations that once enjoyed investment grade credit ratings, but fell on hard times, today most junk bonds are originally issued as high yield securities. Further, emerging markets the world over add to new issuance; the prospective high returns attract many investors.

This low-grade security growth has precipitated two other interesting phenomena: the markets for "syndicated loans" and "leveraged loans." Syndicated loans are package deals provided often by investment banks rather than commercial banks, which traditionally have made such large loans. These loans are aggregated and packaged, and then "securitized," that is, sold as securities. Many of these bonds went bad in the 2007-2008 Banking Crisis. A leveraged loan is one which is made to companies that already may

have low-grade bonds outstanding; such loans are then often syndicated.

**Distressed debt** consists of junk bonds whose companies are in such dire straits that their yields are substantially higher than the risk-free rate. “Defaulted debt” represents corporations who have defaulted on their bond issues. A “default” may consist merely of the violation of its bank lending agreement or its bond’s **indenture**, i.e., the legal document, which sets forth all the terms of the loan, and which may require that the issuer maintain its financial ratios within certain parameters. It may also result, more seriously, from a late or missed interest payment.

Key Terms:

Distressed Debt

Vulture Funds

**Vulture funds** are often operated by hedge funds and attempt to profit off the remains of bankrupt corporations by purchasing the defaulted bonds and repossessing the company’s assets. The vultures then sell off the assets at a profit.

Typically, default rates increase as the economy enters a recession. Default rates are sometimes defined as the dollar amount of defaults relative to the aggregate amounts outstanding. The credit rating agencies define default rates in connection with the number of high-yield issuers who default.

## 13.11 Summary: Interest Rates, the Corporation, and Financial Markets

Corporations are intimately connected with the financial markets because the value of the corporation depends on ever-changing prices reflected in the markets, and because corporations periodically require access to the financial markets in order to acquire capital for growth. In this chapter, we outlined four financial markets: money, capital, derivatives, and foreign exchange markets. The focus was, of course, on securities markets, that is, the markets for stocks and bonds. Securities markets, in turn, are connected to the macroeconomic world, and monetary and fiscal policies are a key component of security markets analysis.

The means by which corporations gain access to the financial markets and its capital raising function is the subject of investment banking. In particular, we are interested in new securities' issuance, or initial public offerings (IPOs), but I-banking serves various other essential roles in our financial sector.

Interest rates affect securities' valuations, most visibly affecting bonds' prices, but the impact on equities is notable also. This is but one example of the connections between macroeconomics and securities markets. In this section, we also glanced at the interconnections between interest rates of various sorts.

There is much to know.

Note:

Review questions for Chapters Twelve through Fourteen will appear at the end of Chapter Fourteen.

# Chapter 14: Equity Valuation and Return Measurement

# 14.1 Chapter Fourteen: Learning Outcomes

Learning Objectives

In this chapter, you will:

- **Calculate** the prices of both Preferred and Common Stocks, and Capital Gains using the Dividend Discount Models.
- **Explore** the various approaches to stock valuation.
- **Uncover** the various sub-elements within the Dividend Discount Model.
- **Provide** both Annual Portfolio Returns and Multi-year Geometric Returns.

## 14.2 The Philosophy of Equity Valuation

Earlier, we read that "the value of an asset is equal to the future cash flows the asset is expected to produce, discounted and aggregated to its present value." We refer to this value or price as "intrinsic value." This *Valuation Premise* shall serve as the starting point for the valuation (i.e., pricing) of equity.

In this section, we will commence with a simple example of stock valuation, one which perfectly corresponds with this statement. We shall then observe that the formulation does not conform to, or is not applicable to, many, or most, equity situations. For example, our starting point will be the application of this TVM premise to Preferred Stocks that have fixed dividends. However, what about other stocks, whose dividends are not constant? For this there will be another model, a variation on the original model.

What about investors who buy stocks for capital growth rather than income? In other words, what about investors who do not care about dividends, but instead purchase stocks for profit? Most American stocks pay no dividends at all! What are such stocks worth? For that too, we shall offer a further refinement of the original model.

To summarize, we shall systematically address shortcomings in the initial model and progressively build more comprehensive and complex models, which encompass all, or many "complaints" about the simplicity of the earlier models. Keep this in mind as we proceed down the equity valuation path.

*For most things are differently valued by those who have them and by those who wish to get them:*

*what belongs to us, and what we give away, seems very precious to us.*

– Aristotle

Nicomachean Ethics, Book IX

(F. H. Peters translation)

## 14.3 Equity Valuation

We look at Equity Valuation from different perspectives.

- Going-Concern value sees the firm from a perpetual operating perspective. It is an ongoing concern that will produce EBIT (or EBITDA) forever.

- Liquidation Value is diametrically opposed to the going concern value. Here we imagine what the firm's assets would fetch were the firm to go out of business and its assets liquidated, i.e., sold off to the highest bidders or available buyers.

- Book Value (BV) represents the accountant's stated net (common) equity divided by the number of shares outstanding (NOSO).

- Relative Value compares a security to another based on some shared or relative metric, such as default risk.

- Market Value (P for Price per Share) is defined as the market value of the firm's equity. Markets can misprice assets on occasion.

- Intrinsic Value (V) represents the "intrinsic" or "internal" worth of the company based on financial analytic formulae and models. You may think of this as what the shares are "really" worth. Such formulae often rely on a discounted cash flow paradigm, e.g., the DDM. The intrinsic value may differ from the market price, thereby offering investors the opportunity to purchase shares cheaply or sell the shares short, if over-valued. Does V = P? If not, what advantage does the investor have?

All the foregoing valuations may differ from one another. In a perfectly efficient market, observed market prices should equal intrinsic values.

## Equity Valuation and the Macroeconomy

*How we value the stock market now and in the*

*future influences major economic and social policy decisions that affect not only investors, but society at large, even the world. If we exaggerate the present and future value of the stock market, then as a society we may invest too much in business start-ups and expansions, and too little in infrastructure, education, and other forms of human capital. If we think the market is worth more than it really is, we may become complacent in funding our pension plans, in maintaining our savings rate, in legislating an improved Social Security system, and in providing other forms of social insurance. We might also lose the opportunity to use our expanding financial technology to devise new solutions to the genuine risks – to our homes, cities, and livelihoods – that we face.*

-Irrational Exuberance by Robert J. Shiller

Preface, p. xii

(Princeton University Press, 2000)

# 14.4 The Dividend Discount Model (DDM): Fixed Dividend or No- Growth Version

We shall assume that the security's *Intrinsic Value* (V) will be equal to the market price (P), i.e., that P = V. Intrinsic value refers to what the security is "truly" worth. To the extent that P ≠ V, one would be presented with the opportunity to earn an extraordinary profit (by either buying cheaply or selling "richly"). Hereforward, we shall use P rather than V, although you may see V used elsewhere.

$$P_0 = \frac{D}{(1+R)^1} + \frac{D}{(1+R)^2} + \frac{D}{(1+R)^3} \ldots + \ \ldots + \ \ldots + \frac{D}{(1+R)^n}$$

A stock is a kind of *perpetuity*; the corporation, as a going concern, is eternal. Dividends will be paid in perpetuity. Unlike bonds, there is no face value to be paid at a specified time in the future. The only cash flows are the dividends.

This formula says that the price of a stock is equal to its aggregated discounted future cash flows, i.e., the dividends discounted to present value. Since, for the moment we assume that dividends are fixed (as in a *preferred* stock), the foregoing, never-ending equation may be simplified algebraically to the following (as was demonstrated for a perpetuity, only the terms have been modified to accommodate stock):

$$P_0 = \frac{D}{R}$$

This simplification works because of the "Law of Limits," discussed earlier in our TVM section. Interestingly, by transposing, the above formula may be re-formulated as:

$$R = \frac{D}{P_0}$$

Note: D ÷P is the *Dividend Yield*!

While corporations pay dividends quarterly, the formula looks at annual dividends. Recall that, in the case of a perpetuity, discounting frequency has no effect on Present Value.

## 14.5 The Dividend Discount Model (DDM): Constant Growth Version

a.k.a. "The Gordon Model"

Of course, not all stocks pay fixed dividends. Let us take a small step forward and assume that the dividend grows – at a constant rate of growth. In this version of the Dividend Discount Model (DDM), the constant dividend growth rate is designated as "G":

$$P_0 = \frac{[D_0(1+G)]}{R-G} = \frac{D_1}{(R-G)}$$

This formula is identical to a constant growth perpetuity. The nomenclature was altered here in order to suit *common* stocks. With some simple algebraic transposition, we can re-formulate the DDM so as to isolate the discount rate to one side:

$$R = \left(\frac{D_1}{P_0}\right) + G$$

Code: $D_0$ = the last reported Annual Dividend (i.e., the sum of the last four, quarterly dividends)

$D_1 = D_0(1 + G)$

G = the imputed growth rate for dividends.

Note, interestingly, that this means that the discount rate is equal to the sum of the dividend yield plus the growth rate in dividends. This formulation presents a percentages view on the model; all terms are percentages.

Note also that when G = 0, we have the "no-growth" version of the DDM. If G > 0, we have the "Constant-Growth" DDM. As discussed in the TVM section, R must exceed G for both practical / financial and mathematical reasons. Remember that when we speak of "growth," we refer to growth in dividends, which, in turn, comes from growth in sales and profits.

Questions:

Given: R = .15  D = $1.50  G = 0

1. What is the price?
2. What happens to P if R goes down?
3. What happens to P if G goes down?

Solutions:

1. P = ($1.50) / .15 = $10
2. Just as in the TVM, as the discount rate decreases, present values (P) increase. So too here!
3. “R – G” increases, and D1 decreases, both of which result in, or cause, a lower price

Note:

This version of the DDM is a.k.a. “The Gordon Model,” named after Myron Gordon. This idea is not without its

controversy or weaknesses, the discussion of which is beyond our current scope.

# 14.6 Dividend Discount Model (DDM) (Problems)

Let us now apply all that we learned regarding calculating Price using the DDM. Solve the following problems. In the course of doing so, explain why Price and Return (at bottom) change from solution to solution.

| | To Solve | Solution |
|---|---|---|
| | Calculate the dollar price ($P_0$) for each problem, given the following: | |
| 1 | $D_0$ = \$1<br>R = .10 | |
| 2 | $D_0$ = \$1<br>R = .05 | |
| 3 | $D_0$ = \$2<br>R = .05 | |
| 4 | $D_0$ = \$1<br>R = .10<br>G = .05 | |
| 5 | $D_0$ = \$1<br>R = .15<br>G = .05 | |
| 6 | $D_0$ = \$1<br>R = .10<br>G = .08 | |
| | Calculate the implicit discount rate (R) for each problem, given the following: | |
| 7 | $P_0$ = \$12<br>$D_0$ = \$1<br>G = .10 | |
| 8 | $P_0$ = \$30<br>$D_0$ = \$1<br>G = .05 | |
| 9 | $P_0$ = \$100<br>$D_0$ = \$2.50<br>G = .10 | |

*Buy when there's blood in the streets.*

– Nathan Mayer Rothschild (1777-1836)

# 14.7 Dividend Discount Model (Solutions)

The following table presents the solutions to the problems on the prior page.

| | | |
|---|---|---|
| 1 | $1 ÷ .10 = | $10 |
| 2 | $1 ÷ .05 = | $20 |
| As R decreases, P increases | | |
| 3 | $2 ÷ .05 = | $40 |
| As D increases, P increases | | |
| 4 | $1 (1.05) ÷ (.10 - .05) = | $21 |
| 5 | $1 (1.05) ÷ (.15 - .05) = | $10.50 |
| As R-G increases, P decreases | | |
| 6 | $1 (1.08) ÷ (.10 - .08) = | $54 |
| As R-G decreases, P increases | | |
| 7 | [($1× 1.1) ÷ $12] + .10 = | 0.1917 |
| 8 | [($1× 1.05) ÷ $30] + .05 = | 0.0850 |
| 9 | [($2.50 × 1.1) ÷ $100] + .10 = | 0.1275 |

- As G increases, R – G decreases, and P increases. G, as the growth rate in dividend, also affects $D_1$ (because $D_1 = D_0 [1 + G]$). As G increases, so too does $D_1$.

- So far, we have assumed that P = V, i.e., Market Price = Intrinsic Value. If however, V > P, we then have an unusual opportunity to achieve an *excess* ("unearned") return; if the opposite pertains, we should sell the stock – if we already own it, or sell it short – if we are aggressive

*Stock prices climb a wall of worry.*

-Anonymous

## 14.8 What About Quarterly Dividends?

No doubt you have noticed that the Dividend Discount Model, as so far presented, assumed that dividends are paid annually. We all know that companies pay dividends in quarterly payments. Does this difference in payment frequency matter? Should we not employ the DDM using quarterly payments? If dividends are paid quarterly, most of the funds will arrive sooner. Is it not a basic principle of the Time Value of Money that if the funds come in sooner the present value is greater, and isn't a stock's price present value? Ought that not affect the way we write the formula?

To adjust for this, we modify the formula and arrive at the following, using the simpler no-growth version:

$$\text{Price} = (D / 4) \div [(1 + R)^{1/4} - 1]$$

To illustrate this, we shall assume that:

D = $1

R = 0.10

When we calculate based on annual dividends, we get:

P + D / R

P = 1 ÷ 0.10 = **$10**

When we calculate based on quarterly dividends, we get:

$P = (D / 4) \div [(1 + R)^{1/4} - 1]$

$P = (1 / 4) \div [(1.10)^{1/4} - 1] =$ **$10.37**

This value is substantially higher!

## On Speech and Deed

**On Accurate Speech**

*Speak clearly, if you speak at all: carve every word before you let it fall.*

-Oliver Wendell Holmes, Sr.

Justice of the Supreme Court

*Speak softly and carry a big stick.*

-Theodore Roosevelt

President of the United States

*… a sage weighs his words carefully…. Every argument and opinion expressed by the Sages is subject to close scrutiny. The same is true for their actions.*

-Rabbi Adin Even-Israel Steinsaltz

Reference Guide to the Talmud, p. 124 (2014)

*The noble man is modest in his speech, but exceeds in his actions.*

-Confucius

*I sez what I means and I means what I sez.*

-Popeye

Famous Sailor And Philosopher

**On Accuracy in Deed**

*Simon Says.*

-Source and date unknown

# 14.9 Components of the Dividend Discount Model

The DDM formula contains several variables whose values must be ascertained in order to solve for Price (P). Here is the formula (again).

$$P = [D_0 (1 + G)] \div (R - G)$$

$$= D_1 \div (R - G)$$

The variables are:

| | |
|---|---|
| P = Market Price | R = The risk-adjusted discount rate |
| $D_0$ = The last recorded dividend | G = The dividend's sustainable growth rate |
| $D_1$ = The next expected dividend | V = Intrinsic value |

Price = Intrinsic Value

We must solve for "P." The market price (P) will equal the security's intrinsic value (V) if the security is efficiently – or correctly – priced in the market. That is what we are trying to uncover with the formula. We will assume here that P = V.

The Last Annual Dividend

$D_0$ is the prior year's dividend, and is thus a known, historical fact. D1 is the next dividend.

Next Year's Dividend

Next year's dividend depends on our expected dividend growth rate, "G."

$$D_1 = D_0 \times (1 + G)$$

Growth Rate in the Dividend

The dividend's growth rate is defined as:

$$G = (D_1 \div D_0) - 1$$

However, we do not know $D_1$, the next year's dividend. Therefore, we need a formula for "G." Here, is the non-intuitive formula for G.

$$G = \text{ROE} \times \text{RR}$$

ROE = Return-on-Equity = (NI ÷ Eq.)

RR = Retention Rate = (NI – D ÷ NI) = (A.R.E. ÷ NI)

A.R.E. = Addition to Retained Earnings = **NI – D**

Therefore:

$$G = (\text{NI} \div \text{Eq.})\ (\text{A.R.E.} \div \text{NI})$$

$$G = (\text{A.R.E.}) \div \text{Eq.}$$

The “G,” which we formulated above represents the growth rate of the firm’s Equity and not of the Dividend! We must demonstrate that the two growth rates are the same. We will examine “G” more closely below and introduce “R.”

# 14.10 A Closer Look at Dividend Growth

In order to fully understand the "g" term in the dividend discount model, a term which is not intuitive, let us have a closer look. (As you go through this example, keep in mind that, as the company retains earnings the balance sheet gets "bigger.") We are given the following information about a company:

**If:**

| | |
|---|---|
| Equity (EQ) | $10,000,000 |
| Net Income (NI) | 1,000,000 |
| Dividends Paid (D) | 100,000 |

**Then:**

| | | | |
|---|---|---|---|
| Return on Equity (ROE) = | NI ÷ EQ = | 1,000 ÷ 10,000 = | .10 |
| Calculated Payout Ratio (PR) = | D ÷ NI = | 100 ÷ 1,000 = | .10 |
| Calculated Retention Rate (RR) = | (NI – D) ÷ NI = | (1,000 – 100) ÷ 1,000 = | .90 |
| G = | (ROE) × (RR) | (.10) × (.90) = | **.09** |

Given this information, G = (ROE) × (RR) = (.10) × (.90) = **.09**. By definition, G, the growth rate in the dividend, <u>must</u> <u>also</u> be defined, far more simply, as: G = [($D_1$) ÷ ($D_0$)] – 1. This simple formula is readily understood. Remember that G = ARE / NI,

or the rate at which the Equity grows. Take note that, here, we have assumed that G, ROE, and RR are constants.

Let's see if the two formulae work out to be the same; if so, we will therefore also know that the "G = (ROE) (RR)" formula makes sense, given the same data. Let's see first how the simple formula, $G = [(D_1) \div (D_0)] - 1$, works out using an *accounting–type* (i.e., chronological) approach.

| | | | |
|---|---|---|---|
| $D_0 =$ | $(EQ_0)$ (ROE) (PR) = | ($10,000) (.10) (.10) = | $100 |
| $EQ_1 =$ | $(EQ_0)$ + [$(EQ_0)$ (ROE) (RR)] = | ($10,000) + [($10,000) (.10) (.90) = | $10,900 |
| $D_1 =$ | $(EQ_1)$ (ROE) (PR) = | ($10,900) (.10) (.10) = | $109 |
| G = | $[(D_1) \div (D_0)] - 1 =$ | [(109) ÷ (100)] – 1 = | **.09** |
| | $(NI_0 - D_0) / Eq._0 =$<br>$(NI_1 - D_1) / Eq._1 =$ | [(1,000 – 100] ÷ 10,000 =<br>[(1,090 – 109] ÷ 10,900 = | |

This gets us the same result – as it should! (In fact, we could not have known the value of $D_1$ had we not had the first formulation!) This notion assumes that the retention rate (and payout ratio), dividend growth rate, and ROE are constants. You'll note that the equity is growing at the same rate of 9%. This latter table says that if we earn and retain earnings, we will be able to pay out more money in dollars as dividends later.

To assist you with the above formulae:

(EQ) (ROE) = (EQ) (NI / EQ) = NI and

(NI) (PR) = (NI) (D/NI) = D

# 14.11 Summary of DDM Variables' Sources

| Datum | How do we know its value? |
|---|---|
| $D_0$ | History; Fact |
| G | $G = ROE \times RR = ARE / Eq.$ |
| $D_1$ | $D_1 = D_0 (1 + G)$ |
| R | $R = R_F + MRP$ |
| P | $V = P_0 = D_1 / (R - G)$ |
| Market Risk Premium (MRP) | $R_M - R_F$ |
| Portfolio Risk Premium (PRP) | $R_P - R_F$ |

*Buy low, sell high.*

-Bernard Baruch (1870 – 1965)

*To buy when others are despondently selling and to sell when others are greedily buying requires the greatest fortitude and pays the greatest reward*

-Sir John Templeton

## 14.12 Value Prediction Problem

You are given the following information:

- Next year's projected EPS for the S&P 500 = $132 per share
- The dividend growth rate has been = 0.04
- The risk-free rate = 0.025 (either the T-Bill or the 10-Year Note, your choice)
- The MRP (Market Risk Premium) = 0.06
- The Return-on-Equity has been = 0.11.

Using the data given, what is the value for the S&P (per share) for next year?

Solution Plan:

Let's use the DDM!

First, write out the DDM formula first to see what variables you already have and which are missing.

$$P_0 = [D_1 \div (R - G)]$$

Then, in no necessary order:

1.G = (ROE) (RR)

2. 0.04 = (0.11) (RR)

3. RR = 0.36

4. PR = 1 – RR

5. PR = 1 – 0.36 = 0.64

6. D = (EPS) (PR)

7. $D_1$ = [($132) (0.64] = $84.48

8. R = $R_F$+ MRP = 0.025 + 0.06 = 0.085

9. $P_0$= $D_1$÷ (R – G)

10. $P_0$ = $84.48 ÷ (0.085 – 0.04) = **$1,877**

## 14.13 A Qualitative Look at The Discount Rate

The discount rate, or "R," for the market ($R_M$) or for a company's stock ($R_S$) is a variable, which itself is determined by general market levels of interest rates, the default risk of the company's bonds, and credit spreads.

R = *f* (general levels of interest rates, default risk, credit spreads)

General levels of interest rates: All interest rates are interrelated. In general, if rates go up for a base rate, such as Treasuries, other rates will follow.

Default risk: If a company's bond default risk increases, the dividend on preferred stock will also be less secure. Discount rates on "Preferreds" will go up and prices will go down. So too will the discount rate and price of common equity follow. The company's ability to pay common stock dividends and to retain additional earnings will be reduced. Remember: Default risk, essentially, has to do with the chance that a corporate borrower, or issuer of bonds, will not pay the interest on its borrowings in full and on time.

Credit Spreads: We know that credit spreads reflect rate differentials between, typically, 10-year Treasury Notes

and 10-Year B-rated Corporates. However, we can create spreads between Notes and anything, such as discount rates on "Preferreds." Thus, spreads may be observed between T-Notes, Preferred-, and Common-Stock Returns. And we already know that spreads narrow or widen according to economic circumstances and outlooks. If spreads widen, required stock returns will go up and prices will go down.

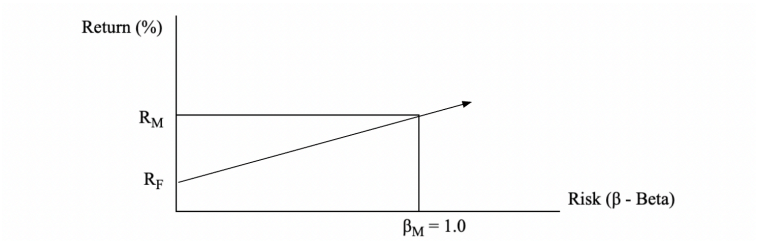
These three variables, when aggregated, will constitute the stock's risk (β). "R" can be stated (again) formulaically:

$$\mathbf{R_M = R_F + MRP}$$

$$\mathbf{MRP = Market\ Risk\ Premium = R_M - R_F}$$

$$\mathbf{R_M = R_F + (R_M - R_F)\ \beta_M}$$

The Market Return (RM) equals the Risk-free Rate (RF) plus a Market Risk Premium.



A specific portfolio's or security's return ($R_P$ or $R_S$) will equal the Risk-free Rate ($R_F$) *plus* a Market Risk Premium ($R_M$ – $R_F$), adjusted for the relative risk ($\beta_S$) of the portfolio. $\beta_P$ (portfolio risk) or $\beta_S$ (individual security risk) can be equal to, greater than, or less than the Market's Risk level ($\beta_M$). Thus, we can also speak of a "Portfolio Risk Premium" ($R_P$ – $R_F$). We can substitute "$R_S$" for "$R_P$" at the individual stock level and use "$R_S$" as the discount rate in the DDM.

## 14.14 Business Ethics: The Small Investor's Experience of Insider Trading

We have just completed a discussion of the elements in securities' valuation. Some individuals try to get an upper hand in their stock selections by engaging in illegal activities. The following is a salient example.

"Insider Trading" has to do with the personal use of material nonpublic information about the future prospects of a company – whether good or bad, in order to garner an unearned profit by trading its securities ahead of any public announcements regarding the pertinent information. An "insider" is an employee, a relative, or friend of an employee. Insider trading, in the United States, is illegal. Don't even think of it.

Is such trading, from a moral point of view, fair or not? Is it fair that an insider may have an exploitable advantage to enrich himself/herself? Is the law "correct" in making it illegal? Imagine the scenario below and respond to the questions that follow:

Suppose we focus on three parties to a series of transactions on a security at a point in time. Mr. X is interested in selling 1,000 shares of ABC corporation; Mr. Y is interested in purchasing 1,000 shares; Mr. Z, an informed corporate insider, is interested in purchasing a large block of the stock. Messrs. X and Y are small investors each of whom has personal reasons for engaging in the transaction.

In a normal market, X would sell to Y at the current price. Both parties would be satisfied at having engaged in the transaction. However, Mr. Z holds some critical information concerning the very favorable portents of the shares and wants to buy before the word gets out. His bid causes the share price to rise and thus Mr. X gets a higher price than he may have otherwise received, all else equal. No harm done.

Due to the rise in share price, Mr. Y pays more than he may have otherwise. Such possible harm may be overcome by the pending rise of the shares once the information is announced. The slight short-term harm is overcome by the longer-term profits Mr. Y shall earn. Mr. Y may nonetheless feel harmed.

However, had Mr. X known about the pending release of favorable information, he may have either waited before selling or changed his mind completely about his decision to sell. Mr. X may feel harmed.

Questions:

1. Justify Mr. Z's insider trading on a moral basis.
2. Denounce Mr. Z's trading – morally speaking.
3. What is your own view of Mr. Z's trading? Find a moral justification for your position – apart from what you stated already.

Bottom Line: It is illegal. Don’t even think about it!

## 14.15 Capital Gains

It is interesting to note that, if G > 0, the model will automatically generate capital gains. Here again is our formula. Below is a problem whose resolution illustrates the model's automatic generation of capital gains.

Question:

Formula:

$$P_0 = [D_0 (1 + G)] / (R – G)$$

$$P_0 = D_1 / (R – G)$$

Given:

| | |
|---|---|
| $D_0$ = \$1 | The Last Dividend |
| R = 10% | The Discount Rate |
| G = 5% | The Dividend's Constant Growth Rate |

What is the price today?

What would the price be in one year?

Solution:

Price Today:

$P_0$= \$1 (1 + .05) / (.10 – .05)

= 1.05 / .05

= \$21

Price in One-Year:

$P_1$ = $D_2$ / (R – G)

$P_1$ = \$1.05 (1 + .05) / (.10 – .05)

= 1.1025 / .05

= \$22.05

We observe that \$22.05 / \$21 = 1.05. That is to say that next year's price will be greater than last year's by 5%, or

the same as the stock's growth rate (again, assuming a constant pay-out ratio).

We often say that a stock is "ahead of itself," if the rate of growth in price exceeds the dividend – or earnings – growth rate (assuming a constant pay-out ratio).

## Capital Gains, Dividend Growth: Some Practice Problems

The following should help summarize some relevant concepts.

1. Complete the empty cells, given the data noted below for a stock. The basic formula for the *Dividend Discount Model* is:

$$P_0 = [(D_0)(1 + G)] \div [R - G]$$

| | |
|---|---|
| $D_0$ = | \$2.50 |
| R = | .17 |
| G = | .10 |

| | |
|---|---|
| $P_0$ = | \$ |
| $P_1$ = | \$ |
| Capital Gain = | \$ |
| Capital Gain = | % |

2. Once again, complete the spreadsheet, given the data noted for a particular stock.

Given:

| | |
|---|---|
| ROE = | .17 |
| Payout Ratio = | .20 |

Solve:

| | | | |
|---|---|---|---|
| Equity $_0$ = | $100,000 | Equity $_1$ = | |
| Net Income $_0$ = | | Net Income $_1$ = | |
| Dividends $_0$ = | | Dividends $_1$ = | |

| | | |
|---|---|---|
| Dividend Growth Rate = | $D_1 \div D_0 - 1$= | |
| G = | (ROE) (RR) = | |

- Explain, in words, what is meant by the term, "G," in question #2.

- Assuming G is a constant (question #1), $P_0$ (1 + G) = $P_1$.

## Capital Gains, Dividend Growth: Some Practice Problems (Solutions)

Problem 1:

| | |
|---|---|
| $D_0$ = | $2.50 |
| R = | .17 |
| G = | .10 |

| | |
|---|---|
| $P_0$ = | $39.29 |
| $P_1$ = | $43.21 |
| Capital Gain = | $3.929 |
| Capital Gain = | 10% |

Problem 2:

| | |
|---|---|
| Equity $_0$ = | $100,000 |
| Net Income $_0$ = | $17,000 |
| Dividends $_0$ = | $3,400 |

| | |
|---|---|
| Equity $_1$ = | $113,600 |
| Net Income $_1$ = | $19,312 |
| Dividends $_1$ = | $3,862 |

| | | |
|---|---|---|
| Dividend Growth Rate = | $D_1 \div D_0 - 1$= | (3,862/3,400) – 1 = .136 |
| G = | (ROE) (RR) = | (.17) (.80) = .136 |

## 14.16 Portfolio Return (Weighted Averages)

Now that we know all about individual bond and stock returns, what about portfolios of securities? A portfolio is a collection of individual securities.

If the market values of the portfolio's constituent securities were the same, the portfolio weights would be equal, and the simple arithmetic- and weighted-averages would be the same. In calculating the simple arithmetic return, one would take the observed returns and divide by the number of observations. Here, the weights are unequal, so we must employ a weighted average calculation as below.

$$R_P = \sum_{i=1}^{n} (w_i) \times (R_i)$$

<u>Example</u>: Calculate the (historical or expected) *weighted average return* for the following portfolio, which consists of three securities (A, B, and C) and has the market values and individual returns as noted.

| Security | Market Values | Weights | Historical/Expected Returns | Sum |
|---|---|---|---|---|
| A | $100,000 | | .10 | |
| B | $200,000 | | .12 | |
| C | $300,000 | | .16 | |
| Portfolio ABC | | 1.00 | | |

Note: Portfolio risk is not simply a weighted-average of the portfolio constituents' respective risks – due to "covariance" among securities.

## Portfolio Return (Solution to Problem)

Solution (table form):

| Security | Market Values | Weights | Expected Returns | Sum |
|---|---|---|---|---|
| A | $100,000 | 100/600 = 1/6 | .10 | .0167 |
| B | $200,000 | 200/600 = 1/3 | .12 | .0400 |
| C | $300,000 | 300/600 = 1/2 | .16 | .0800 |
| Portfolio ABC | $600,000 | 1.00 | | .1367 |

Solution (by formula):

$$R_P = 1/6\ (.10) + 1/3\ (.12) + 1/2\ (.16) = 0.1367$$

Short-cut solution:

$$R_P = [(100)\ (.10) + (200)\ (.12) + (300)\ (.16)] \div 600 = 0.1367$$

In the short-cut solution, we do not take the weights first. Instead, we multiply the dollar values by each expected return and, at the end, divide by the entire portfolio value.

The 0.1367 return means that the investment would increase by 13.67% *in one year*. If you had invested $1,000, after one year's time, you would have one thousand dollars plus $136.70 for a total of $1,136.7. Some of the return, presumably, would come from income (rent, interest, or dividends) and some from capital growth (price appreciation). The constituent security returns, as given here, were themselves represented without any further detail as to the breakdown of income and growth portions.

If the weights had all been equal (i.e., .3333, in this case), we could have calculated a simple average by adding the sum of the returns and dividing by ($n$ =) 3. In other words, a simple average implies equal weights.

If one of the constituent security's returns were negative, we would of course get a different result. Let us say that the return on Security "A" were -.10, rather than positive. In this case, the portfolio return would be:

$$R_P = 1/6\ (-.10) + 1/3\ (.12) + 1/2\ (.16) = .1033$$

Another way in which we could have calculated this would have been as follows:

$$0.1367 - (2)\ (.0167) = .1033$$

## 14.17 The Geometric Average Return: Multi-year Returns

Generally, we quote *return* in annual terms. In order to calculate the return for multiple years, we must arrive at a reasonable, average annual return figure.

Suppose you observe the following three (historical or projected) *annual* returns:

.10 .25 .35

The simple, arithmetic average return would be:

$$[.10 + .25 + .35] \div 3 = .2333$$

For the average to be valid in terms of the time value of money, its future value should equal the product of the three observations. However,

$$(1.2333)^3 \neq (1.10)(1.25)(1.35)$$

Again, the conceptually correct average must be consistent with the time value of money and its "$(1 + R)^n$" format. By modifying the line above and asking what the correct average rate, "R," *should* be, we arrive at:

$$(1 + R)^3 = (1.10)(1.25)(1.35)$$

$$(1 + R) = [(1.10)(1.25)(1.35)]^{1/3}$$

$$R = [(1.10)(1.25)(1.35)]^{1/3} - 1$$

$$R = .228981$$

Thus, the average multi-period return is 0.2290. This calculation is referred to as the "geometric average" and is consistent with the manner in which we do the time value of money. The general notation for this formula requires the use of *product summation*notation – "Π" (as opposed to using the usual sigma summation notation, Σ). The notation reads as follows:

$$\text{Geometric Average} = [\Pi\ (1 + R_i)^{1/n}] - 1$$

<u>Question</u>: What would this average be if the 10% observation were negative?

<u>Answer</u>: 0.1495. How did you get this?

<u>Note</u>: Should there be a negative return in the mix as above, the same method should be used as always. The following should make common sense. For example, should one experience a 50% loss and a 100% gain in consecutive years, the geometric average return would be: $[(1 + \{-0.50\}) \times (1 + 1)]^{½} - 1 = 0.0$.

Had you instead calculated the *simple* average, you would have gotten: $[(-0.50) + (1.0)] \div 2 = 25\%$. That cannot be correct!

# 14.18 Chapters 12-14: Review Questions

**Chapters 12 – 14: Review Questions**

1. How are the Return to the Investor and Cost to the Issuer related?

2. What is the cost of debt? Are taxes included? What is the relevant formula?

3. What is meant by "Technical Default"?

4. What are the two elements in the Cost of (Common) Equity Capital?

5. The investors' "Required Return on Equity" ($R_M$) consists of what elements?

6. What is meant by "Market Risk Premium"?

7. What is meant by "Portfolio Risk Premium"?

8. What is meant by "Rational Expectations"?

9. Define each of the following *Risks*:

    - Liquidity

- Credit
- Inflation
- Sovereign / Country / Political
- Foreign Currency

With respect to Inflation Risk, utilize the phrases "Nominal" and Real" properly.

10. How are Price and Reinvestment Risks related? Explain.

11. Who are the two major credit rating agencies?

12. Do the agencies agree with one another on all ratings? Explain.

13. What is the difference between "Investment Grade" and "High-yield"?

14. How do credit ratings relate to default risk?

15. To what do "Indenture" and "Covenants" refer?

16. What is a "lien"? What relevance does it have to bonds?

17. What two variables are depicted on the "Yield Curve," and on what axes are they depicted?

18. If the bond category is not stated on the Yield Curve, what is the default option?

19. Define "Liquidity Preference." What impact does this concept have on the "normal" slope of the Yield Curve?

20. What is meant by an “Inverted” Yield Curve? Why does it come about? How long does it usually last?

21. Discuss each of the four Yield Curve theories.

22. Observed rates are stated below. Fill in the Spot Rates. Graph it.

| Maturity | YTM | Spot Rate |
|---|---|---|
| 1 | 2.00% | |
| 2 | 2.50% | |
| 3 | 3.25% | |
| 4 | 4.00% | |
| 5 | 3.75% | |

23. Draw in a fictitious Yield Curve for Treasuries of your liking. Then, draw in a B-rated Corporate Bond Yield Curve. Posit a Credit Spread for the Ten-year bonds.

24. Using the graph you drew in the prior question, show how Credit Spreads may expand or contract.

25. Why do Credit Spreads expand and contract?

26. What is meant by a “Flight to Quality”?

27. Define the three forms of income. To which financial instruments does each refer?

28. Explain the “Valuation Premise.”

29. Calculate the “Holding Period Return” (HPR), given the following:

- Initial cost: $12 million
- Income over holding period: $1.3 million

- Sale Price: $11 million

30. What is the key deficiency of the HPR?

31. Define each of the following. Can you find another name for each?

- Face Value
- Coupon
- Market Yield

32. What is meant by “Par, Discount. and Premium”? Why are bonds priced one or the other of these ways?

33. Given the following, calculate the price of this bond? Your answer should be stated in percentage terms.

- Coupon: 4%
- Term-to-maturity: 10 Years
- Discounting Frequency: Semi-annual
- Yield-to-maturity: 8%

34. What would be the price for the bond in the prior question if its coupon were 0%? Explain in words why the price is higher or lower.

35. Why does compounding frequency matter in pricing bonds?

36. What two variables determine a bond’s market yield? Explain your terms.

37. Why may it be said that the “true” price of a bond is its Market Yield and not its dollar price?

38. Is it reasonable to ascribe the Valuation Premise to equities, or just to bonds? Why?

39. Define each of the following terms relative to equity valuation:

- Going-concern Value
- Liquidation Value
- Book Value
- Market Value
- Intrinsic Value
- Relative Value

40. Why are equities so important to our Macroeconomy?

41. Given the following, calculate the stock's Intrinsic Value, and its current and prospective Dividend Yield.

- Last Dividend = $3.20
- Discount Rate = 8%
- Growth Rate = 3.5%

42. In addition to a mathematical explanation, why must a stock's discount rate exceed its dividend growth rate?

43. What categories of equity are best suited to the no-growth and constant growth Dividend Discount Models (DDM)?

44. Does it matter that the DDM is based on yearly dividends when stocks pay dividends quarterly? Explain.

45. You are given the following. Calculate the stock's dividend growth rate.

- Common Equity = $100 million
- Return on Equity = 10%
- Number of Common Shares Outstanding = 5 million
- Payout Ratio = 20%

46. If the Equity Discount Rate in the prior question is 10%, what is the Intrinsic Value of the stock?

47. What will be next year's price for this stock?

48. Explain each of the three variables that determine the Equity Discount Rate.

49. What is the relationship between the growth rates of the dividends and the price of the stock? Explain.

50. Given the following data, what is the expected portfolio return? (Note the signs.)

| Security | Weights | Expected Return |
|---|---|---|
| 1 | 5% | (5%) |
| 2 | 10% | 8% |
| 3 | 20% | 9% |
| 4 | 30% | 10% |
| 5 | 35% | 12% |

51. Following are the annual returns for Joseph's portfolio for the last five years.

a. What are his arithmetic and annualized geometric returns?

b. Does it matter that the data are not ordered chronologically?

c. Why are the two measures related as such, i.e., where one is higher than the other? Which calculation is more conceptually accurate?

| | | | | |
|---|---|---|---|---|
| (5%) | 8% | 9% | 10% | 12% |

Solution to Question #22:

| Maturity | YTM | Spot Rate |
|---|---|---|
| 1 | 2.00% | A = 2.0% |
| 2 | 2.50% | B = 3.0024% |
| 3 | 3.25% | C = 4.7687% |
| 4 | 4.00% | D = 6.2827% |
| 5 | 3.75% | E = 2.7559% |

$A = {}_1r_1 = 2.0\%$

$B = {}_2r_1 = (1.02)\ (1 + B) = 1.025^2$

$C = {}_3r_1 = (1.02)\ (1.030024)\ (1 + C) = 1.0325^3$

$D = {}_4r_1 = (1.02)\ (1.030024)\ (1.047687)\ (1 + D) = 1.04^4$

$E = {}_5r_1 = (1.02)\ (1.030024)\ (1.047687)\ (1.062827)\ (1 + E) = 1.0375^5$